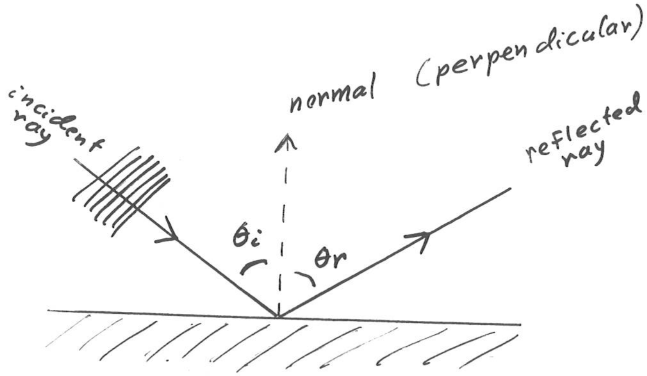
Specular Reflection



$$\theta_i = \theta_r$$

angle of incidence = angle of reflection

The incident and reflected rays are in the same plane as the normal vector.

Refraction

- The speed of light in a medium is less than the speed of light in vacuum.

$$v \le c$$

- In fact, $v = \frac{c}{n}$ where $n \ge 1$. n is called the index of refraction.

$n_{air} \approx 1$
$n_{vac} = 1$
$n_{H_2O} = 1.33$
$n_{Glass} = 1.33$

- The frequency of light does not change in a medium.

- Because $\lambda f = v$, the wavelength changes. λ is shorter in a medium than in vacuum.

big density $\Rightarrow$ n large $\Rightarrow$ v small

Ex What is the speed of light in water?

$c = 3 \times 10^8$ m/s

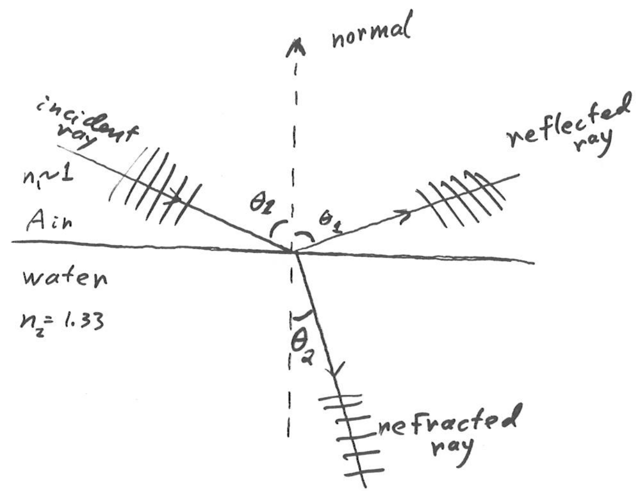
$n_{H_2O} = \frac{4}{3}$ no units!

$$v_{H_2O} = \frac{c}{n_{H_2O}} = \frac{c}{4/3} = \frac{3}{4}c$$

$$= \boxed{2.25 \times 10^8 \text{ m/s}}$$

$$v_{Air} = \frac{c}{n_{Air}} \approx \frac{c}{1}$$

All of this has the effect of bending (refracting) a light ray.

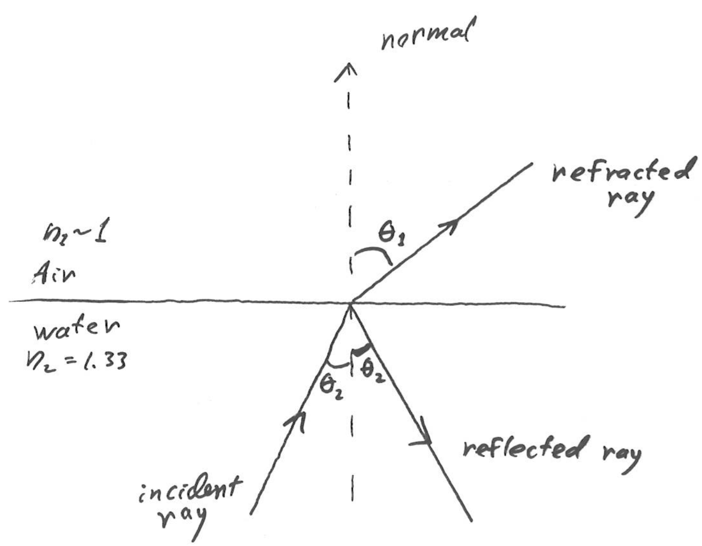


Snell's Law

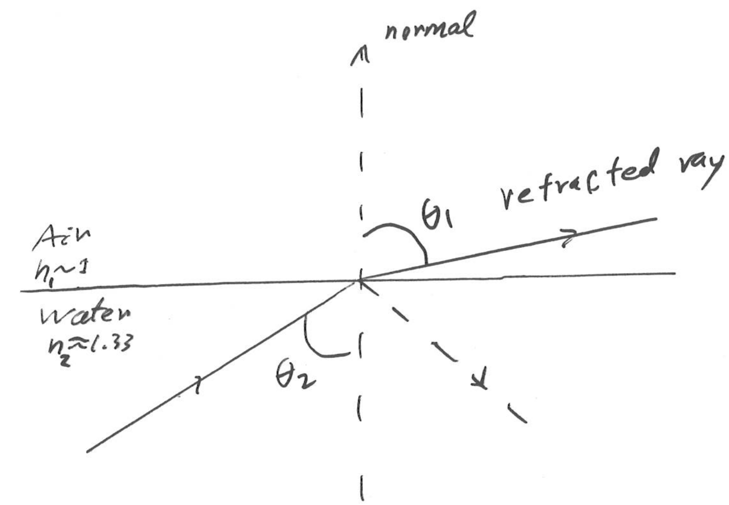
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$n_1 < n_2$
$\sin\theta_2 < \sin\theta_1$
$\theta_2 < \theta_1$

The incident ray can also originate in the denser medium (the one with larger n).

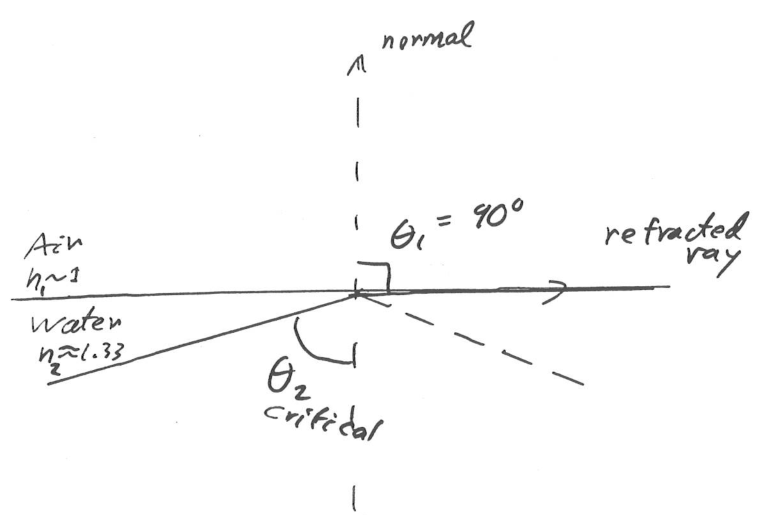


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

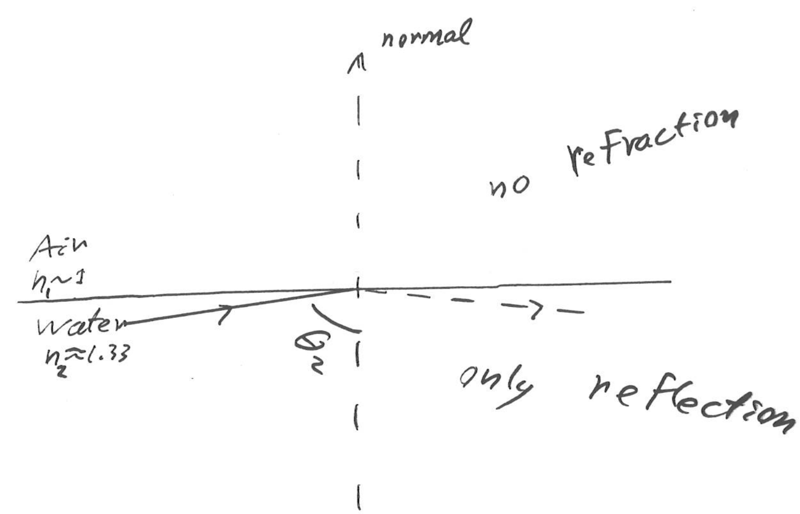


Total internal reflection

— no refracted ray

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



... And only when the incident ray is in the denser medium

$n_2 > n_1$

Ex: What is the critical angle for total internal reflection at a water → air interface?

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$n_1 \sim 1$ (Air) $\quad n_2 \sim 1.33$ (water)

when $\theta_2 = \theta_{2\,critical}$ then $\theta_1 = 90^\circ$

$$\sin\theta_1 = \sin 90^\circ = 1$$

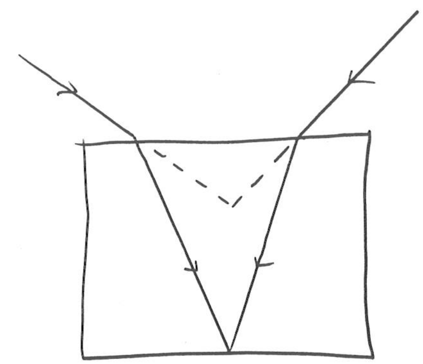
$$n_1 (1) = n_2 \sin(\theta_{2\,critical})$$

$$\frac{n_1}{n_2} = \sin(\theta_{2\,crit.})$$

$$\sin^{-1}\left(\frac{n_1}{n_2}\right) = \theta_{2\,critical}$$

$$\sin^{-1}\left(\frac{1}{1.33}\right) \cong 49^\circ$$

The Glass Block



Objects appear <u>higher</u> than the bottom of the block.

Spherical Mirror Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$f > 0$ concave mirror

$f < 0$ convex mirror

d_o always positive

$d_i > 0$ on object side (real side)

$d_i < 0$ on other side (virtual side)

Magnification

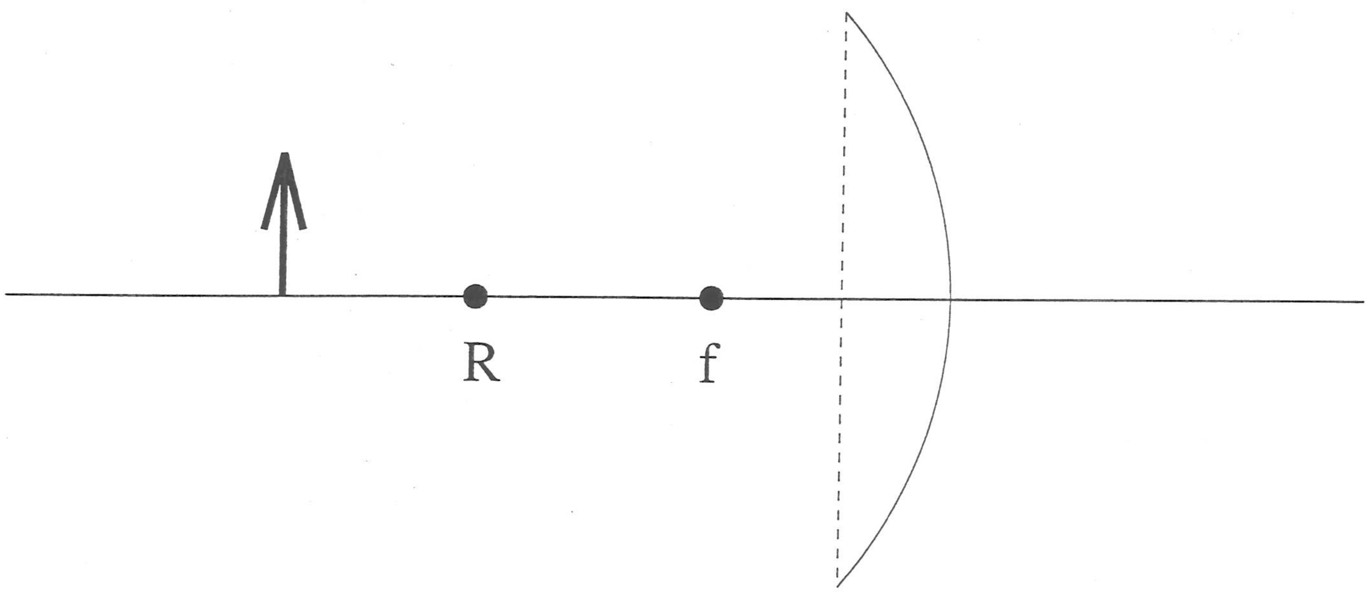
$$M = -\frac{d_i}{d_o}$$

$M < 0$ image is inverted

$M > 0$ image is upright

$|M| > 1$ enlarged $|M| < 1$ reduced

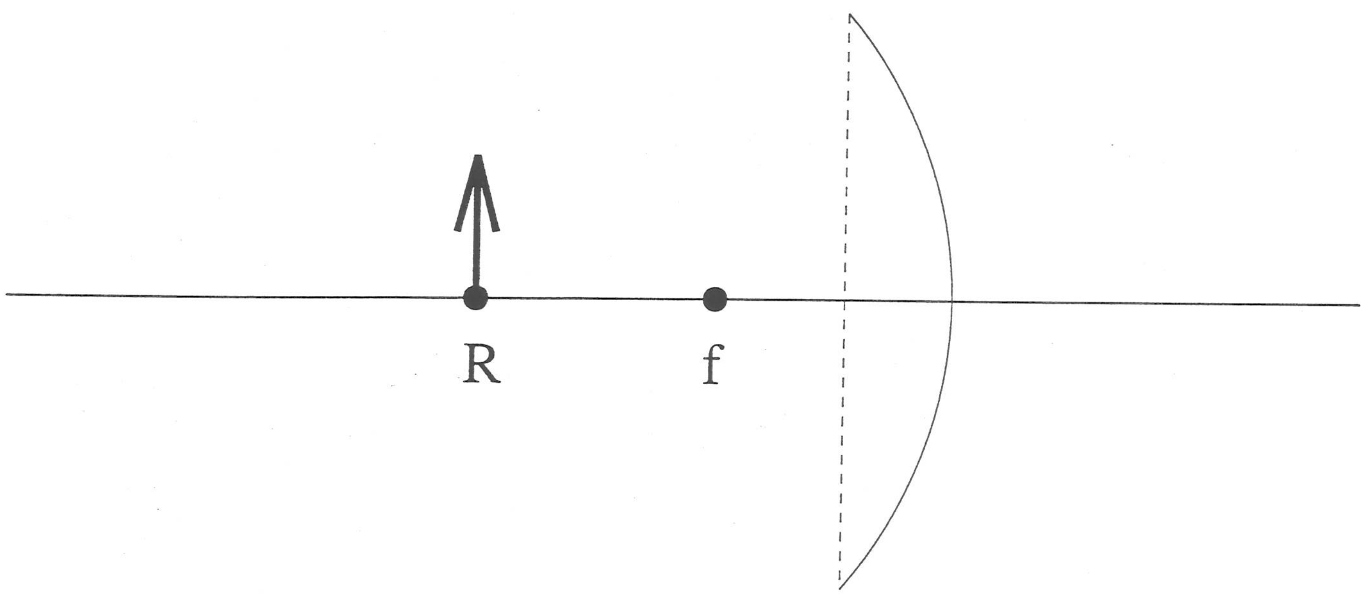
Concave Mirror



Object ______

Image ______

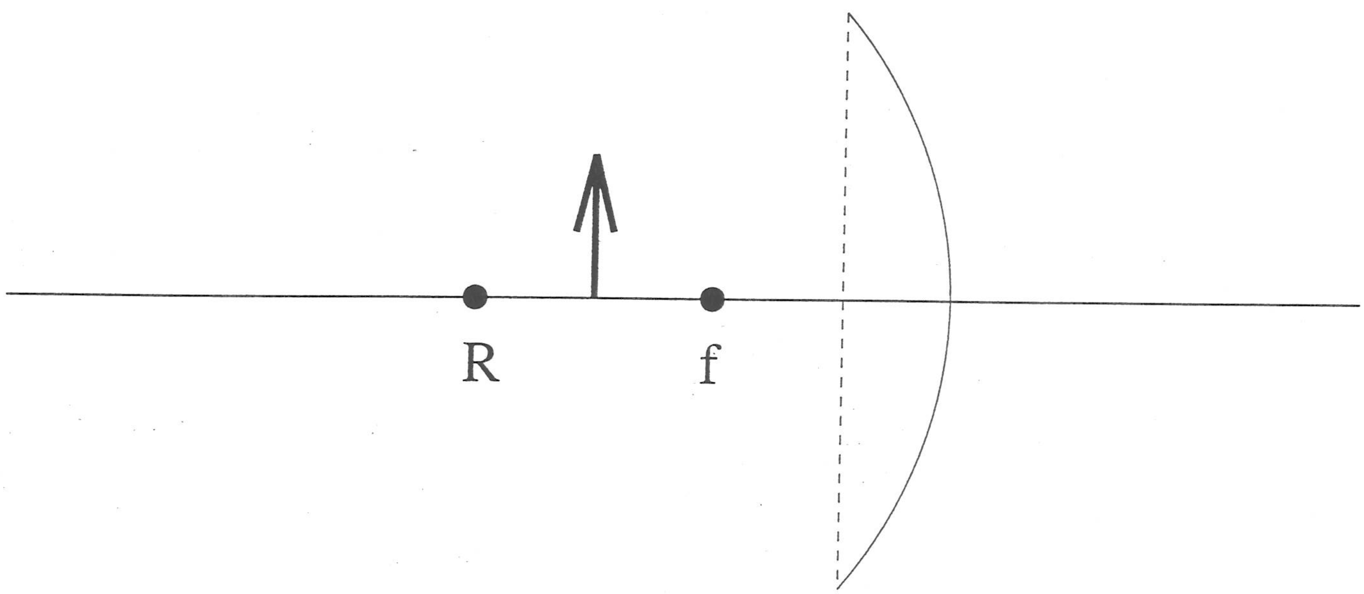
Concave Mirror



Object

Image

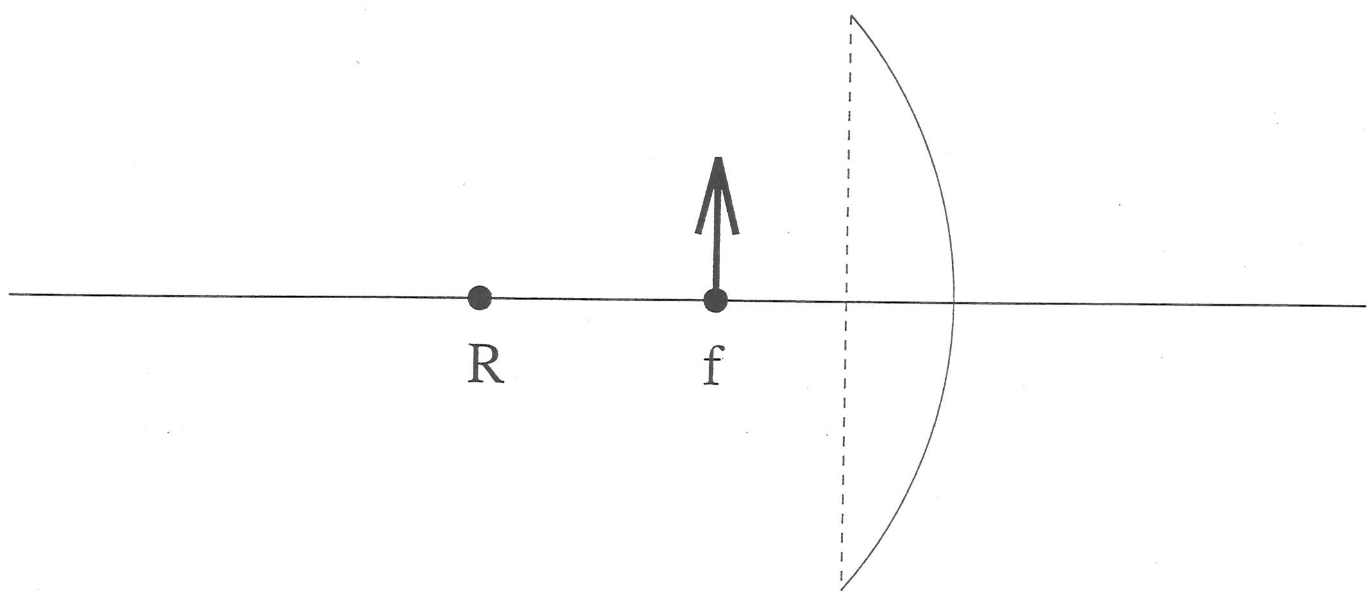
Concave Mirror



Object ______

Image ______

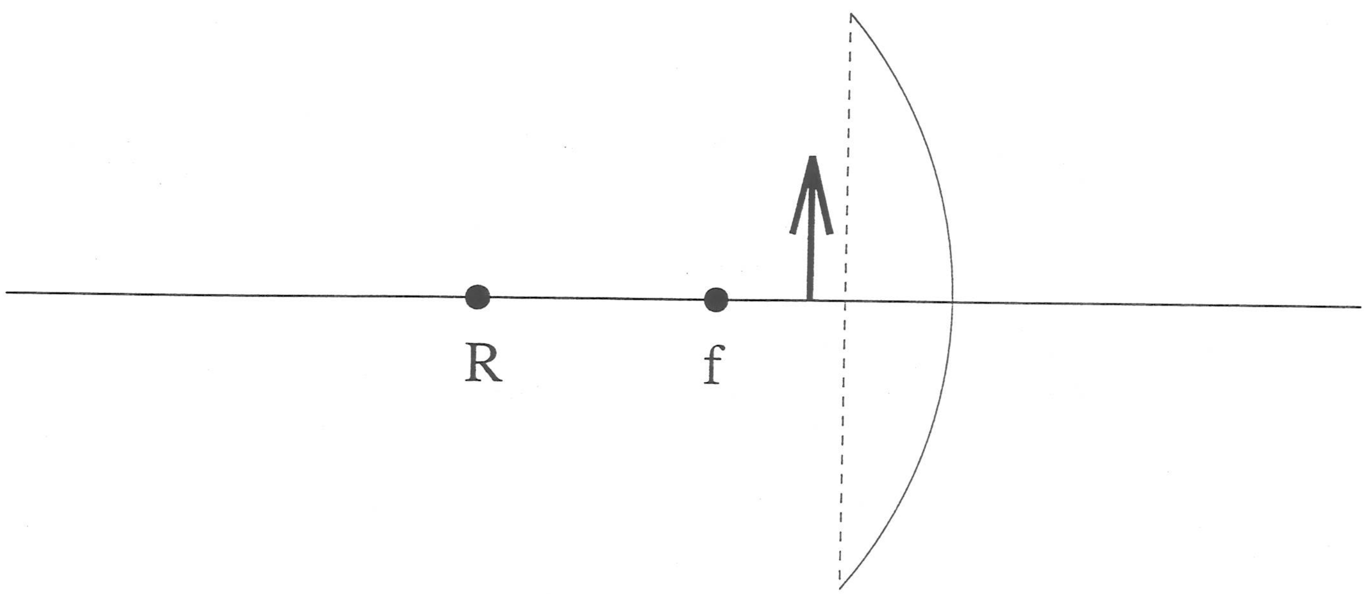
Concave Mirror



Object

Image

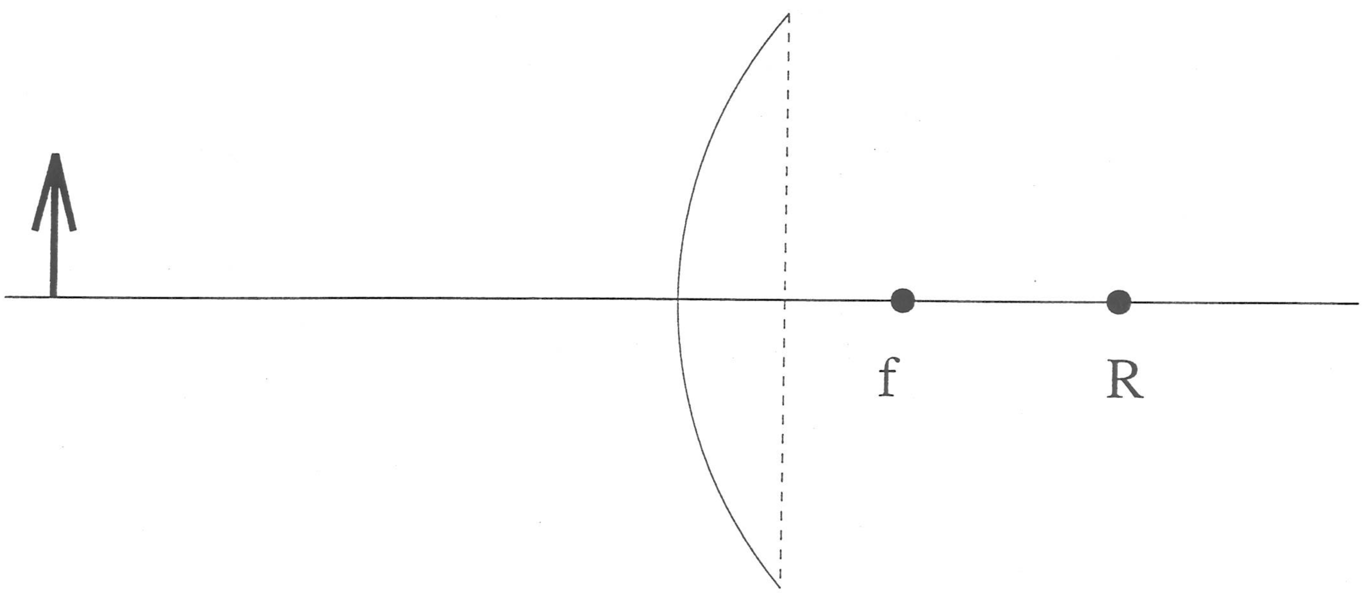
Concave Mirror



Object ______

Image ______

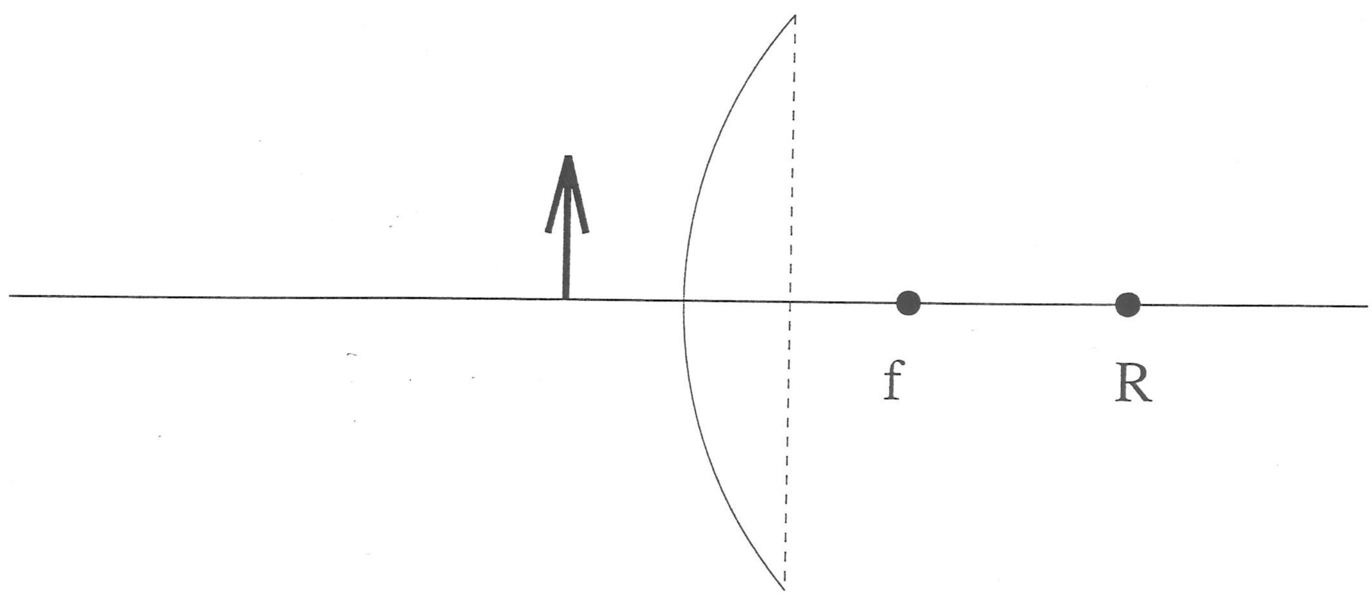
Convex Mirror



Object

Image

Convex Mirror



Object ________

Image ________

Spherical Mirror and Lens Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$f > 0$ concave mirror

$f < 0$ convex mirror

d_o always positive

$d_i > 0$ on object side (real side)

$d_i < 0$ on other side (virtual side)

Magnification

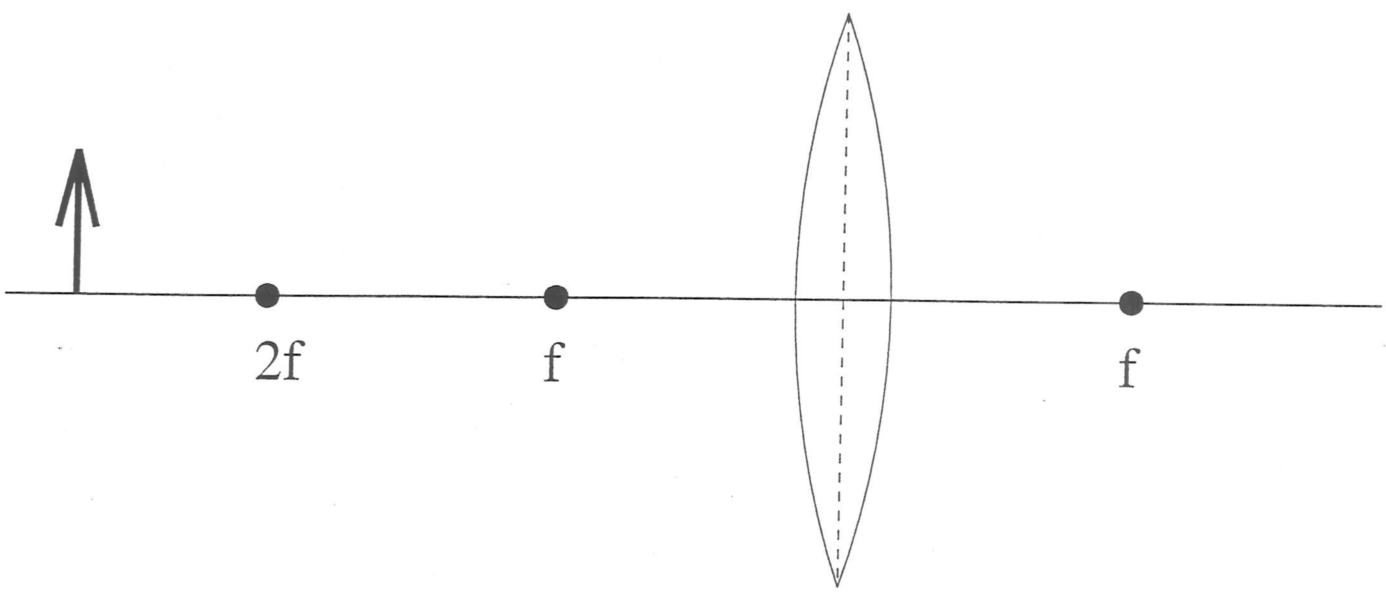
$$M = -\frac{d_i}{d_o}$$

$M < 0$ image is inverted

$M > 0$ image is upright

$|M| > 1$ enlarged $|M| < 1$ reduced

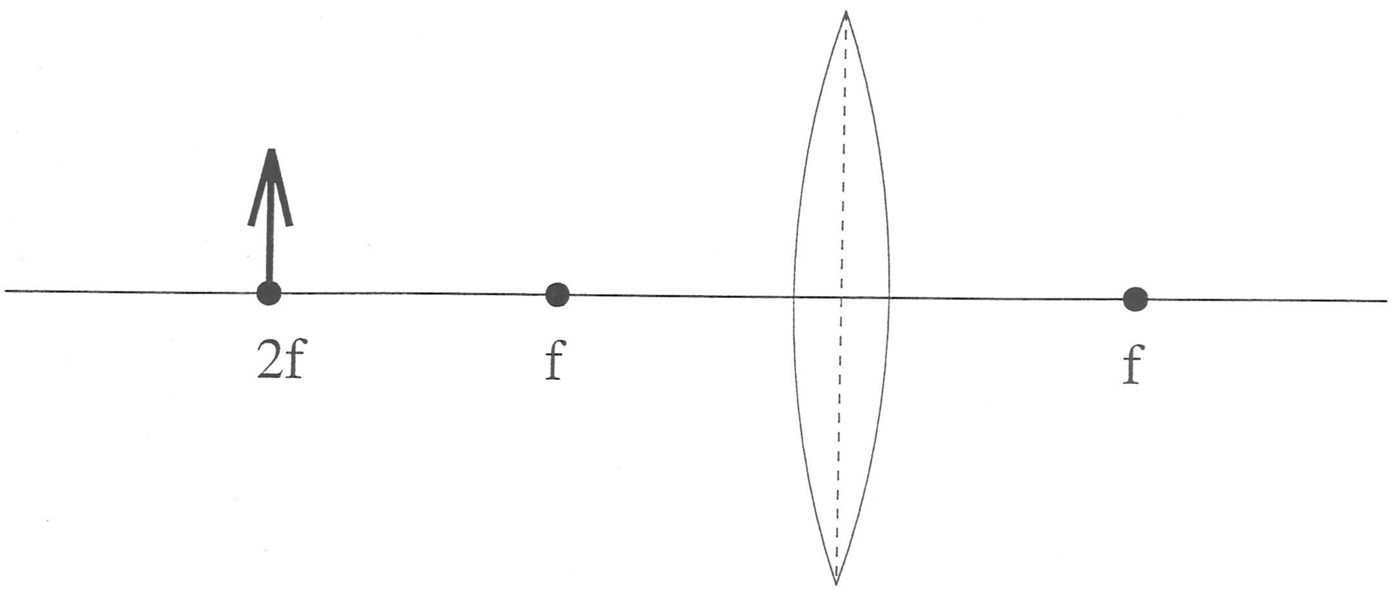
Converging Lens



Object ________

Image ________

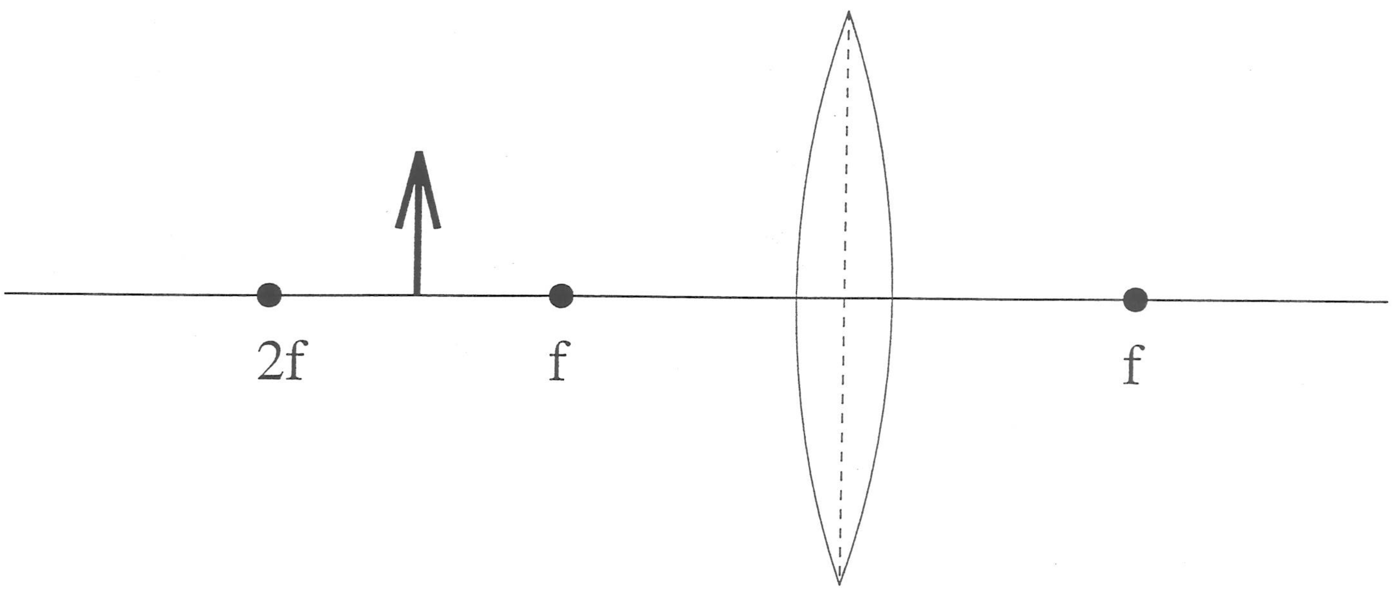
Converging Lens



Object ______

Image ______

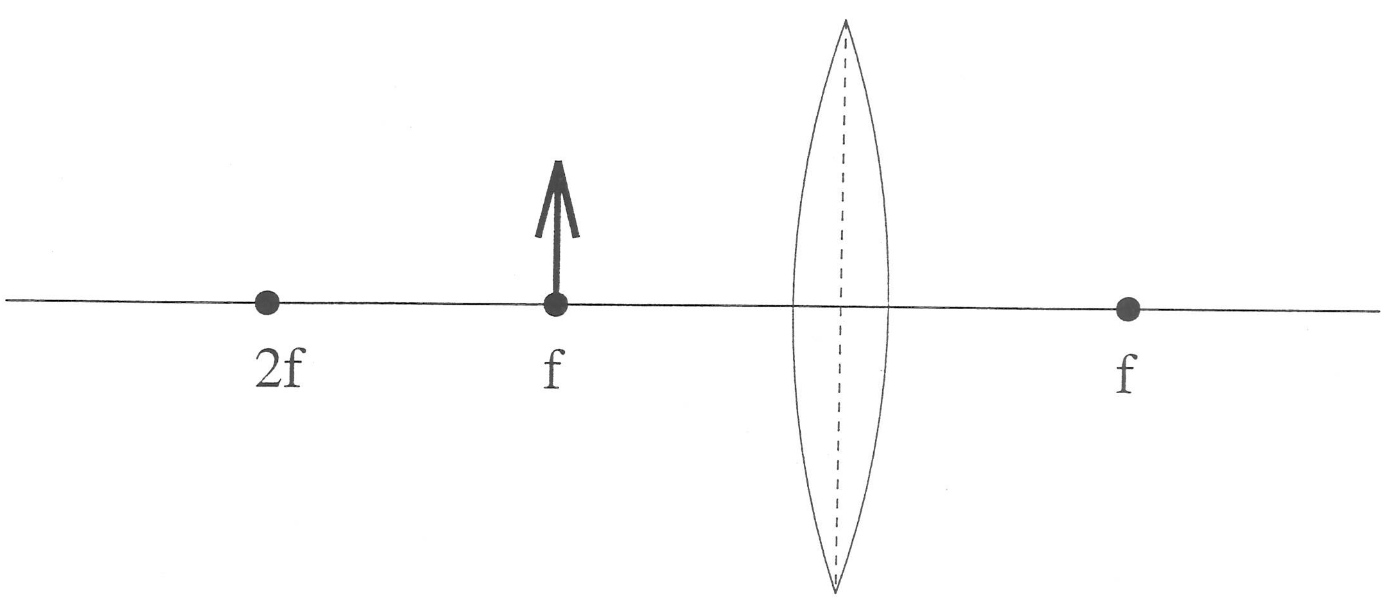
Converging Lens



Object ________

Image ________

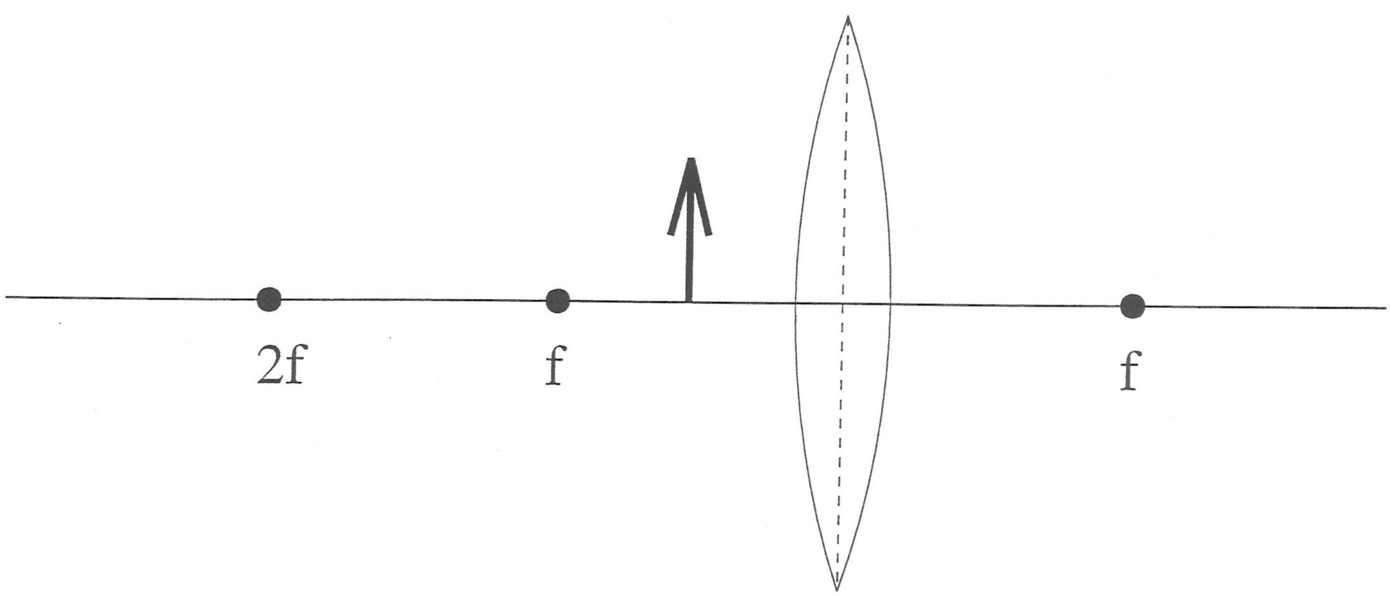
Converging Lens



Object ______

Image ______

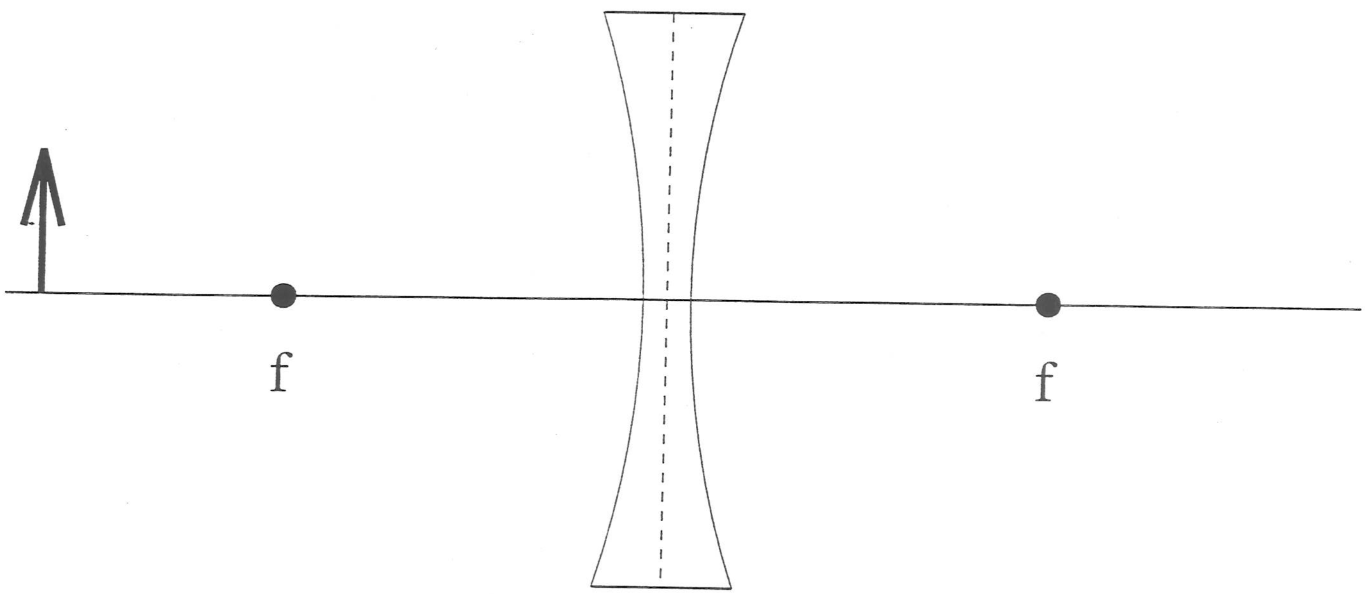
Converging Lens



Object ________

Image ________

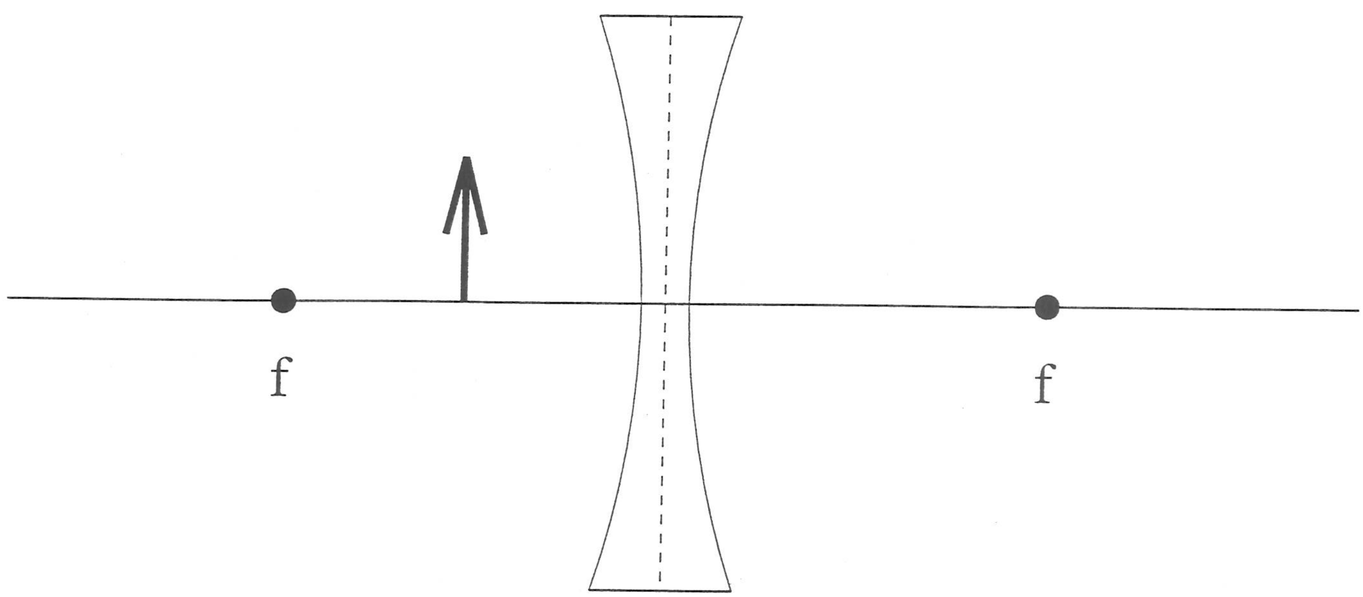
Diverging Lens



Object ______

Image ______

Diverging Lens



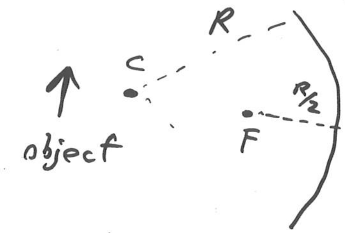
Object ________

Image ________

Fresnel Lens

Differences between Spherical Mirrors and Spherical Lenses

~~concave~~ any mirror



$\boxed{C = 2F}$

$M = -\frac{d_i}{d_o}$

real side

virtual

positive d_o

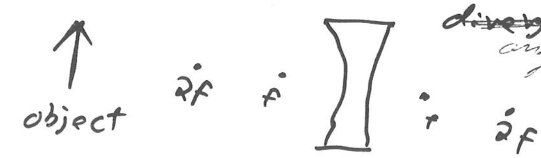
positive d_i

d_i negative

$f > 0$ concave

$f < 0$ convex

~~diverging~~ any lens



$C \neq 2f$

virtual side

real side

$f > 0$ converging

$f < 0$ diverging

positive d_o

d_i negative

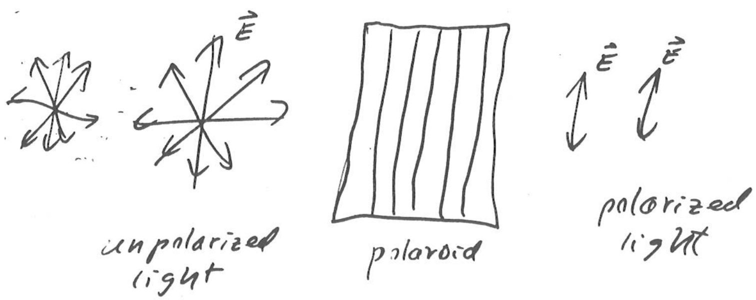
d_i positive

M same $= -\frac{d_i}{d_o}$

Polaroids

Light from the Sun, a lightbulb, a match, etc. is unpolarized, that is, it contains $\vec{E}$ fields pointing in all directions at random.

Think of a polaroid as a picket fence that only allows those $\vec{E}$ fields aligned along the pickets to pass through.



If the polaroid direction (picket direction) and the $\vec{E}$ field direction make an angle θ, then only

$$E_{max} \cos\theta \quad \text{gets through}$$

and the component of $\vec{E}$ that does get through is now polarized along the polaroid direction.

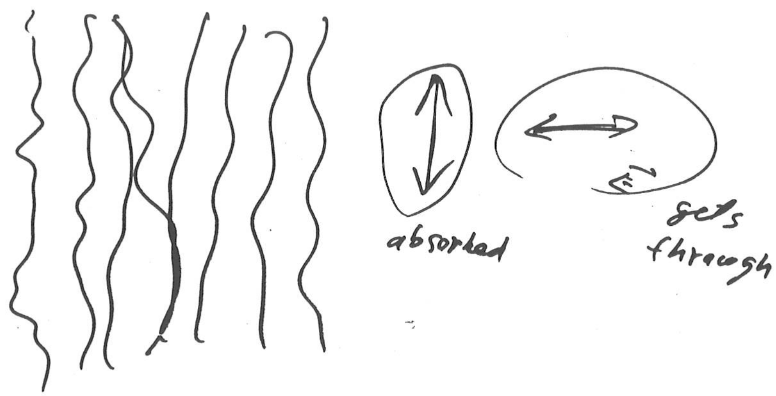
The intensity of radiation is proportional to $|\vec{E}|^2$

$$I_{max} = \frac{1}{c\mu_0} E_{rms}^2 = \frac{1}{c\mu_0} \frac{E_{max}^2}{2}$$

so the intensity (what your eye detects) that gets through the polaroid is

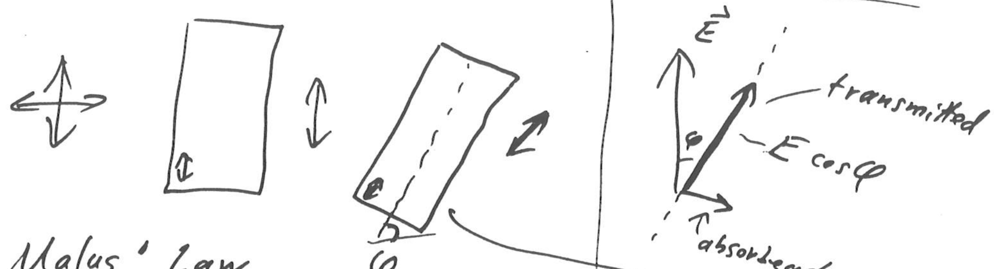
$$I_{max} \cos^2\theta$$

Polarization: direction of Electric field perpendicular to direction of travel



$$I_{unpolarized} = \vec{E}^2 = \vec{E}\cdot\vec{E}^2 = E_x^2 + E_y^2$$

$$I_{polarized} = E_x^2 = \frac{1}{2} I_{unpolarized}$$



Malus' Law
$I = I_0 \cos^2\varphi$

One polaroid

What happens to the original intensity Imax after unpolarized light passes through one polaroid?

$$\frac{I_{max}}{2}$$

Crossed polaroids

What happens to Imax as unpolarized light is passed through 2 polaroids set at 90° to each other?

0

Stacked polaroids

A third sheet is inserted between 2 crossed polaroids 45° from each. Imax?

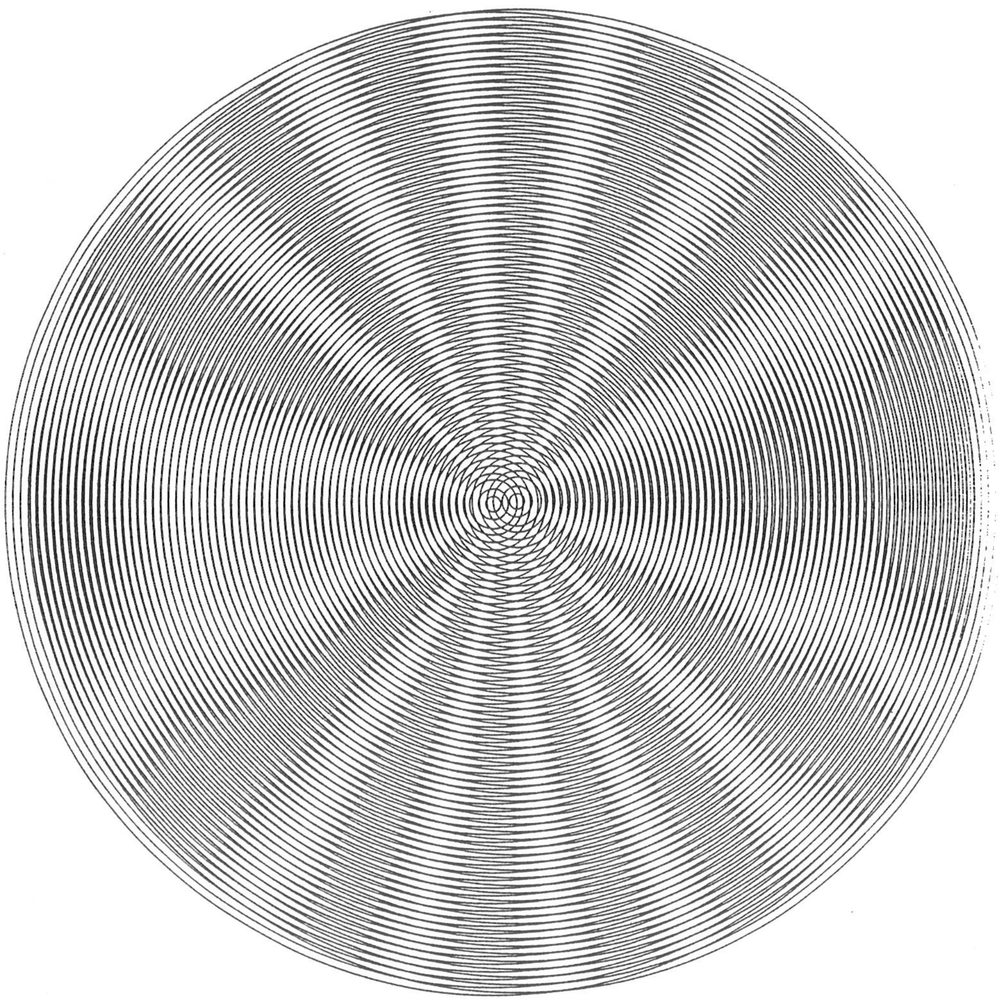
$I_m \updownarrow \quad \frac{I_{max}}{2} \swarrow\!\!\nearrow \quad \left(\frac{I_{max}}{2}\right)\cos^2(45°) = \frac{I_{max}}{4} \longleftrightarrow$

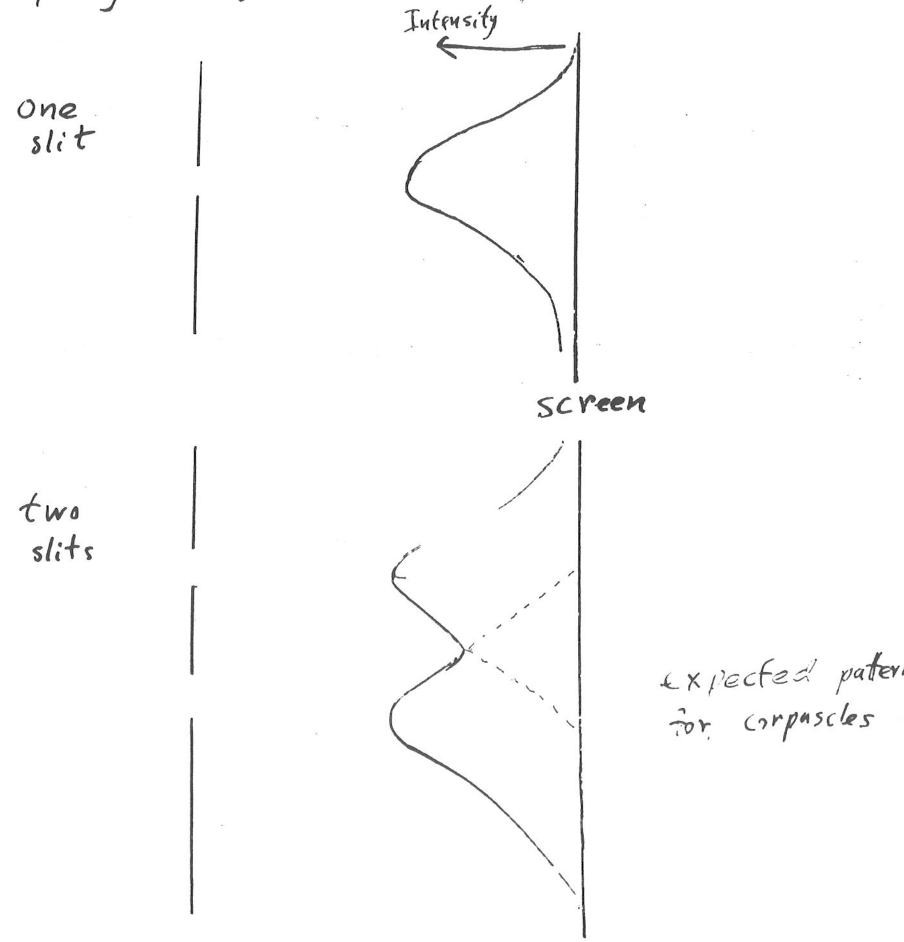
$$\left(\frac{I_{max}}{4}\right)\cos^2(45°) = \frac{I_{max}}{8}$$

Interference

Newton : Light is made of particles or corpuscles.

Young : Light is made of waves.

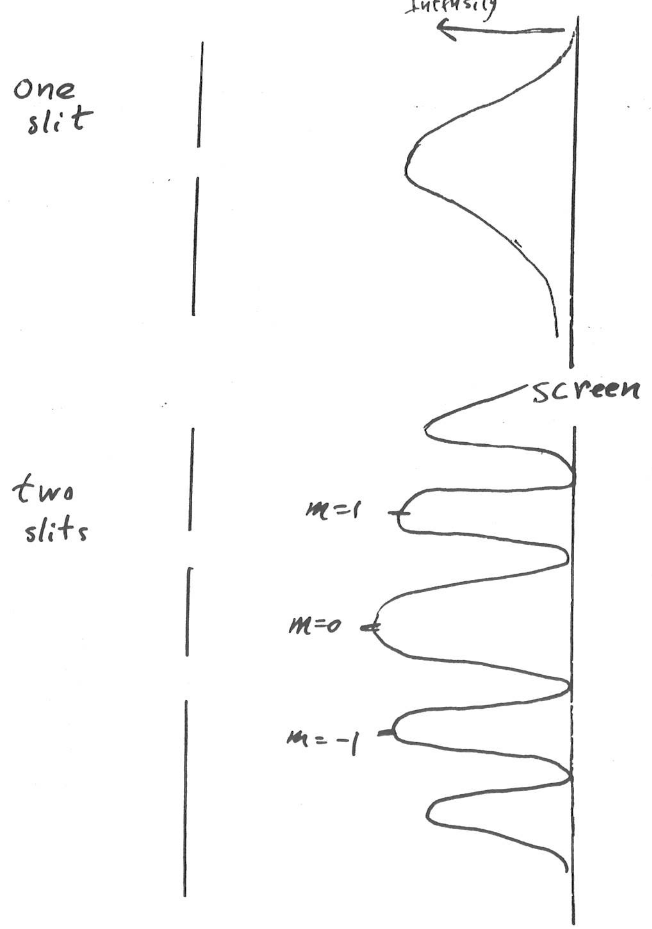




Interference

Newton : Light is made of particles or corpuscles.

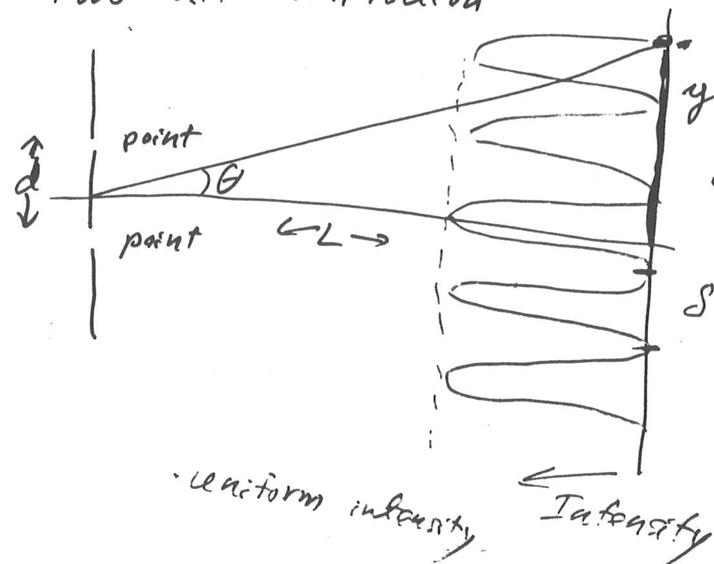
Young : Light is made of waves.



actual pattern for interfering <u>waves</u>.

(particles do not interfere.)

Two slit diffraction



$\sin\theta \sim \frac{y}{L}$

$\delta = \begin{cases} m\lambda \\ (m+\frac{1}{2})\lambda \end{cases}$

$\delta = d \sin\theta$

path difference δ tells you where bright spot occur

Intensity function — requires

$E_{tot} = E_1 + E_2$

Path difference $\delta = d \sin\theta$

Constructive Interference Bright $\delta = d\sin\theta = m\lambda$

$m = 0, \pm 1, \pm 2, \ldots$

Destructive Interference Dark $\delta = d\sin\theta = (m+\frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2, \ldots$

Small angle approximation

for $\theta \lesssim 0.1$ rad ($\approx 5°$)

$\theta \approx \sin\theta \approx \tan\theta = \frac{opp}{adj} = \frac{y}{L}$ (radian only!)

$y = L\sin\theta = \begin{cases} Lm\lambda/d & \text{const. bright} \\ L(m+\frac{1}{2})\lambda/d & \text{destr. dark} \end{cases}$

$y \ll L$ $\quad \lambda \ll d$ $\quad m$ = order

$y \sim 10\,cm$, $L \sim 10\,m$, $\lambda \sim 700\,nm$, $d \sim 1\,mm$

Phase Difference φ

when $\delta = 0$, $\varphi = 0$

when $\delta = \lambda$, $\varphi = 2\pi$

$\frac{\delta}{\varphi} = \frac{\lambda}{2\pi} \Rightarrow \varphi = \frac{2\pi\delta}{\lambda}$

$= \frac{2\pi d \sin\theta}{\lambda}$

Intensity

$$I = S_{AVG} = \frac{E_0^2}{2\mu_0 c}$$

$\nwarrow$ Poynting vector

$$E_{tot} = E_1 + E_2$$

$$= E_0 \sin(\omega t) + E_0 \sin(\omega t + \varphi)$$

$$= 2E_0 \cos\left(\frac{\varphi}{2}\right) \sin\left(\omega t + \frac{\varphi}{2}\right)$$

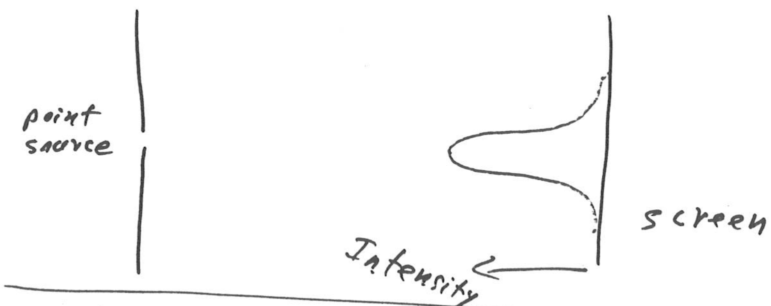
max is $2E_0$ - constr. - $\varphi = 0, 2\pi, 4\pi, \ldots$

min is 0 - destr. - $\varphi = 180° = \pi$ rad, 3π, $5\pi, \ldots$

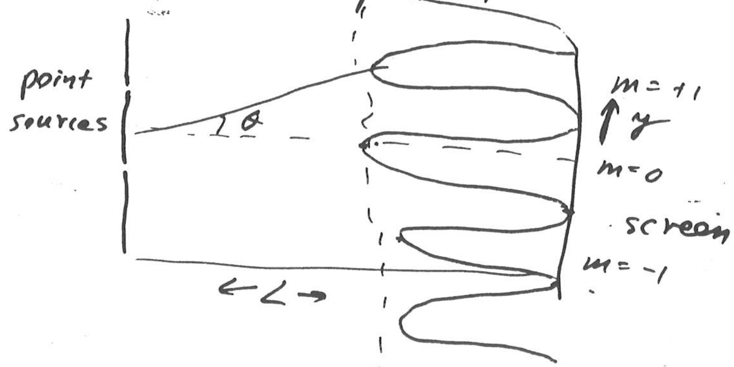
$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$= I_0 \cos^2\left(\frac{\pi d y}{L\lambda}\right)$$

Previously, single slit pattern



double slit pattern



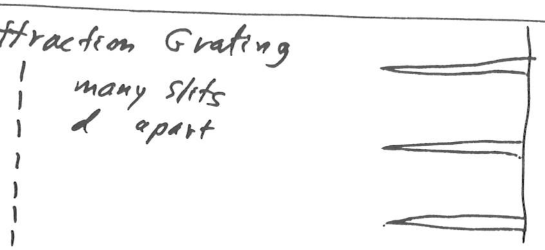
$\sin\theta = \frac{y}{L}$

constant intensity $\begin{cases} \text{in } \theta \\ \text{in } y \end{cases}$

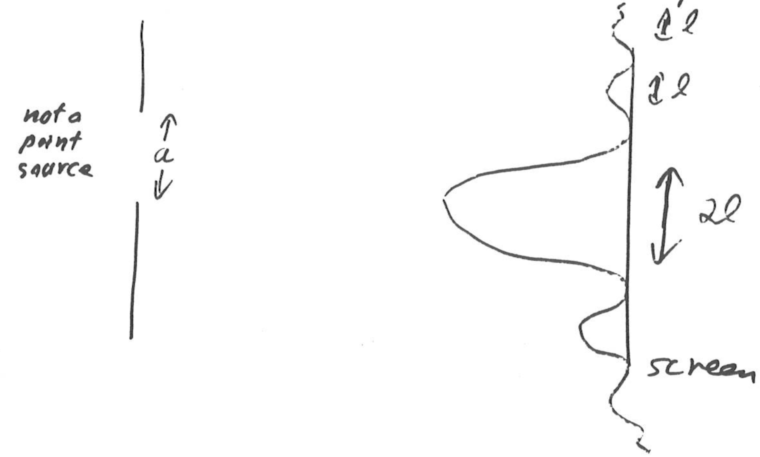
$E_{tot} = E_1 + E_2 = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi)$

$I \propto E_{tot}^2$

Diffraction Grating



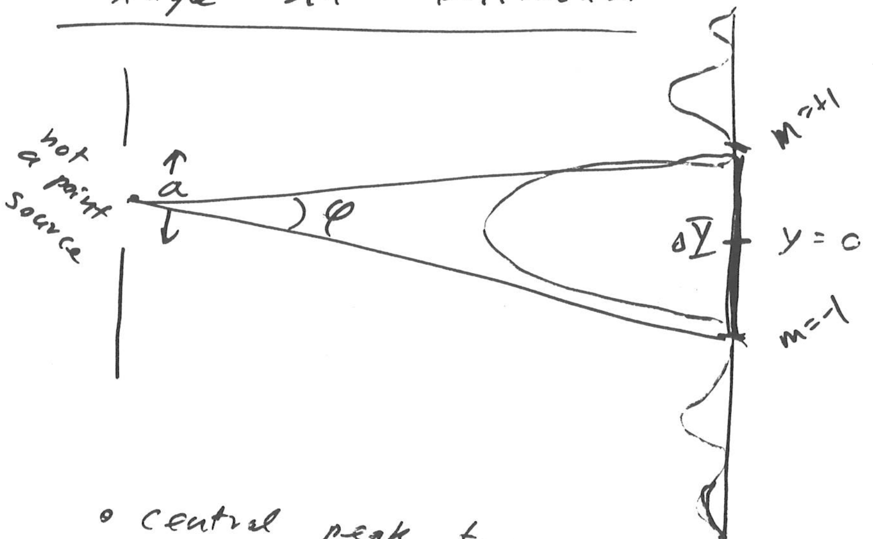
Single slit Diffraction pattern



Observations

- side peaks are not as bright as the central peak.
- central maximum is <u>twice</u> as wide as other maxima.

Single Slit Diffraction



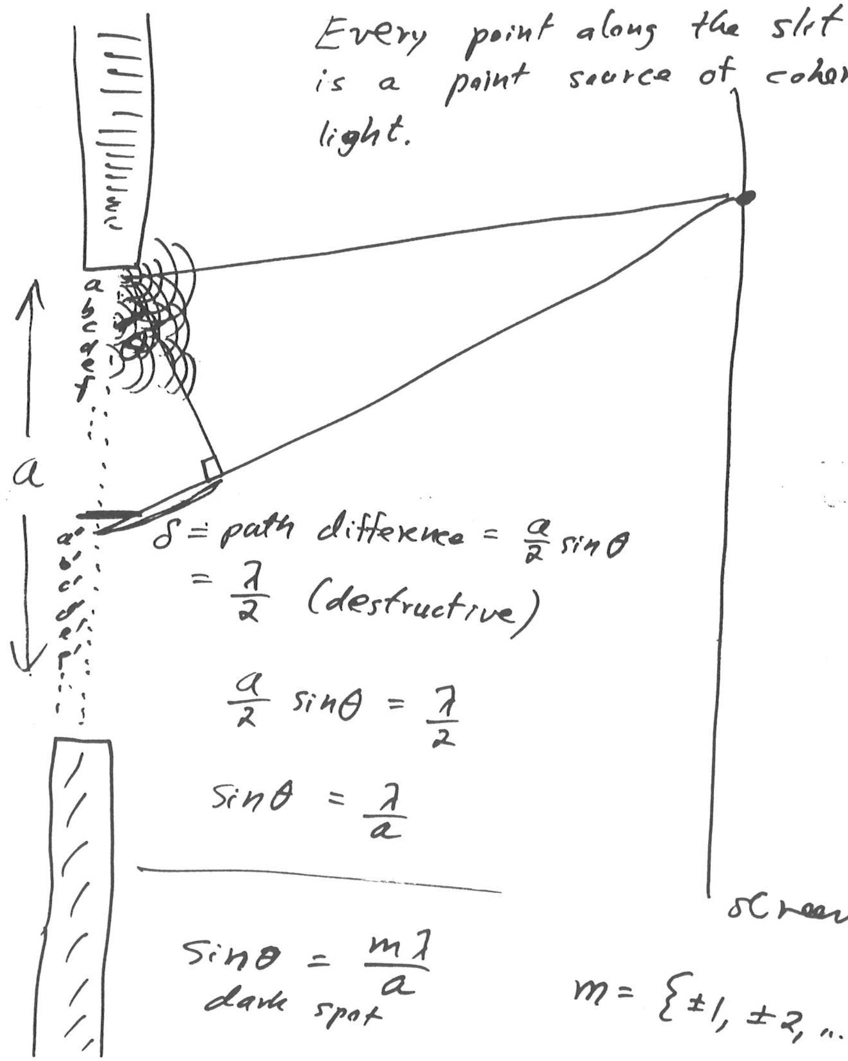
- central peak twice as wide as others
- Intensity falls away from $y=0$

$$\varphi = 2 \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$\Delta Y = L\varphi$$

Huygen's Principle

Every point along the slit is a point source of coherent light.



δ = path difference $= \frac{a}{2}\sin\theta$

$= \frac{\lambda}{2}$ (destructive)

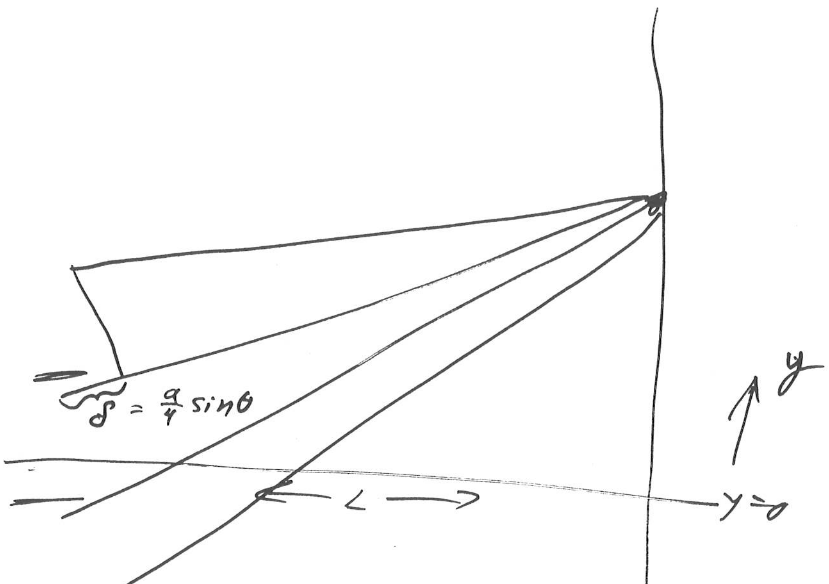
$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$\sin\theta = \frac{\lambda}{a}$$

$$\sin\theta = \frac{m\lambda}{a}$$

dark spot

$m = \{\pm 1, \pm 2, \ldots\}$



destructive : $\delta = \frac{\lambda}{2}$

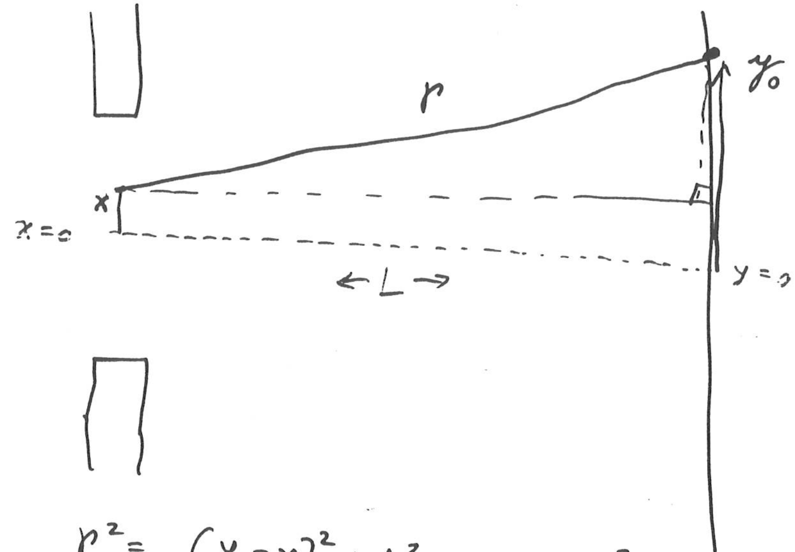
$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$

$\sin\theta = \frac{2\lambda}{a}$

In general
dark spots at

$\sin\theta = \frac{m\lambda}{a} = \frac{y}{L}$

$\boxed{m \neq 0}$



$$r^2 = (y_0 - x)^2 + L^2$$
$$= R^2 + x^2 - 2xy_0$$
$$= R^2\left(1 + \cancel{\frac{x^2}{R^2}} - \frac{2xy_0}{R^2}\right)$$
$$= R^2\left(1 - \frac{2x}{R}\sin\theta\right)$$

$R^2 \equiv y_0^2 + L^2$

$\left(\frac{x}{R}\right)^2 \sim 0$ $\quad \frac{y_0}{R} \sim \sin$

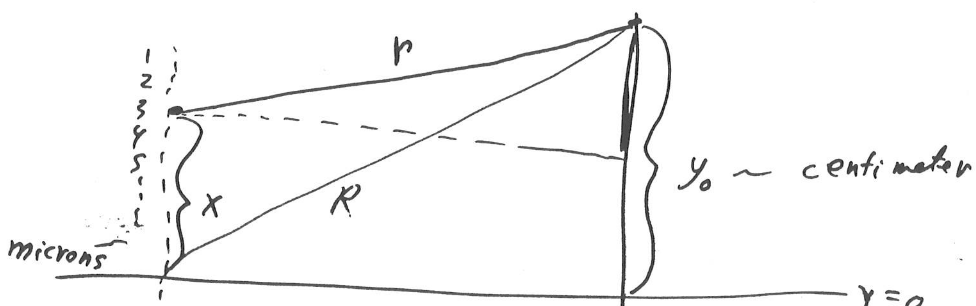
$$r = R\sqrt{1 - \frac{2x}{R}\sin\theta}$$

Binomial theorem $(1-z)^n \approx 1 - nz + \dots$ $\quad z \ll 1$

$$r \approx R\left(1 - \frac{x}{R}\sin\theta\right)$$

Intensity

$$E_{tot} = dE_1 + dE_2 + dE_3 + \dots\dots$$
$$= \sum_n dE_n \Rightarrow \int dE$$



$$r^2 = (y_0 - x)^2 + L^2 \qquad R^2 = y_0^2 + L^2$$
$$= y_0^2 + x^2 - 2xy_0 + L^2$$
$$= R^2 + x^2 + -2xy_0$$
$$= R^2\left(1 + \frac{x^2}{R^2} + \frac{-2xy_0}{R^2}\right)$$

$$y_0 \gg x$$

$$r = R\sqrt{1 + \frac{x^2}{R^2} - \frac{2xy_0}{R^2}}$$

$$\frac{x^2}{R^2} \to 0$$

$$\frac{y_0}{R} \approx \sin\theta \sim \frac{y_0}{L}$$

binomial

$$r \approx R\left(1 - \frac{x}{R}\sin\theta\right)$$

$$\underset{\text{at screen}}{E_{tot}} = dE_1 + dE_2 + \dots\dots = \sum_n dE_n = \int dE$$

$$dE = \frac{E_0}{a}\,dx\,\sin(kr - \omega t) \qquad k = \frac{2\pi}{\lambda}$$

$$dE = \frac{E_0}{a}\sin(kR - \omega t - kx\sin\theta)\,dx$$

$$E_{tot} = \int dE = \int_{x=-\frac{a}{2}}^{+a/2} \frac{E_0}{a}\sin(kR - \omega t - kx\sin\theta)\,dx$$

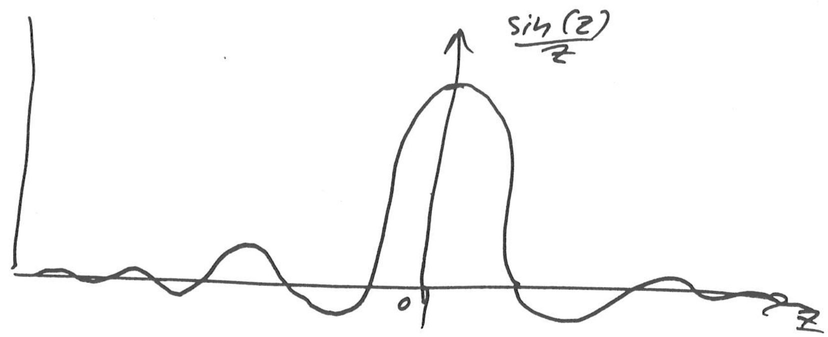
$$= \frac{E_0}{a}\left.\frac{-\cos(kR - \omega t - kx\sin\theta)}{-k\sin\theta}\right|_{x=-a/2}^{+a/2}$$

$$= \frac{E_0}{ak\sin\theta}\left[\cos\left(kR - \omega t - \frac{ka}{2}\sin\theta\right) - \cos\left(kR - \omega t + \frac{ka}{2}\sin\theta\right)\right]$$

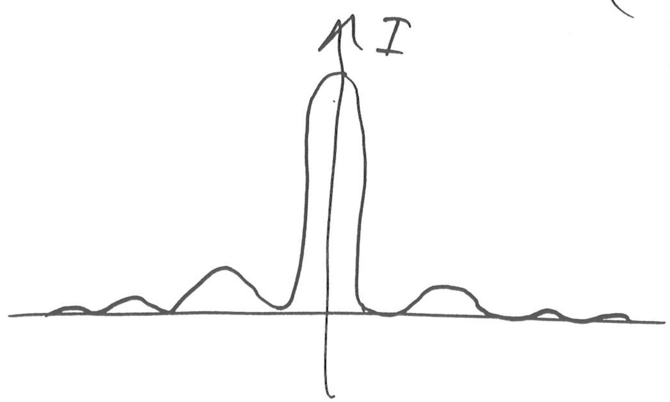
$$= \frac{E_0}{ak\sin\theta}\,2\sin(kR - \omega t)\sin\left(\frac{ka\sin\theta}{2}\right)$$

$$= \left[E_0\sin(kR - \omega t)\right]\frac{\sin\left(\frac{ka\sin\theta}{2}\right)}{\frac{ka\sin\theta}{2}}$$

$$E_{tot} = E_0 \sin(kR - \omega t) \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}$$

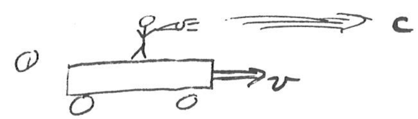


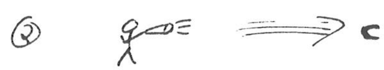
Intensity: $I \propto \vec{E}^2 \propto \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$



Michelson and Morley...

... cannot find the aether, so the speed of light is measured to be the <u>same</u> in all inertial reference frames.





Observer ② measures the speed of his light beam to be c, and observer ② measures the speed of ①'s light beam to be c as well!

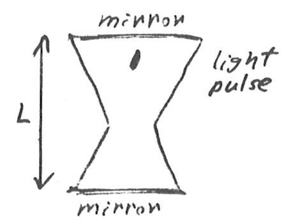
<u>Consequence</u>: ① and ② cannot have watches and meter sticks that agree.

Space and time are <u>not</u> absolute.

Different observers will get different measurements.

The Light Clock

This is a theoretical device used in <u>Gedanken</u> (or <u>thought</u>) experiments.

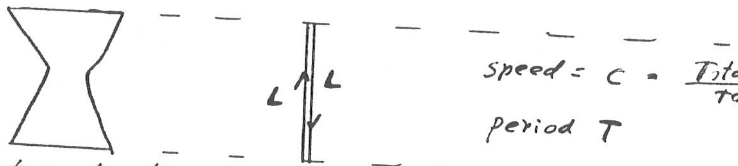


The period T is the time it takes the light pulse to complete <u>one full cycle</u> (tick-tock).

The path length covered by the pulse in one period T is $2L$.

We will derive results using the light clock, but the results will be valid for <u>any</u> clock (springs, pendula, biological, etc.).

Time Dilation



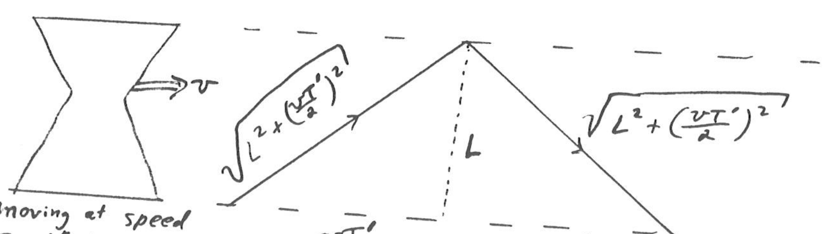
at rest with respect to us

speed = c = $\dfrac{\text{Total path length}}{\text{total time}}$

period T

$$c = \frac{2L}{T} \Rightarrow L = \frac{cT}{2}$$

We will <u>assume</u> that the length of the clock perpendicular to the direction of motion is unchanged. We will <u>prove</u> this shortly.



moving at speed v with respect to us

period of this <u>moving</u> clock = T'

speed of the light pulse = c = $\dfrac{\text{Total path length}}{\text{total time}}$

$$c = \frac{2\sqrt{L^2+\left(\frac{vT'}{2}\right)^2}}{T'}$$

A Little Algebra

$$c = \frac{2\sqrt{L^2+\left(\frac{vT'}{2}\right)^2}}{T'}$$

$$c^2 = \frac{4L^2 + v^2T'^2}{T'^2} \qquad L = \frac{cT}{2} \leftarrow \text{not } T'$$

$$c^2 = \frac{c^2T^2 + v^2T'^2}{T'^2}$$

$$c^2T'^2 = c^2T^2 + v^2T'^2$$

$$T'^2(c^2-v^2) = c^2T^2$$

$$T' = \sqrt{\frac{c^2}{c^2-v^2}}\, T = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, T$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \equiv \gamma$$ (gamma factor) dimensionless

$$T' = \gamma T$$

$$0 \le v < c$$

$$1 \le \gamma < \infty$$

v	γ
.5c	1.155
.75c	1.511
.9c	2.294
.99c	7.10
.999c	22.36
.999999c	707.1

$$T' = \gamma T$$

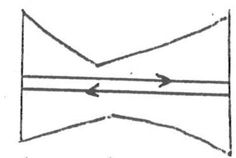
<u>Consequence:</u> Moving clocks run slow compared to clocks at rest with respect to the observer. The faster a clock moves, the slower it runs.

T = <u>Proper Time</u> $\equiv$ time recorded by a clock at rest. Proper time passes quickest of all.

<u>Paradox #1</u>: Suppose that you and I are moving relative to one another. I observe <u>your</u> clock to be running slow, but you observe <u>my</u> clock to be running slow.

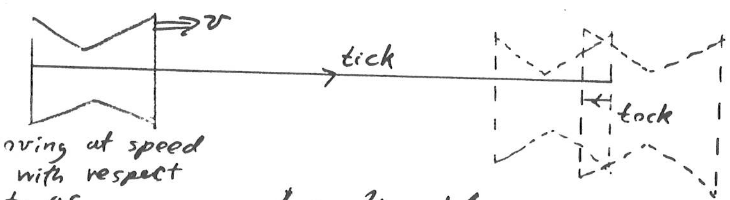
(By the way, I see you moving in slow motion and you see me moving in slow motion.)

Length Contraction



Length = L
period = T
speed of pulse = $c = \frac{2L}{T}$

$\Rightarrow T = \frac{2L}{c}$



Length = L'

period = $T' = \gamma T = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, T$

the clock is moving, so it is running <u>slow</u>.

speed of light pulse = c

$$\left.\begin{aligned} t_{tick} &= \frac{L'}{c-v} \\ t_{tock} &= \frac{L'}{c+v} \end{aligned}\right\} \text{observed by \underline{us}}$$

$$\text{period}: T' = t_{tick} + t_{tock} = \frac{L'}{c-v} + \frac{L'}{c+v}$$

More Algebra

$$T' = \frac{L'}{c-v} + \frac{L'}{c+v}$$

$$T' = \frac{L'}{c^2-v^2}\left[(c+v) + (c-v)\right] = \frac{2cL'}{c^2-v^2}$$

$$\frac{T}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2cL'}{c^2-v^2} \quad \text{but} \quad T = \frac{2L}{c}$$

$$\frac{\cancel{2}L}{c\sqrt{1-v^2/c^2}} = \frac{\cancel{2}cL'}{c^2-v^2}$$

$$L'\left(\frac{c^2}{c^2-v^2}\right) = \frac{L}{\sqrt{1-v^2/c^2}}$$

$$L'\left(\frac{1}{1-v^2/c^2}\right) = \frac{L}{\sqrt{1-v^2/c^2}}$$

$$L' = L\sqrt{1-v^2/c^2} = \frac{L}{\gamma}$$

$$T' = \gamma T$$
$$L' = \frac{L}{\gamma}$$

$$L' = \frac{L}{\gamma}$$

Consequence: Moving objects shrink in the direction of motion by the gamma factor.

E.g. A meter stick moving at .999999c along its length is $\frac{1\,m}{707.1} = 1.414$ mm long as observed by us. Of course, if an observer were moving at .999999c with the meter stick, that observer would record a length of 1 meter.

L = Proper Length ≡ the length recorded by an observer not moving with respect to the object.

Proper Length is the longest of all.

Paradox #2: Suppose that you and I are moving relative to one another. I observe you to be shrunk in the direction you're moving. You observe me to be shrunk in the same direction.

Proof ...

... that dimensions perpendicular to the direction of motion are unaffected.

Watch readings and meter stick measurements may disagree among moving observers, but all observers can agree on the result of a yes/no experiment this is independent of space + time (coin toss).

Questions like: Which occurred first?, or which is longer? Cannot be answered consistently by all observers.

Suppose that you are traveling in a cylindrical space ship with a radius just small enough to fit through a hole in the Great Barrier. (Measurements of ship and hole radii at rest.)

If perpendicular dimensions are shrunk (like the parallel dimension) then:

2 will observe a shrinking cylinder that will easily pass through the hole.

1 will observe a shrinking hole that is too tiny to fit through.

The question is: Does the ship crash into the barrier?

2 Predicts no crash

1 Predicts a crash

But both observers 1 and 2 must agree.

The only way to resolve this problem is to say that perpendicular dimensions do not change.

The Twins Paradox

One of a set of identical twins remains on Earth while the other flies out to Alpha Centauri (4 light-years distant) at very near the speed of light:

$$0.99999999999999999999c$$

and returns at the same speed.

Result: The Earth-bound twin ages a little over 8 years, but the twin in the rocket ages about 7 minutes.

This is not the paradox. This effect has been measured using extremely accurate atomic clocks.

This is the paradox:

The Earth-bound twin observes: the twin in the rocket to be slowed down due to time dilation. For every minute that passes on Earth, the Earth-bound twin observes only a fraction of a second elapses on the rocket.

But

The twin in the rocket observes: the twin on Earth to be slowed down due to time dilation. For every minute that passes on the rocket, the rocket twin observes only a fraction of a second elapses on Earth.

From the rocket twin's point of view, the Earth is flying away at near light-speed and then returning. Why isn't the Earth-bound twin younger than the rocket twin? What is different about the twin in the rocket?

Answer: The rocket twin has changed inertial reference frames and has undergone an acceleration.

Proof of Special Relativity

An elementary particle called the "muon" has an average lifetime of $2\,\mu s$ before it decays. (measured at rest – proper lifetime)

If Special Relativity were incorrect, then the average distance that a muon could travel would be

$$(3\times10^8\ m/s)(2\times10^{-6}\ s) = 600\ m$$

But muons are created in the upper atmosphere by cosmic ray collisions, and muons reach the Earth's surface

100,000 m below!

How does Special Relativity explain this?

Explanation #1

From the point of view of an Earth-bound observer:

The speed of the muons is close to (but less than) c.

The muon lifetime is time dilated.

Suppose the gamma factor is 1000.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1000$$

Then the moving muon lifetime is

$2\ ms = 1000\ (2\,\mu s)$ on average.

Can these reach the Earth's surface?

$$d_{AVG} = (3\times10^8\ m/s)(2\ ms) = 600,000\ m$$

Yes!

Explanation #2

From the point of view of the muon:

The Earth is approaching at close to (but less than) c.

The muon lifetime is $2\,\mu s$.

But the distance to the Earth's surface is <u>length</u> <u>contracted</u>.

If $\gamma = 1000$, then the distance from the top of the atmosphere to the surface is

$$\frac{100{,}000\,m}{\gamma} = 100\,m$$

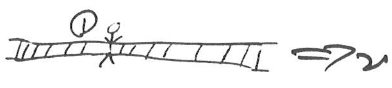
Can the muons reach the Earth's surface

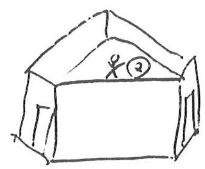
$$d_{AVG} = (3\times10^8\,m/s)(2\,\mu s) = 600\,m > 100\,m$$

Yes!

The Loss of Simultaneity

Observer ① is carrying a ladder of proper length 20 ft at a high speed toward a barn of proper width 10 ft. Observer ② is at rest with respect to the barn.

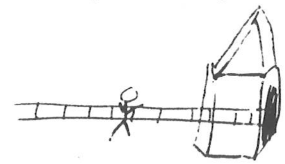




② will observe the ladder to be Lorentz contracted so that it will easily fit completely within the barn. While the shrunken ladder is inside, ② closes both doors **at the same time**, then reopens them to let ① run through the barn.

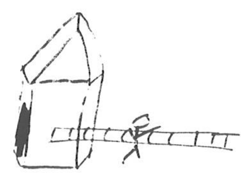
① Will observe the barn to be Lorentz contracted so that it will **not** contain the entire ladder. How does ① observe the closing doors?

Answer: ① observes the far barn door close first.



Then ① observes the far barn door open again.

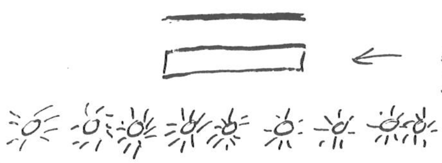
Then ① observes the first barn door close.



Then ① observes the first barn door open again.

Consequence: Events that are simultaneous in one frame will **not** be simultaneous in a frame moving relative to the first.

If we are in the rest frame of the bulbs, what do we observe?

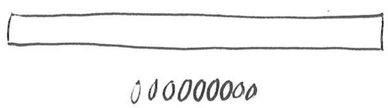


Lorentz contracted yard stick is only 1 foot long

All the bulbs go off at the same time.

Result: a 1 foot shadow on film.

If we are in the rest frame of the yard stick, what do we observe?



Proper length yard stick 1 yard long

Lorentz contracted array of bulbs

If the bulbs go off at the same time in <u>this</u> frame, the shadow image will be 3 yards long. But this is a contradiction — there is only the <u>one</u> photo.

<u>Answer:</u> As observed from the rest frame of the yard stick, the right most bulb goes off first, then the next, until finally the leftmost bulb goes off last.

<u>Result:</u> The shadow image on film is $\frac{1}{3}$ yard long. This is the same result obtained in the rest frame of the bulbs.

Relativistic Velocity Addition

Galilean	Einsteinian
$v_{ac} = v_{ab} + v_{bc}$	$v_{ac} = \dfrac{v_{ab} + v_{bc}}{1 + \dfrac{(v_{ab})(v_{bc})}{c^2}}$
$v_{ac} = v_{ab} - v_{cb}$	$v_{ac} = \dfrac{v_{ab} - v_{cb}}{1 - \dfrac{(v_{ab})(v_{cb})}{c^2}}$

① If two velocities less than c are added, then the resultant velocity is also less than c.

② If either velocity is equal to c, then the resultant velocity is also equal to c.

E.g. A spaceship flying away from Earth at 0.8c fires a missile forward at 0.8c with respect to the spaceship. What is the speed of the missile with respect to Earth?

X Galilean: $v_{m,E} = v_{m,s} + v_{s,E}$

$$= 0.8c + 0.8c = 1.6c$$

✓ Einsteinian: $v_{m,E} = \dfrac{v_{m,s} + v_{s,E}}{1 + \frac{(v_{ms})(v_{sE})}{c^2}}$

$$= \frac{0.8c + 0.8c}{1 + \frac{(0.8c)(0.8c)}{c^2}} = \frac{1.6c}{1 + (0.8)^2}$$

$$= \frac{1.6c}{1 + 0.64} = \frac{1.6}{1.64}c = \boxed{0.975c}$$

less than c!

E.g. A spaceship flying away from Earth at 0.8c turns on its headlights. The pilot sees the light speed away at c. What is the speed of the light with respect to the Earth?

Galilean: $v_{l,E} = v_{l,s} + v_{s,E}$

$$= c + 0.8c = 1.8c$$

Einsteinian: $v_{l,E} = \dfrac{v_{l,s} + v_{s,E}}{1 + \frac{(v_{l,s})(v_{sE})}{c^2}}$

$$= \frac{c + 0.8c}{1 + \frac{(c)(0.8c)}{c^2}} = \frac{1.8c}{1 + 0.8} = \frac{1.8}{1.8}c = \boxed{c}$$

Light always travels at the speed of light, regardless of the speed of the source or of the observer.