Electric Potential Energy

Recall from Mechanics:

Gravitational Potential Energy

$$\Delta U_{grav} \equiv -W_{grav,\, i\to f} = -\int_i^f \vec{F}_{grav}\cdot d\vec{s} = U_f - U_i$$

change in P.E. (unique)

Work done by the force of gravity on some object with mass.

$$U_{grav}(\vec{r}) = -\int \vec{F}_{grav}\cdot d\vec{s}$$

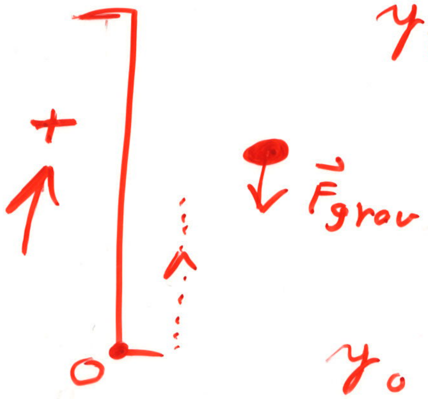
indefinite integral

Potential Energy Function

(defined up to an arbitrary constant of integration)

Consequence: The zero of $U_{grav}(\vec{r})$ can be chosen arbitrarily.

Ex. Find ΔU_{grav} for a stone of mass $\underline{m}$ lifted up to the top of a tower of height $\underline{h}$.



$y_f = h$

$y_0 = 0$

$\vec{F}_{grav} = mg(-\hat{j})$

$d\vec{s} = dy(\hat{j})$

$\hat{j}\cdot\hat{j} = 1$

$$W_{grav} = \int_i^f \vec{F}_{grav}\cdot d\vec{s} = \int_{y=0}^{h} mg(-\hat{j})\, dy\, \hat{j}$$

$$= \int_{y=0}^{h} -mg\, dy = -mgy\Big|_0^h = -mgh$$

$$\Delta U_{grav} = -W_{grav} = +mgh$$

$$U_{grav}(y) = -\int \vec{F}_{grav}\cdot d\vec{s}$$

$$= \int +mg\, dy = +mgy + \text{constant of int.}$$

Electric Potential Energy

$$\Delta U_{elec} \equiv - W_{\substack{elec \\ i \to f}} = - \int_i^f \vec{F}_{elec} \cdot d\vec{s}$$

unique

Work done by the Coulomb force on some object with charge.

$$U_{elec}(\vec{r}) = - \int \vec{F}_{elec} \cdot d\vec{s}$$

The zero of $U_{elec}(\vec{r})$ can be chosen arbitrarily.

$$\boxed{\text{The usual choice is: } \quad U_{elec}(\vec{r}) = 0 \ \text{ at } \ |\vec{r}| = \infty}$$

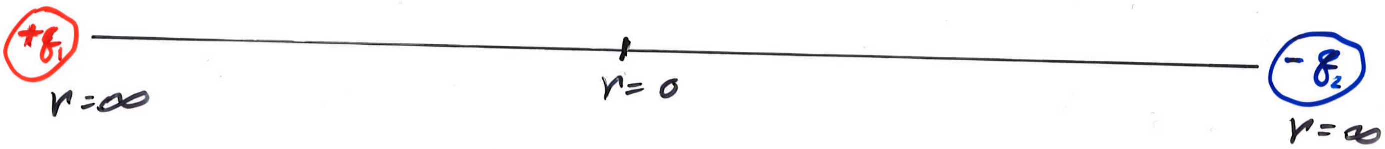
Note: Both the force of gravity and the Coulomb force are CONSERVATIVE. The work W and the potential energy are independent of the path from i to f.

The electric potential energy of a configuration of charges with the choice $U(\infty) = 0$ is the negative of the work done <u>on</u> the charges <u>by</u> the electric field as the charges are brought together from infinitely far apart.

<u>NOT</u> the work that <u>you</u> do to assemble them!

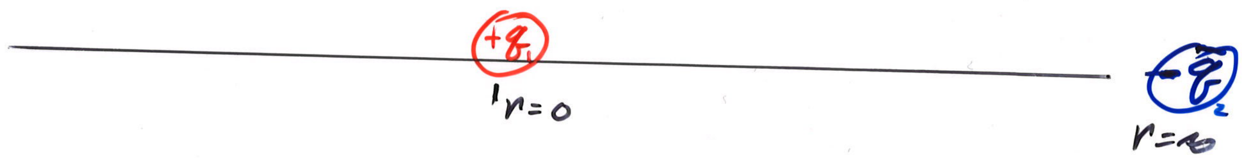
This is the electric field due to the charges <u>already</u> assembled, not including the one you are bringing in from infinity.

Ex. What is the potential energy U of two charges $(+q_1)$ and $(-q_2)$ separated by distance $\underline{d}$?



work done by field?

$W_{elec} = 0$ no charges assembled yet



work done by field?

$\hat{r}$

$$W_{elec} = \int_{r=\infty}^{d} \vec{F}_{elec} \cdot d\vec{s}$$

$\vec{F}_{elec}$ on $-q_2$ due to $+q_1$

← choose a simple path — straight line

$$= \int_{r=\infty}^{d} \left[\frac{1}{4\pi\epsilon_0} \frac{(+q_1)(-q_2)}{r^2} (\hat{r}) \right] \cdot \left[dr\ \hat{r} \right]$$

$$\boxed{\begin{array}{c} \hat{r}\cdot\hat{r} = 1 \\ \hat{s}\cdot\hat{s} = 1 \end{array}}$$

$$= \frac{q_1 q_2}{4\pi\epsilon_0} \int_{r=\infty}^{d} \left(-\frac{1}{r^2} \right) dr$$

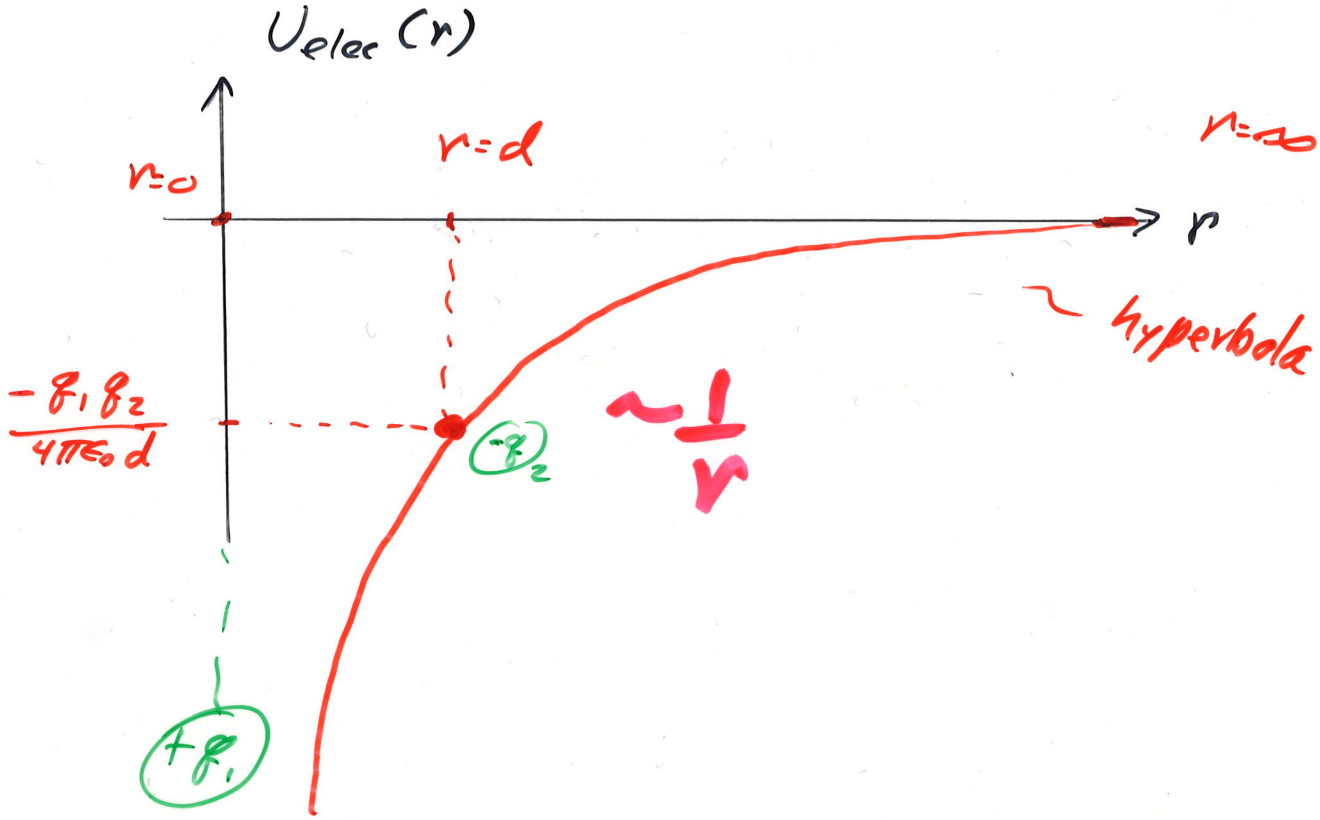
$$= \frac{q_1 q_2}{4\pi\epsilon_0} \left[+\frac{1}{r} \right]_{\infty}^{d}$$

$$= \frac{q_1 q_2}{4\pi\epsilon_0} \left[\frac{1}{d} - \cancelto{0}{\frac{1}{\infty}} \right] = \frac{q_1 q_2}{4\pi\epsilon_0 d}$$

This work is positive.

$$\Delta U_{elec} = - W_{elec} = \frac{-q_1 q_2}{4\pi\epsilon_0 d} = \frac{(+q_1)(-q_2)}{4\pi\epsilon_0 d}$$

$$U(r) = -\int \vec{F}_{elec} \cdot d\vec{s} = \frac{-q_1 q_2}{4\pi\epsilon_0 r} + \text{const}$$



Think of this as a hill on a golf course. All systems seek to lower their potential energy as much as possible. How can charges $(+q)$ and $(-q)$ decrease their potential energy?

Electric Potential

Voltage

$$V(\vec{r}) \equiv \frac{U_{elec}(\vec{r})}{q}$$

same arbitrariness in choosing the zero.

Electric Potential is Electric Potential Energy per unit charge.

$$\Delta U_{elec} = -\int_i^f \vec{F}_{elec} \cdot d\vec{s}$$

$$\Delta V = \frac{\Delta U_{elec}}{q} = -\int_i^f \frac{\vec{F}_{elec}}{q} \cdot d\vec{s}$$

$$= -\int_i^f \vec{E} \cdot d\vec{s}$$

Electric field is Electric Force per unit charge

Ex What is the electric potential $\underline{V}$ a distance $\underline{d}$ away from a point charge $(+q_1)$ with the choice $V=0$ at infinity?

$$U_{\substack{elec \\ +q_1 \leftrightarrow -q_2}} = \frac{(+q_1)(-q_2)}{4\pi\epsilon_0 d}$$

$$V_{\substack{due \\ to\ +q_1}} = \frac{U_{elec}}{(-q_2)} = \frac{+q_1}{4\pi\epsilon_0 d}$$

$$V_{\substack{due\ to \\ -q_2}} = \frac{U_{elec}}{+q_1} = \frac{-q_2}{4\pi\epsilon_0 d}$$

$V \cdot q = U$

(Voltage)(Charge) = Energy

$1\,V \quad 1\,C = 1\,J$

$1\,V = 1\,\frac{J}{C}$

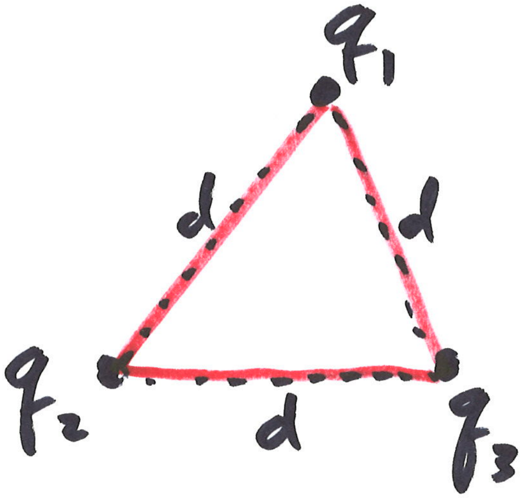
$(1\,V)(1\,e) = 1\,eV$

$= 1$ electron-volt

$= (1\,V)(1.6\times10^{-19}\,C)$

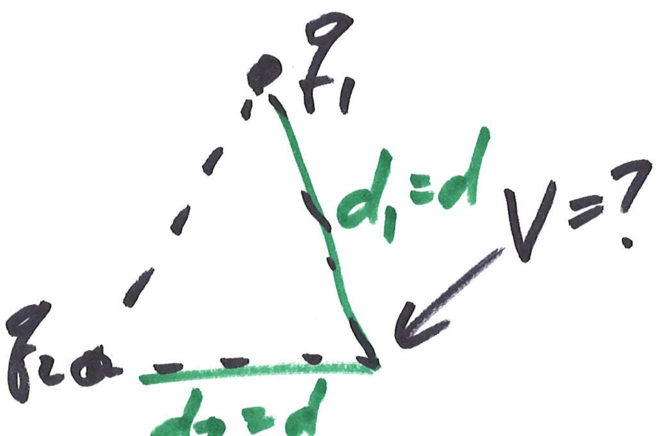
$= 1.6\times10^{-19}\,J = 1\,eV$

Find U_{Elec} for this configuration (with the choice $U(\infty)=0$)



$$U = \sum_{(j>i)}^{N} \sum_{i=1} \frac{k q_i q_j}{d_{ij}}$$

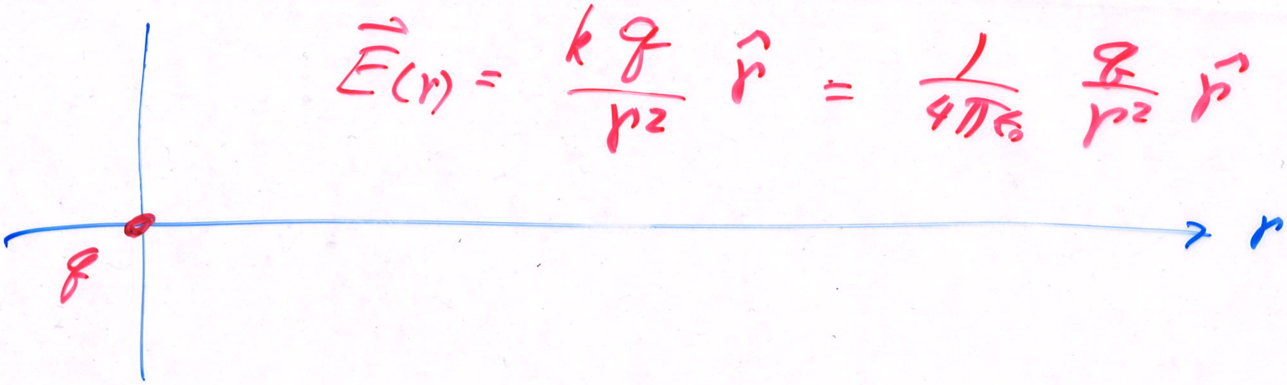
$$U = \frac{k q_1 q_2}{d} + \frac{k q_2 q_3}{d} + \frac{k q_1 q_3}{d}.$$

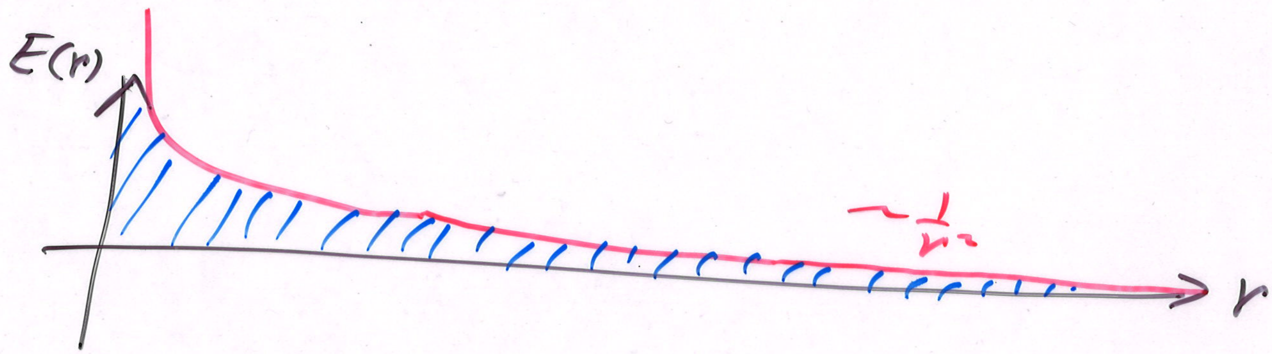


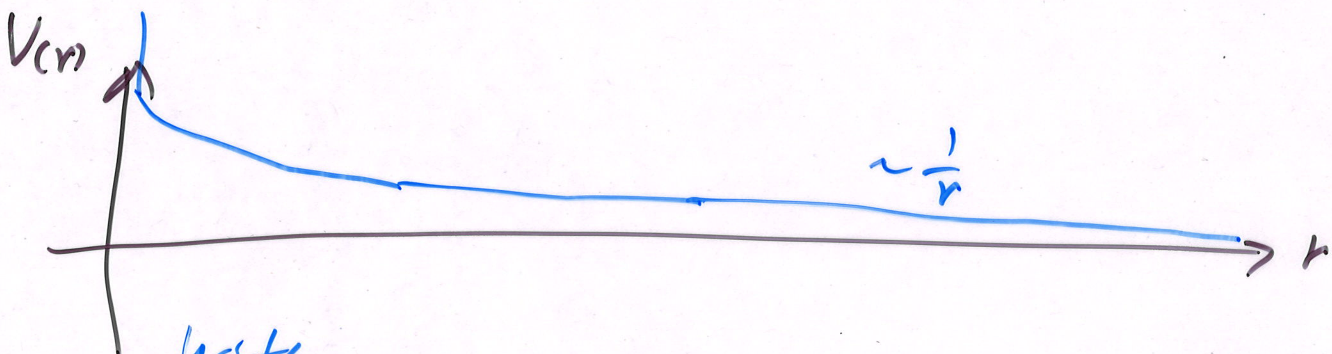
$$V = \sum_{i=1}^{N} \frac{k q_i}{d_i}$$

$$V = \frac{k q_1}{d} + \frac{k q_2}{d}.$$

$$\vec{E}(r) = \frac{kq}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$







With choice $V=0$ at $r=\infty$

$$V(r) = \frac{kq}{r} + C \qquad C=0$$

$$\Delta V_{1\to 2} = -\int_1^2 \vec{E}\cdot d\vec{s} = V_2 - V_1$$

$$E_x = -\frac{\partial}{\partial x} V$$

$$\vec{E} = (E_x, E_y, E_z)$$

$$E_y = -\frac{\partial}{\partial y} V$$

$$E_z = -\frac{\partial}{\partial z} V$$

$$V(x,y,z) = 2x^2y + yz$$

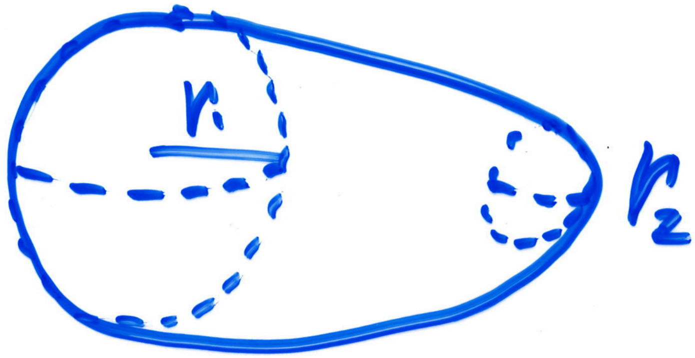
$$E_x = -\frac{\partial}{\partial x}(2x^2y + yz) = -(4xy + 0)$$

$$E_y = -\frac{\partial}{\partial y}(2x^2y + yz) = -(2x^2 + z)$$

$$E_z = -\frac{\partial}{\partial z}(2x^2y + yz) = -(0 + y)$$

$$\vec{E}(x,y,z) = -4xy\,\hat{i} - (2x^2+z)\,\hat{j} - y\,\hat{k}$$

Egg-shaped Conductor (Metal)



$r_1 > r_2$

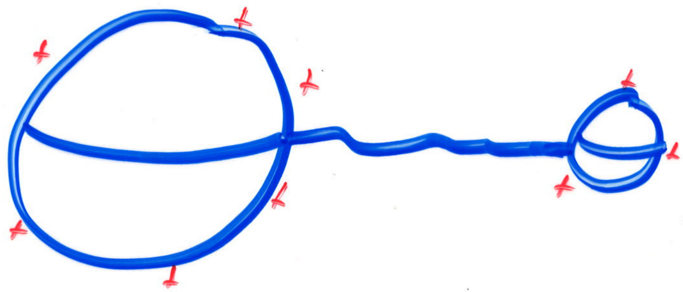
less $\vec{E}$

more $\vec{E}$

r_1

more charge

small σ



r_2

less charge

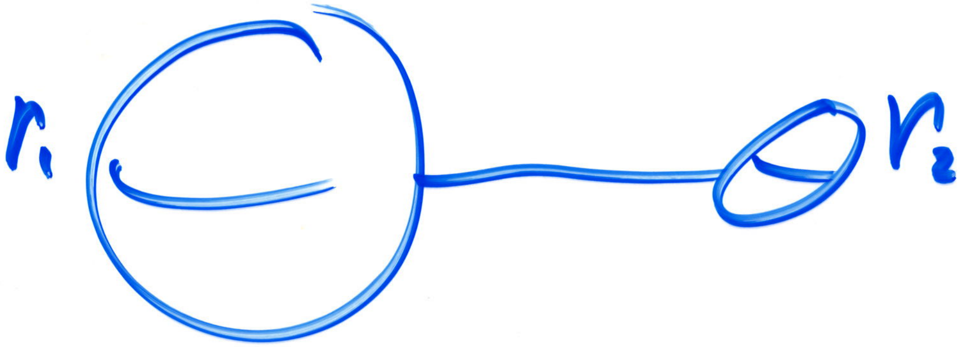
Large σ

Conductors are equi~~potials~~potentials

$$V_1 = \frac{kq_1}{r_1} = \frac{kq_2}{r_2} = V_2$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow \boxed{\frac{q_1}{q_2} = \frac{r_1}{r_2}} > 1$$

Surface Charge Density σ



$$\sigma_1 = \frac{q_1}{4\pi r_1^2} \neq \frac{q_2}{4\pi r_2^2} = \sigma_2$$

$$\sigma_1 < \sigma_2$$

$$E_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2} < \frac{q_2}{4\pi\epsilon_0 r_2^2} = E_2$$

$$E = \frac{\sigma}{\epsilon_0}$$ outside a conductor

just outside