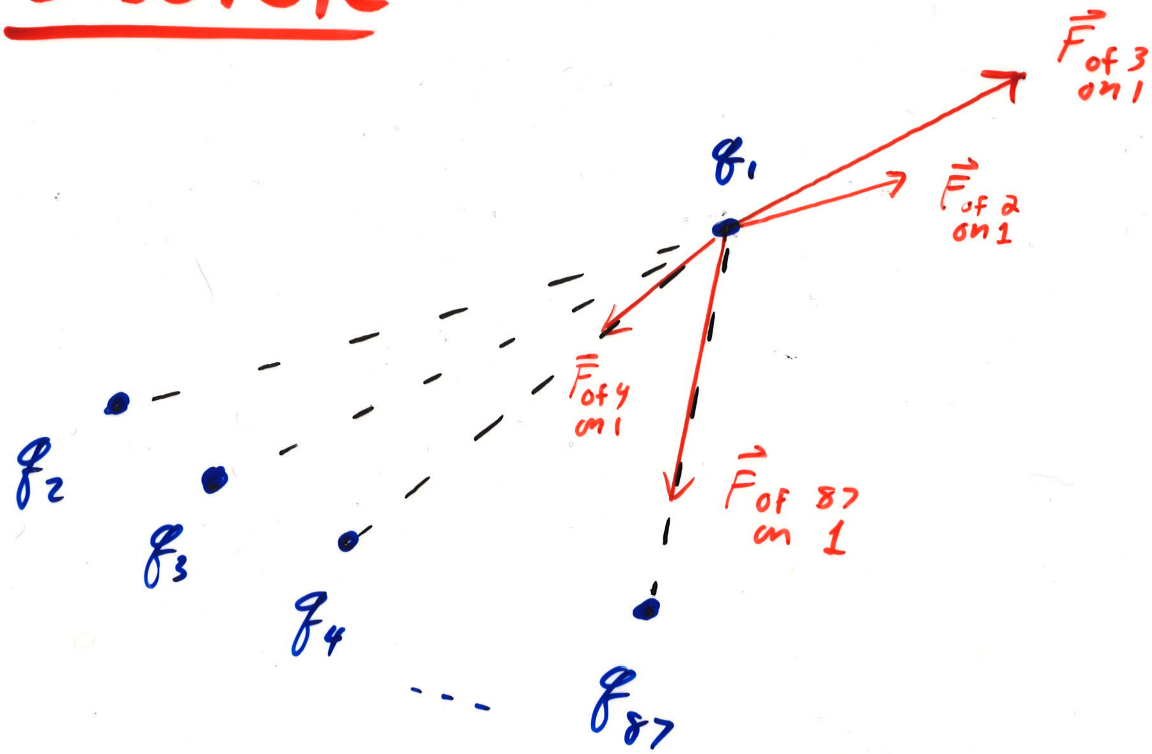
Superposition

Discrete

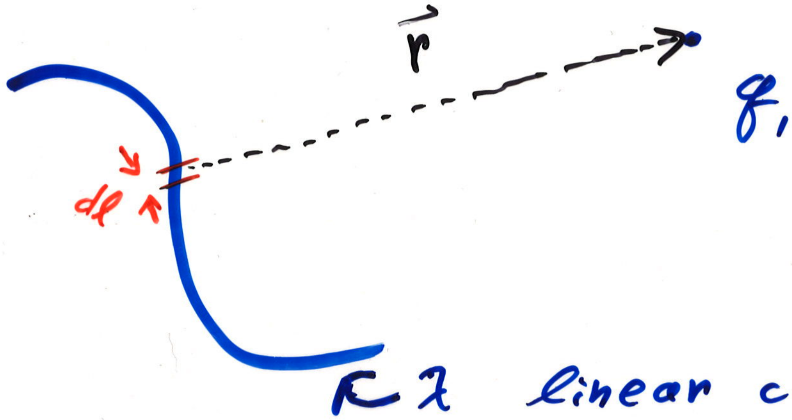


$$\vec{F}_{\substack{\text{total}\\ \text{on } 1}} = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^{87} \frac{q_i}{(r_{i,1})^2}\, \hat{r}_{i,1}$$

points from charge i to charge 1.

Superposition

Continuous



$\leftarrow \lambda$ linear charge density
"charge per meter"

$$dq = \lambda \, d\ell$$

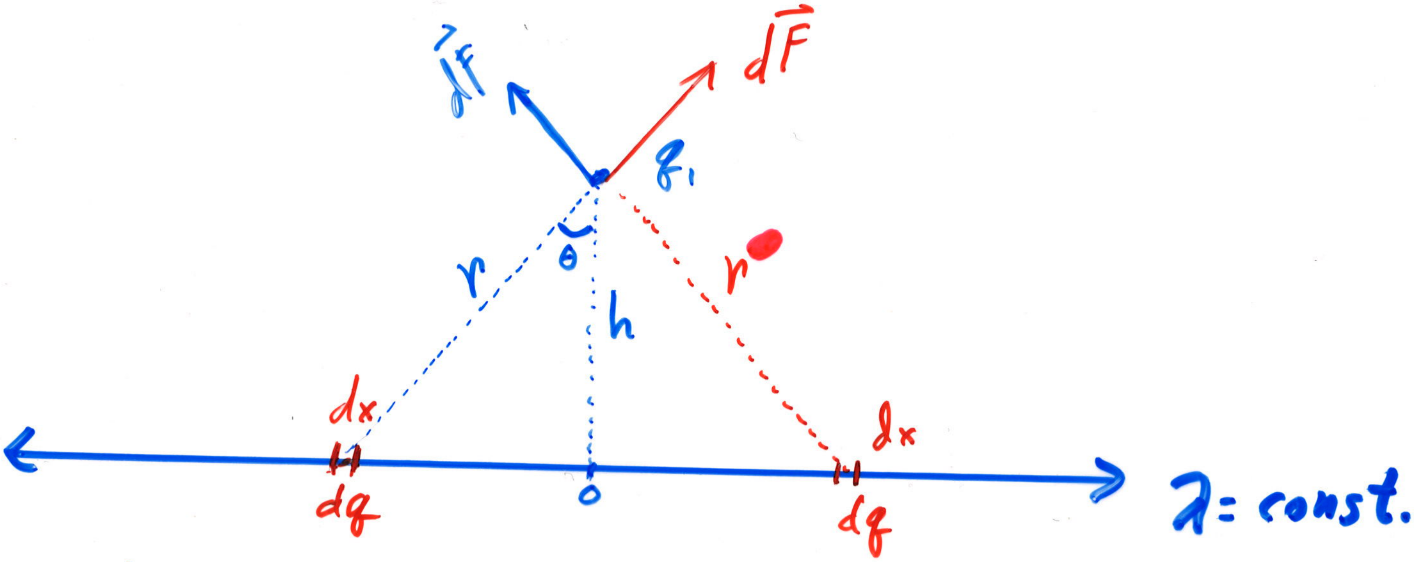
$$\vec{F}_{\substack{total\\ on\ 1}} = \frac{q_1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

$$\vec{F}_{\substack{total\\ on\ 1}} = \frac{q_1}{4\pi\epsilon_0} \int \frac{\lambda}{r^2} \hat{r} \, d\ell$$

$$\vec{F}_{\substack{on\\ 1}} = \int d\vec{F}_{\substack{on\\ 1}} = \frac{q_1}{4\pi\epsilon_0} \int dq \frac{1}{r^2} \hat{r}$$

$$d\vec{F}_{\substack{on\\ 1}} = \frac{1}{4\pi\epsilon_0} \frac{dq \, q_1}{r^2} \hat{r}$$

Ex Find the force on a point charge q_1 a distance h from an infinite uniform line of charge, with linear charge density λ.



$dq = \lambda dx$

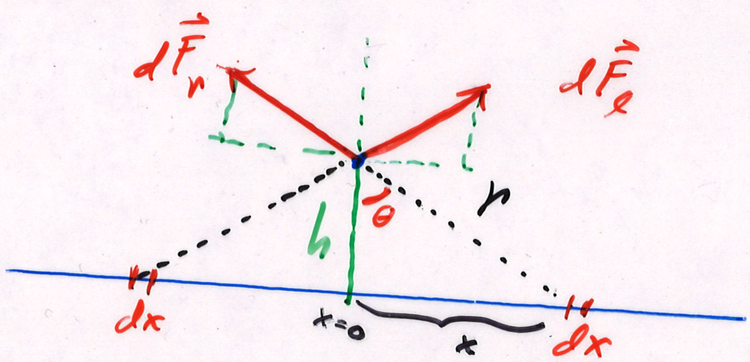
$$|d\vec{F}| = \frac{k\, dq\, q_1}{r^2} = \frac{k \lambda q_1\, dx}{r^2}$$

Find F_{TOTAL_y}

$F_{TOTAL_x} = 0$

$d\vec{F}$ has both x and y components.

For each infinitesimal charge dq on the right of q_1, there is a corresponding dq on the left of q_1.



The x-components will cancel, leaving a force in the y-direction only.

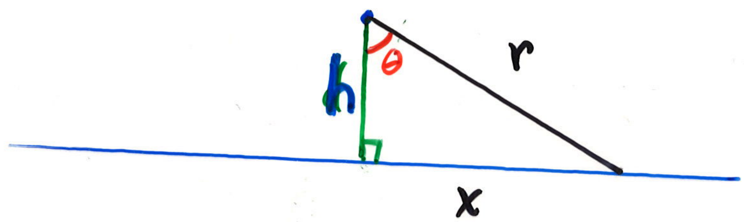
$$dF_y = dF \cos\theta$$

$$F_y = \int dF_y = \int_{x=-\infty}^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q_1 \lambda \cos\theta}{r^2} dx$$

$$F_x = 0$$

r, θ, and x are related:

$$r = \sqrt{x^2 + h^2}$$



$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{h} \quad \Rightarrow \quad x = h\tan\theta$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{h}{r} \quad \Rightarrow \quad r = \frac{h}{\cos\theta}$$

We need dx, so differentiate both sides of

$$x = h\tan\theta$$

$$dx = h\,\frac{1}{\cos^2\theta}\,d\theta$$

Finally, $x = -\infty$ corresponds to $\theta = -\frac{\pi}{2}$

$x = +\infty$ corresponds to $\theta = +\frac{\pi}{2}$

$$\left[\, x = h\tan\theta \quad , \quad \tan\frac{\pi}{2} = +\infty \,\right]$$

$$F_y = \int_{x=-\infty}^{+\infty} \frac{1}{4\pi\epsilon_0} \frac{q_1 \lambda \cos\theta}{r^2} \, dx$$

$$= \frac{q_1 \lambda}{4\pi\epsilon_0} \int_{\theta=-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos\theta}{\left(\frac{h}{\cos\theta}\right)^2} \, h \frac{1}{\cos^2\theta} \, d\theta$$

$$= \frac{q_1 \lambda}{4\pi\epsilon_0 \, h} \int_{-\frac{\pi}{2}}^{+\pi/2} \cos\theta \, d\theta$$

$$= \frac{q_1 \lambda}{4\pi\epsilon_0 \, h} \sin\theta \Big|_{-\pi/2}^{+\pi/2} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{2 q_1 \lambda}{h}}$$

$$\vec{F}_{total} = \frac{1}{4\pi\epsilon_0} \frac{2 q_1 \lambda}{h} \hat{j}$$

Charge is Quantized ↗ discrete

proton charge: $+e = +1.6\times10^{-19}\,C$

electron charge: $-e = -1.6\times10^{-19}\,C$

When dealing with lines of charge, we treat the charge as continuous. This approximation is valid as long as "e" is small compared to the other charges in the system.

The approximation fails in Atomic Physics.

Quarks

u = up quark, charge $+\frac{2}{3}e$

d = down quark, charge $-\frac{1}{3}e$

never free

proton = (uud) neutron = (udd)

electron = electron

What is the force on a charge $2q_1$?

$$\left[\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{\cancel{h}}\right](2q_1)$$

$15.7\ q_1$?

$$\left[\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{\cancel{h}}\right](15.7\ q_1)$$

The Force on the charge at position 1 is

(Electric Field at position 1) q_1

The Electric Field is the "Force per unit charge" at position 1.

Since the Force is a <u>vector</u>, the Electric Field is also a <u>vector</u>.

The Electric Field exists at position 1 <u>even if</u> there is no charge at position 1 to feel a force.

(The Electric Field at position 1 is created by all of the <u>other</u> charges in the problem.)

Mechanical Analog:

The gravitational field (acceleration, or "force per unit mass") of the Earth exists even if there is no small mass m_1 to feel the attractive force.

If you jiggle the other charges (2 → 87) you will create waves in the electric field. These waves travel at the speed of light, and are called radiation.

Radiation is extremely complicated, so we will first study statics, in which the charges will not move.

Units

MKS units of the electric field are

$$[E] = \frac{N}{C} = \frac{\text{newtons}}{\text{coulomb}} = \frac{V}{m}$$

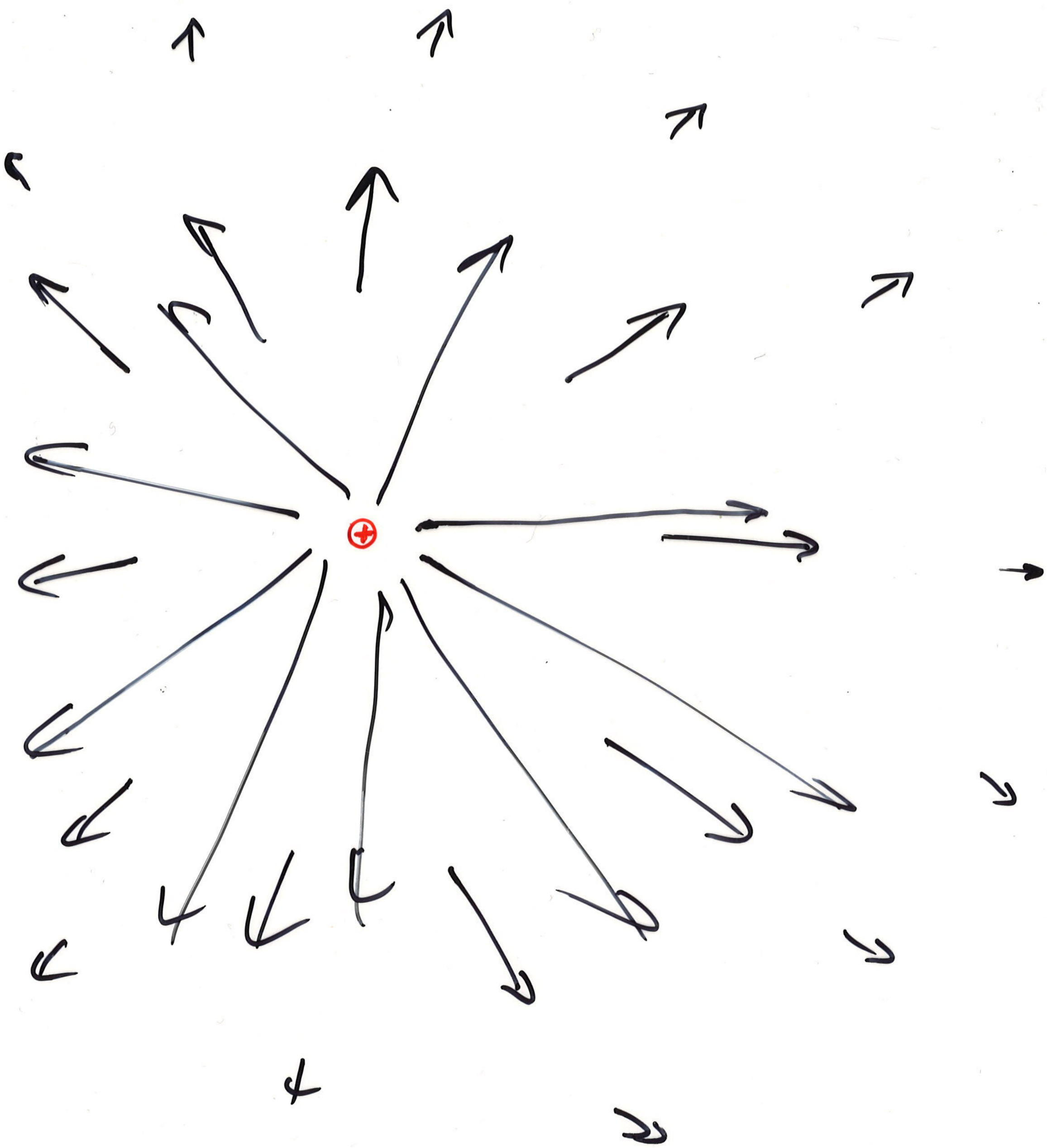
The $\vec{E}$ field is a vector, so we can represent it graphically with directed lines.

Convention: $\vec{E}$ field lines point away from positive charges and toward negative charges.

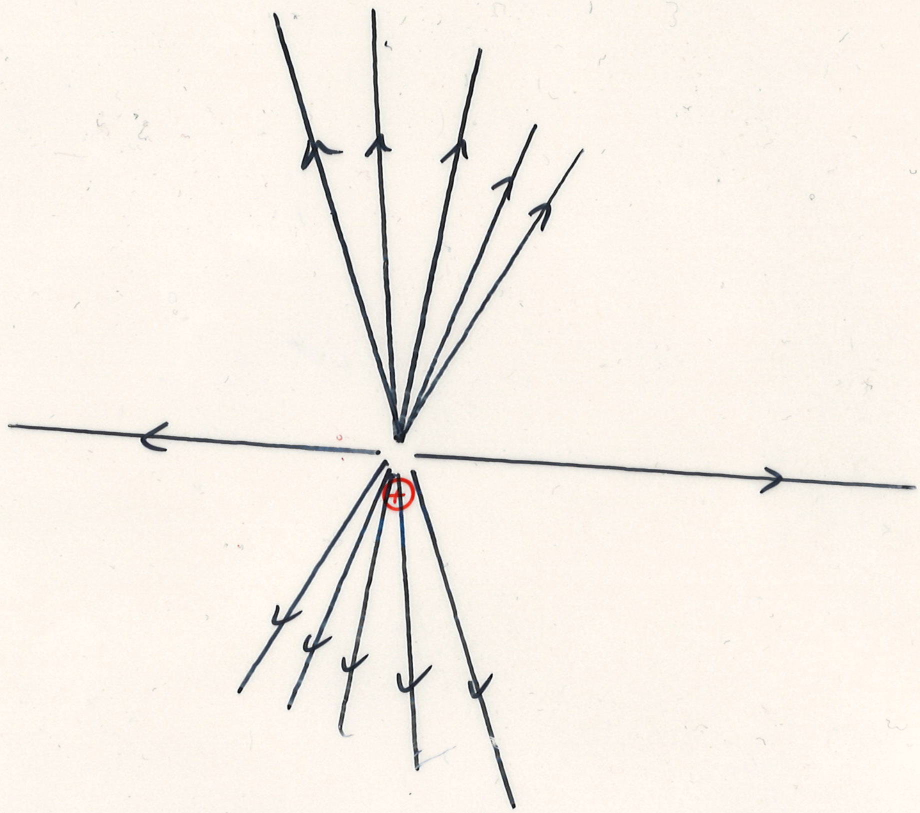
We will use a infinitely small positive "test charge" to map the electric field.

$\vec{F}$orce on a positive test charge is in the direction of $\vec{E}$ field.

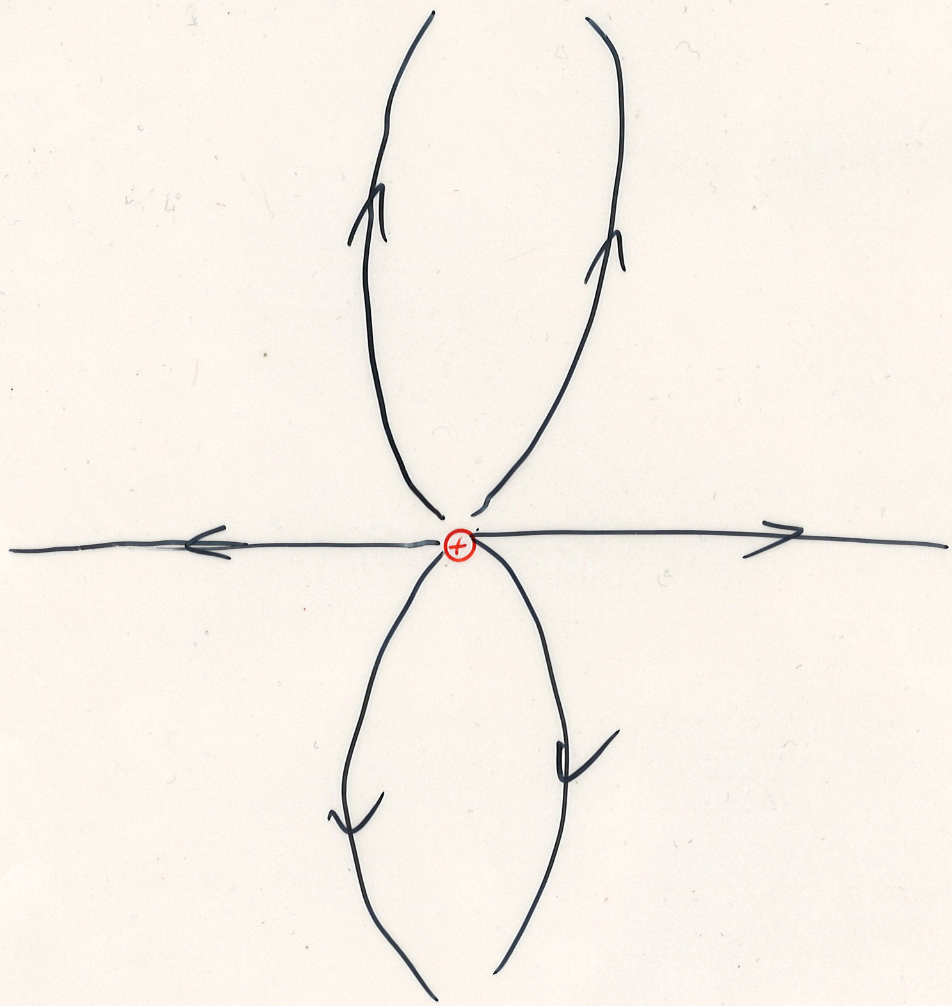
$\vec{E}$ field due to a positive point charge:



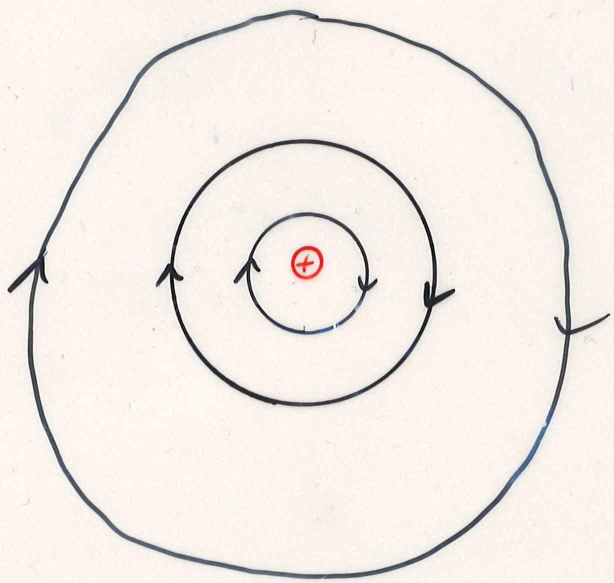
A symmetry argument!



A symmetry argument:



A symmetry argument:

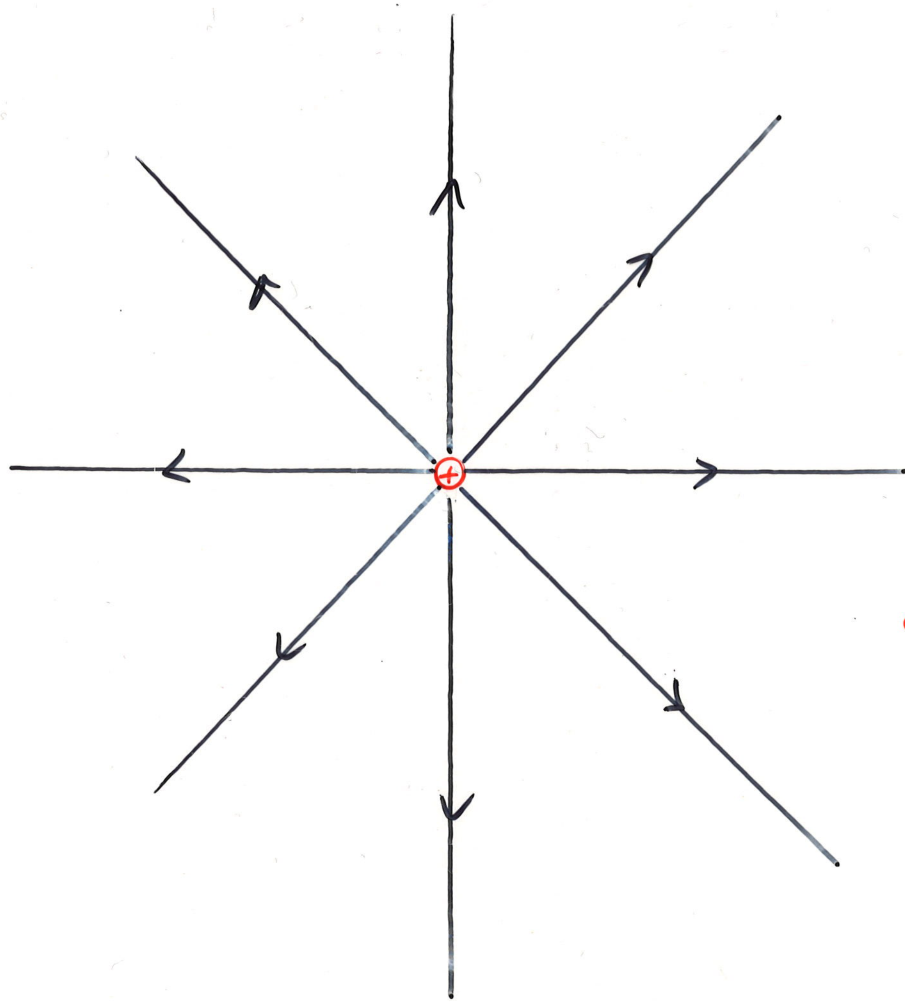


Use a test charge to eliminate this possibility.

The $\vec{E}$ field lines are radially outward.

They are infinitely long.

They are radially symmetric (not bunched up on one side).



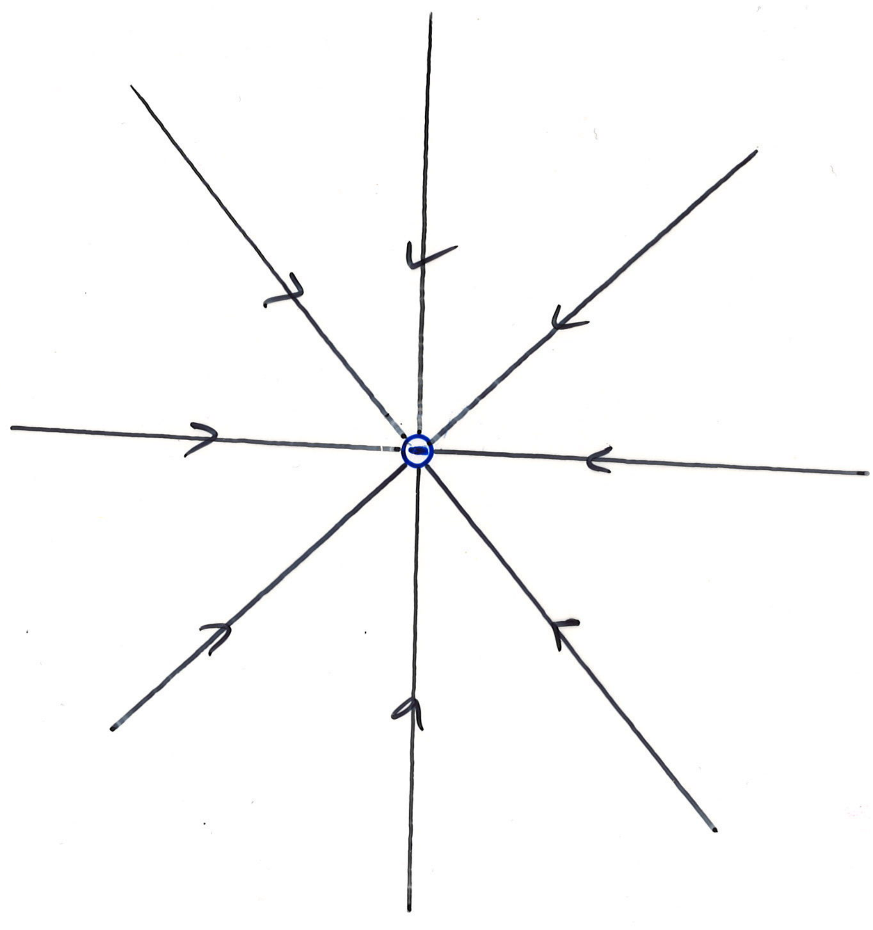
density of lines = strength of $\vec{E}$ field.

How many $\vec{E}$ field lines are there?

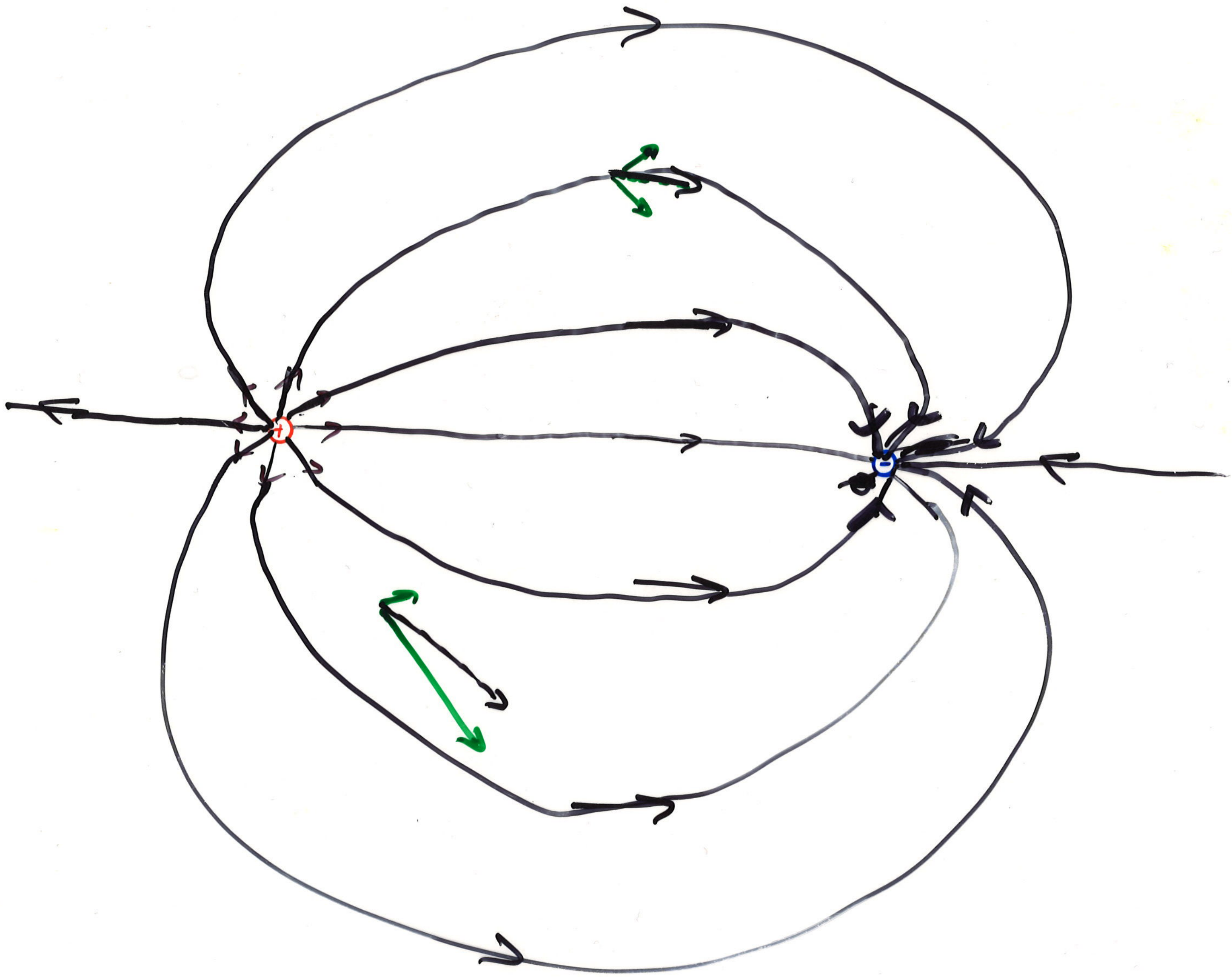
Infinitely many.

Draw as many as you like.

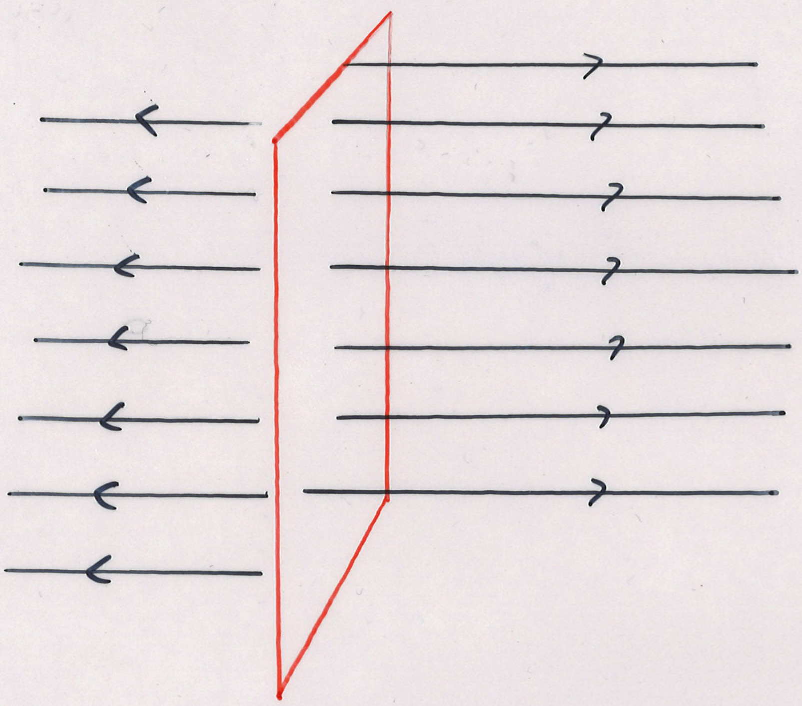
$\bar{E}$ field due to a negative point charge



$\vec{E}$ field for a pair of positive and negative point charges: (Dipole) of the same magnitude



$\vec{E}$ field due to an infinite uniformly positively charged plate: $\sigma = \frac{\text{charge}}{\text{area}}$



uniformly spaced