Capacitance

What is a capacitor?

A device that stores energy and discharges it as we need it.

How do we make one?

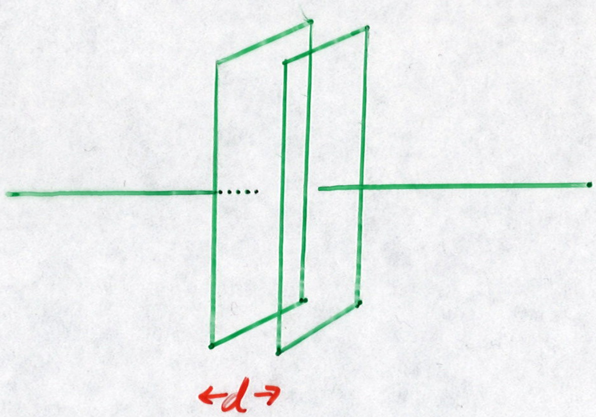
Two isolated conductors of any size and shape will work

The conductors are called "plates" even if they are not plate-like.

A special case:

The Parallel-Plate Capacitor

This is constructed from two conducting sheets, each of area A. The plates are parallel to each other and separated by distance d.



Voltage difference between plates =

Net charge on capacitor =

Charge on each plate =

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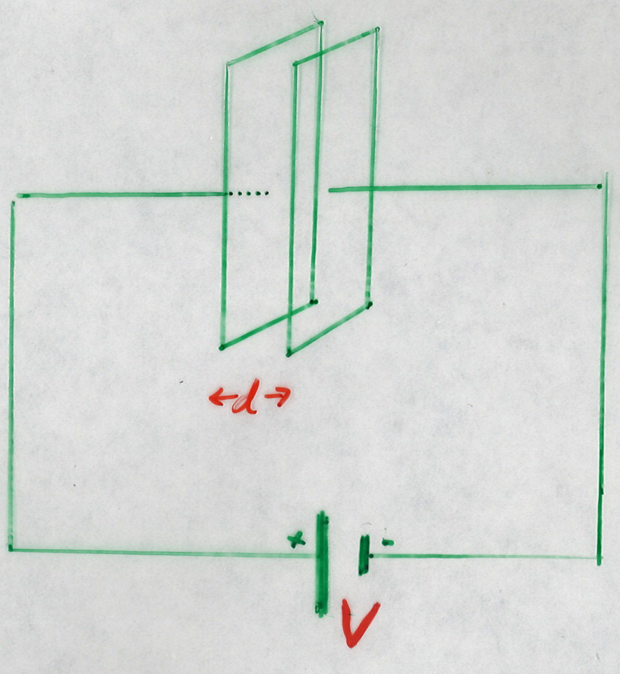


Now connect the capacitor to a battery.

Voltage difference between plates = V

Net charge on capacitor = 0

Charge on each plate = $+q$, $-q$

The charge on each plate is proportional to the applied voltage:

$$q = CV$$

The constant of proportionality is called the capacitance.

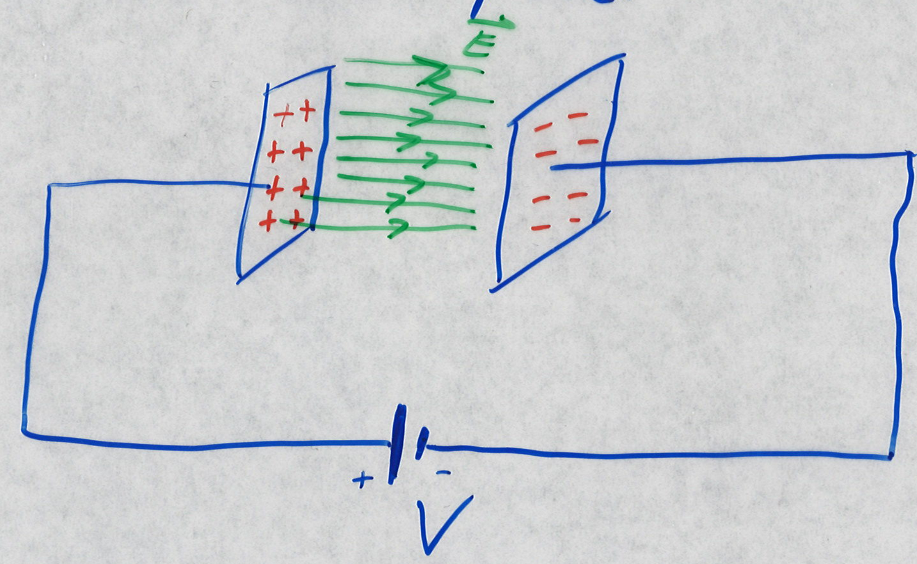
The capacitance, C, depends only on the geometry of the plates.

- their area
- their separation
- their shape (if not parallel-plate)

For parallel-plate, just A and d!

What happens while a capacitor is charging?

Electrons are being <u>removed</u> from one plate and are being <u>deposited</u> on the other plate.



Eventually, the electric field that is created between the charged plates prevents any more charge from accumulating on the plates.

When this occurs, the potential difference between the plates is equal to V, the battery voltage.

Units

The MKS unit of capacitance is the Farad (F).

$$1 \text{ Farad} = 1 \, \frac{\text{Coulomb}}{\text{Volt}} \quad , \quad 1\,F = 1\,\frac{C}{V}$$

1 Farad is a <u>huge</u> capacitance!

$mF = 10^{-3} F$

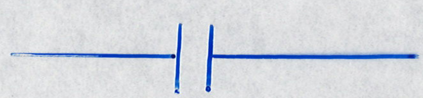
$\mu F = 10^{-6} F$

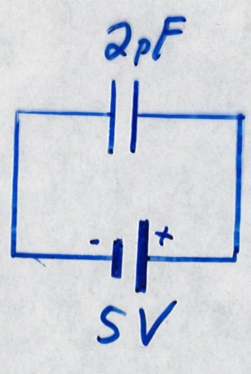
$nF = 10^{-9} F$

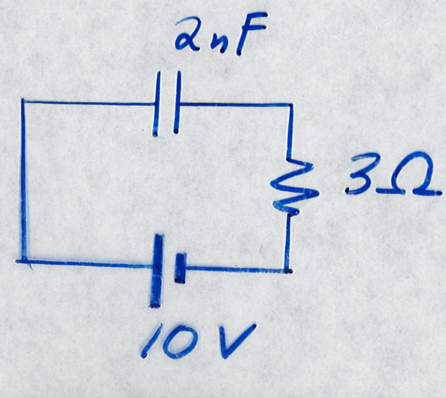
$pF = 10^{-12} F$

Most devices use pF capacitors.

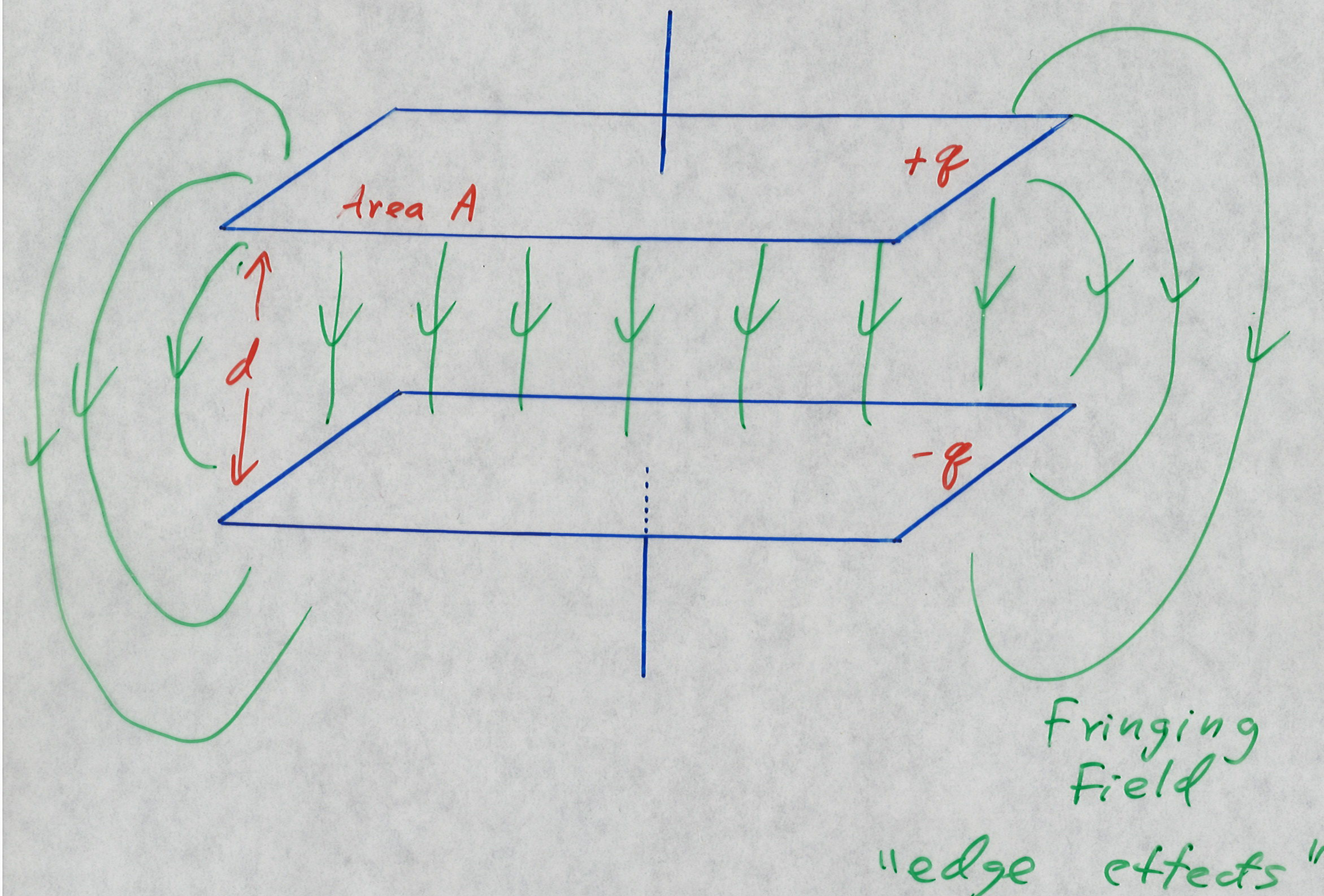
Graphical Symbol





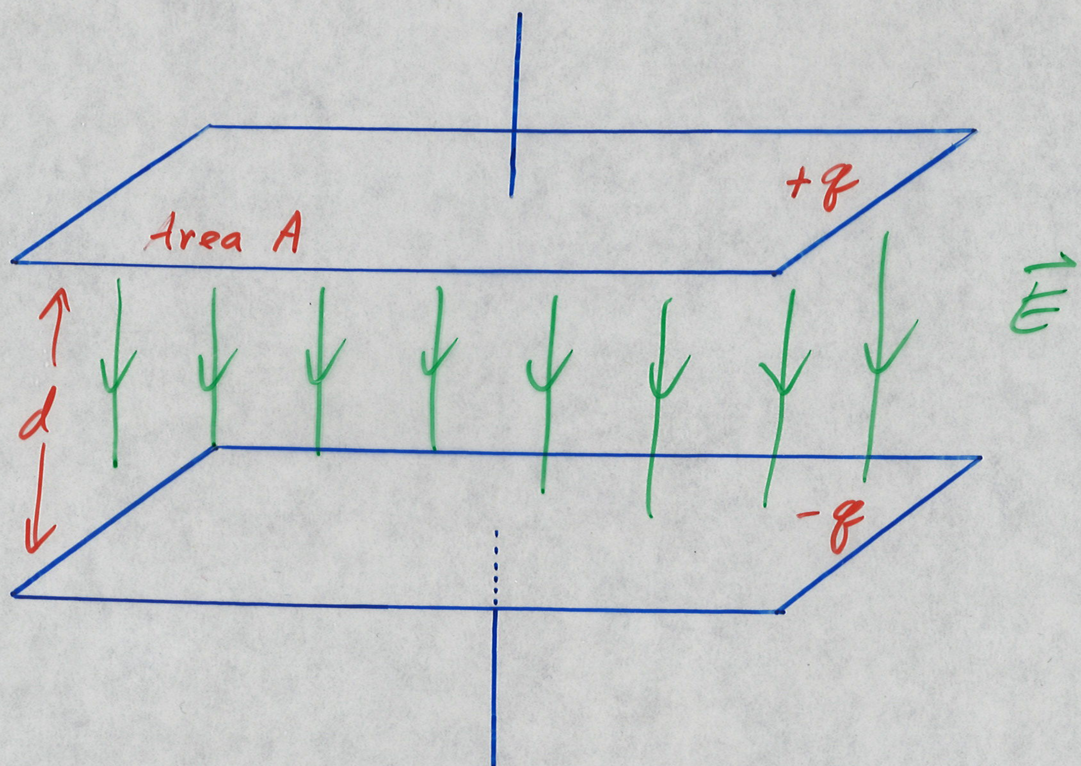


What is the electric field between the parallel plates of a capacitor?



What does the electric field look like?

What is the electric field between the parallel plates of a capacitor?

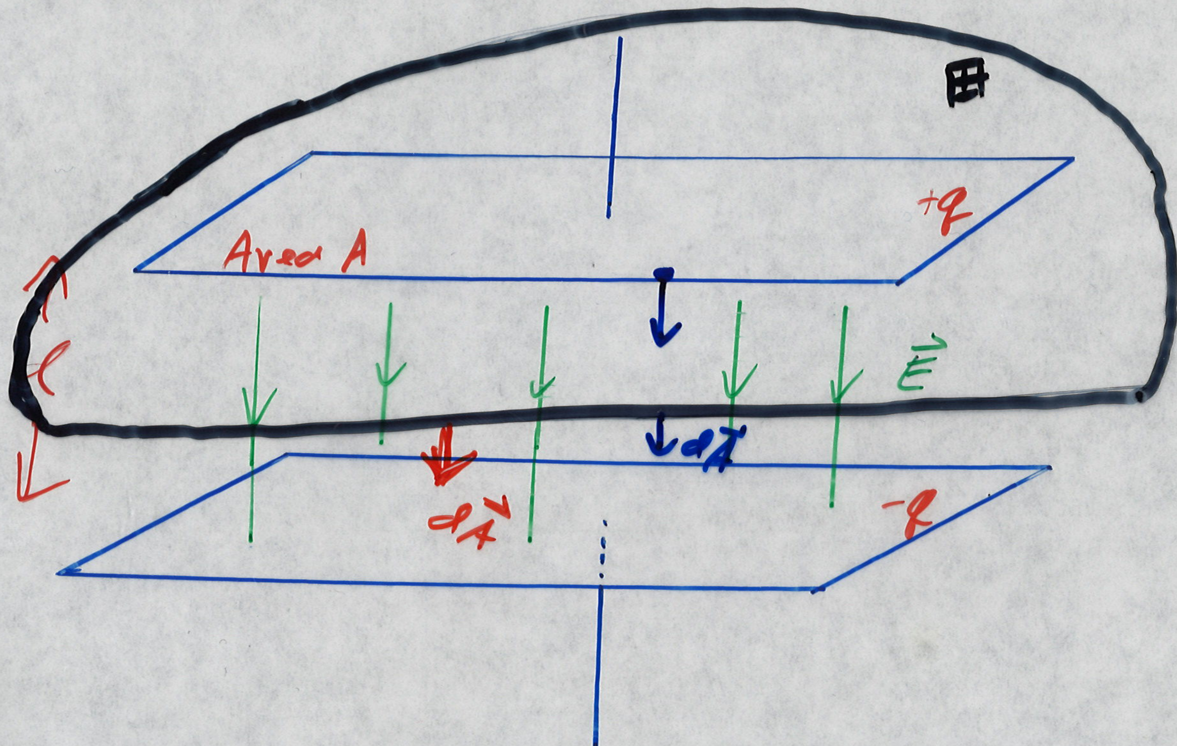


What does the electric field look like?

$|\vec{E}|$ = constant between the plates,

$\vec{E} = 0$ outside the plates.

How do we calculate the $\vec{E}$ field?



Choose a Gaussian surface around one of the plates. The surface shown here is a plane between the plates, and some arbitrary shape outside.

Gauss' Law: $\epsilon_0 \oiint \vec{E} \cdot d\vec{A} = Q_{inside} = +q$

$\vec{E} \parallel d\vec{A}$, $|\vec{E}| = \text{const}$ $\Bigg\}$ $\epsilon_0 E A = +q$

$$E = \frac{q}{\epsilon_0 A}$$

What is the voltage in terms of the electric field?

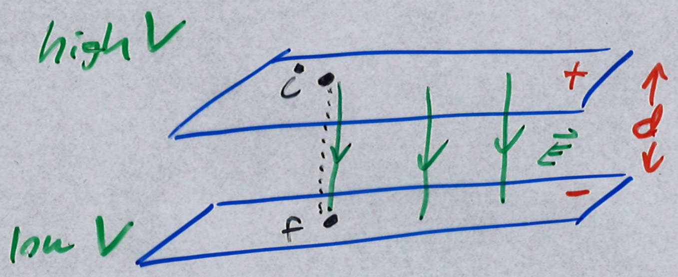
$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

i = some point on the + plate

f = some point on the − plate

We will choose the path of the line integral to follow an $\vec{E}$ field line.

Then $d\vec{s} \parallel \vec{E}$



The "V" in $q = CV$ is the absolute value of the potential difference ΔV.

$$V = |\Delta V| = \int_+^- \vec{E} \cdot d\vec{s} = \int_+^- E\,ds = Ed$$

$$E = \frac{q}{\epsilon_0 A} \qquad V = Ed$$

$$V = \frac{qd}{\epsilon_0 A} \quad \text{or} \quad q = \left(\frac{\epsilon_0 A}{d}\right) V$$

But $q = CV$

therefore $C_{\text{parallel plate}} = \dfrac{\epsilon_0 A}{d}$

geometry
A, d

Ex. What is the area of a 1 Farad capacitor made from two parallel plates separated by 1 mm?

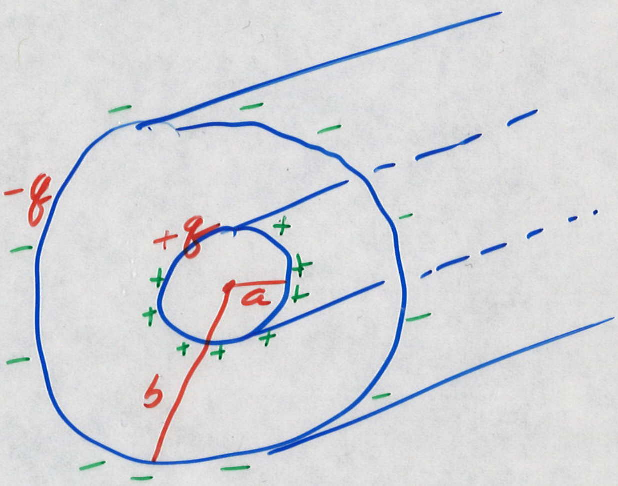
$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d} \qquad A = \frac{Cd}{\epsilon_0}$$

$$A = \frac{(1\ \text{Farad})(1\ \text{mm})\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)}{8.85 \times 10^{-12}\ \text{F/m}} = 1.13 \times 10^{8}\ \text{m}^2$$

$$A = 10\ \text{km} \times 10\ \text{km}$$

LS

Cylindrical Capacitor

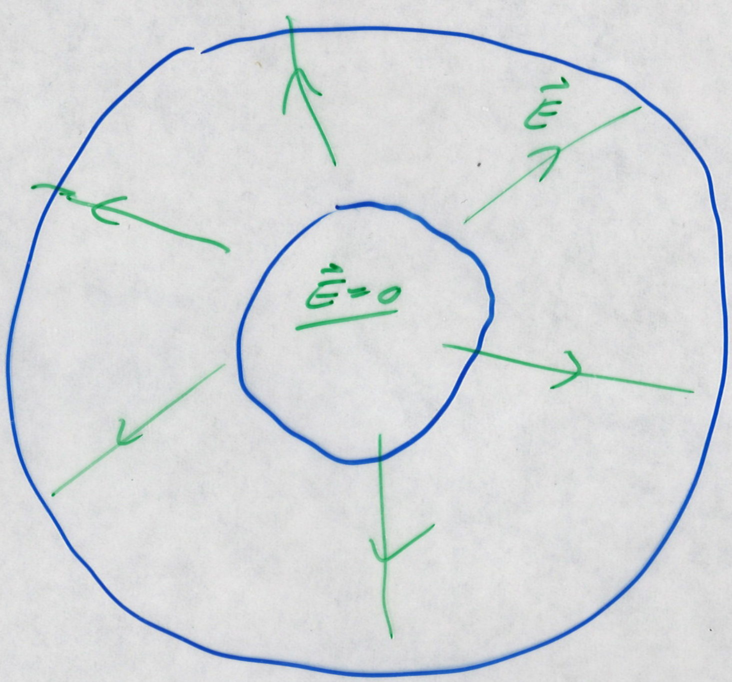


Two coaxial cylinders of radii a and b.

We assume the length L is large compared to a or b.

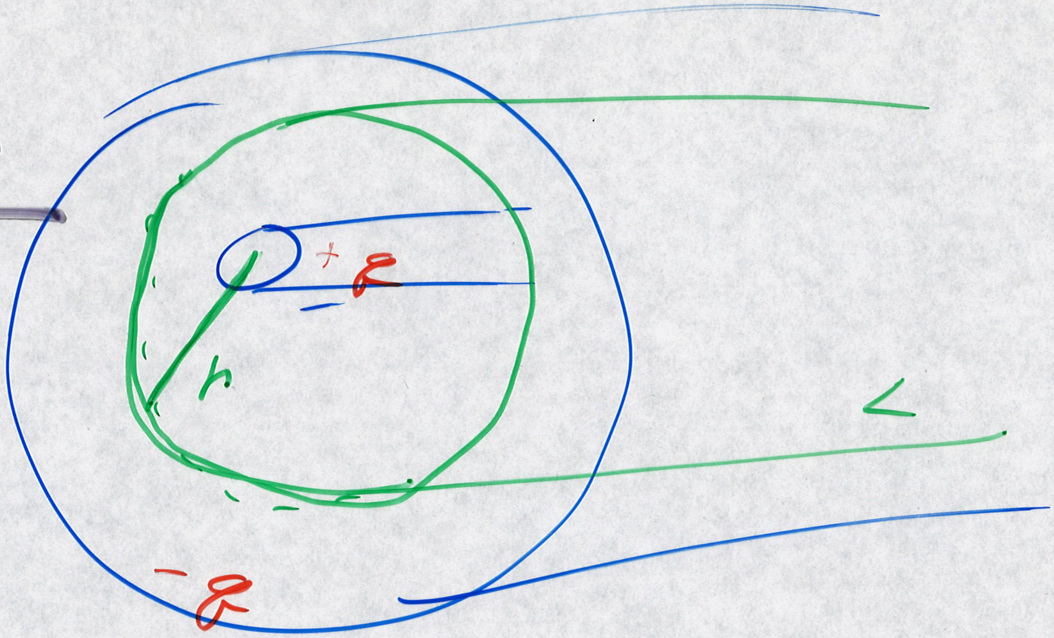
What is the electric field everywhere?

1. Find $\vec{E}$
2. Find ΔV
3. Find C

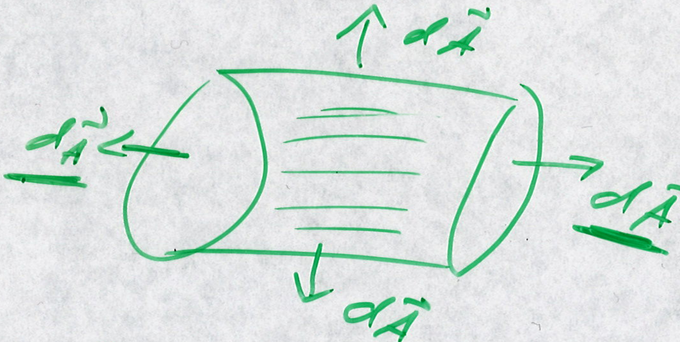


$\vec{E}=0$

$a<r<b$



Gauss' Law $\epsilon_0 \oiint \vec{E}\cdot d\vec{A} = Q_{inside} = +q$



$\epsilon_0 E(r)\, 2\pi r L = +q$

$\iint_{lateral} d\vec{A} = 2\pi r L$

$$E_{(r)} = \frac{q}{2\pi r L \epsilon_0}$$

$\Delta V = -\int_{+}^{-} \vec{E}\cdot d\vec{s}$ choose path to be a radius from a to b.

$$V = \int_a^b \frac{q}{2\pi r L\epsilon_0}\, dr = \frac{q}{2\pi L\epsilon_0}\int_a^b \frac{1}{r}\, dr$$

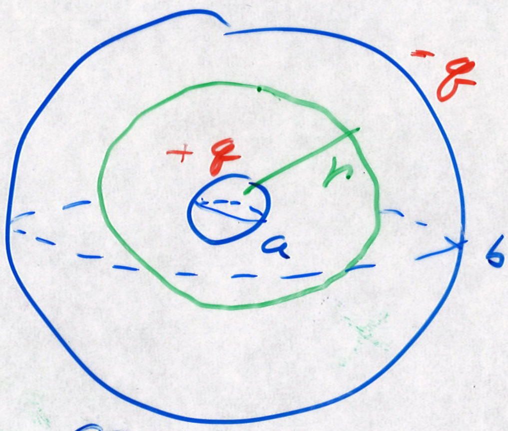
$$V = \frac{q}{2\pi L\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$\ln(b) - \ln(a) = \ln\left(\frac{b}{a}\right)$

$$q = CV \qquad C = \frac{q}{V}$$

$$\underline{C_{\text{cylind}}} = \frac{q}{\frac{q}{2\pi L \epsilon_0} \ln\left(\frac{b}{a}\right)} = \frac{2\pi L \epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

Spherical Capacitor.



1) find $E(r)$

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{A} = +q$$

$$\epsilon_0 E(r)\, 4\pi r^2 = +q$$

$$\oiint d\vec{A} = 4\pi r^2 \qquad E(r) = \frac{q}{4\pi \epsilon_0} \frac{1}{r^2}$$

$\underline{a < r < b}$

2) find ΔV

$$\Delta V = -\int_{+}^{-} \vec{E} \cdot d\vec{s} = -\int_a^b \frac{q}{4\pi \epsilon_0} \frac{1}{r^2}\, dr$$

If $\vec{E} \parallel d\vec{s}$, then $\vec{E} \cdot d\vec{s} = |E|\,|ds|$

$$V = |\Delta V| = \int_a^b \frac{q}{4\pi\epsilon_0} \frac{1}{r^2}\, dr$$

$$\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Bigg|_a^b = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$q = CV$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]} =$$

$$C_{spherical} = 4\pi\epsilon_0 \frac{1}{\frac{1}{a} - \frac{1}{b}}$$

$$= 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$$

Capacitance for a <u>single</u> sphere

$b \to \infty$

$$C_{\substack{single \\ sphere}} = \underline{\underline{4\pi\epsilon_0 a}}$$

radius = a

Capacitance of a head:

$$C_{\substack{\text{single}\\\text{sphere}}} = 4\pi\epsilon_0 r$$

$$= 4\pi\left(8.85\times10^{-12}\,\frac{F}{m}\right)(0.15\,m)$$

$$= 16.7\times10^{-12}\,F \quad = 16.7\,pF$$

Capacitance of the Earth:

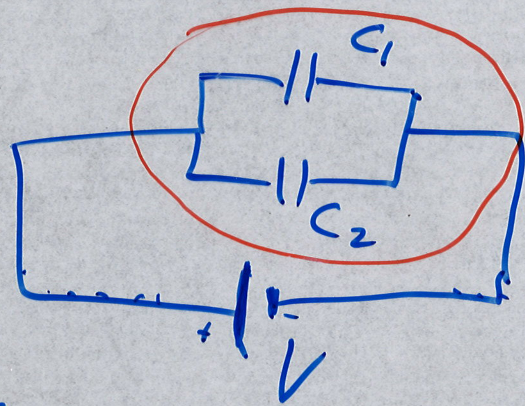
$$C_{\substack{\text{single}\\\text{sphere}}} = 4\pi\epsilon_0 R_E$$

$$= 4\pi\left(8.85\times10^{-12}\,\frac{F}{m}\right)(6.37\times10^{6}\,m)$$

$$= 7.08\times10^{-4}\,F \quad = 708\,\mu F$$

Equivalent Capacitance

Parallel



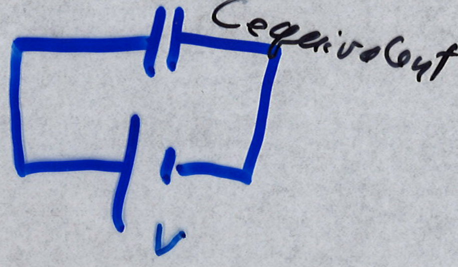
Potential across $C_1 = V$
Potential across $C_2 = V$
} meaning of "parallel"

$q_1 = C_1 V$ $\qquad$ $q_2 = C_2 V$

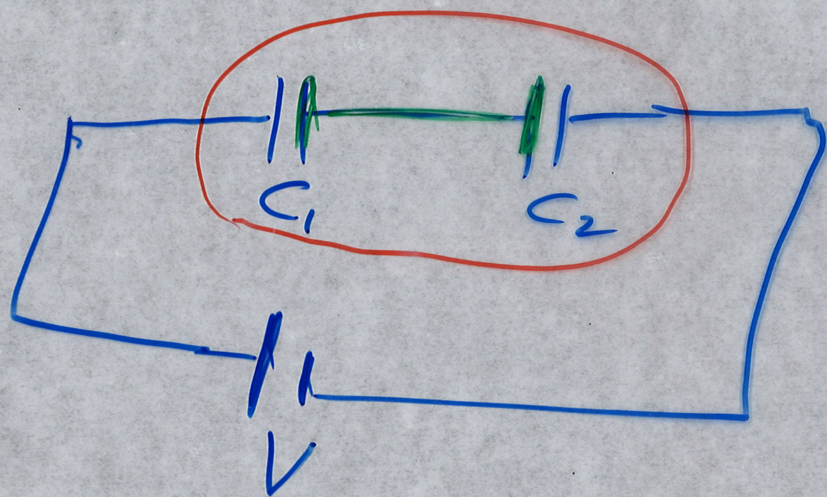
$q_{total} = q_1 + q_2 = C_{equiv} V$

$C_1 V + C_2 V = C_{equiv} V$

$C_{equiv\ parallel} = C_1 + C_2$



In Series



$q_1 = q_2 = q$

$q_1 = C_1 V_1$

$q_2 = C_2 V_2$

$V_1 = \frac{q}{C_1}$

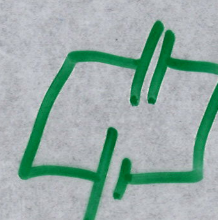
$V_2 = \frac{q}{C_2}$

Total Potential drop across circuit

$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$q = C_{equiv} V$

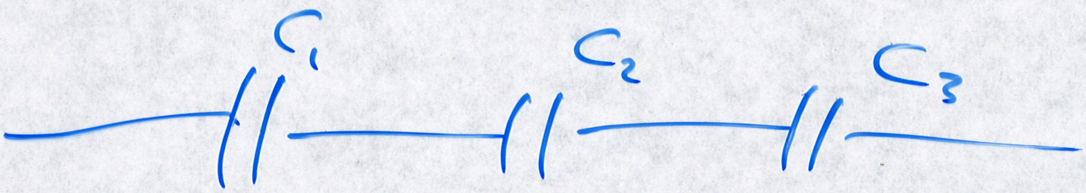
$q = \left[\left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}\right)\right] V$



$C_{equiv\ series} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$

$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C_{\substack{equivalent\\series}} = \frac{C_1 C_2}{C_1 + C_2}$$



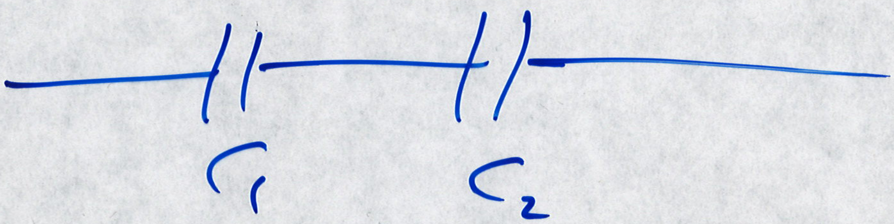
$$C_{equiv} = \frac{C_1 C_2 C_3}{C_1 + C_2 + C_3}$$ ✗

$$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$ ✓

$$C_{\substack{parallel\\plate}} = \frac{\epsilon_0 A}{d}$$

$C_1 = 1\ nF$

$C_2 = 1\ nF$



$$C_{\substack{equiv\\ series}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10^{-9}F)(10^{-9}F)}{10^{-9}F + 10^{-9}F}$$

$$= \frac{10^{-18}F^2}{2 \times 10^{-9}F} = \frac{1}{2} \times 10^{-9}F$$

$$= 0.5\ nF$$

$C_{\substack{equiv\\ series}} <$ either C_1 or C_2

Energy Stored in a Capacitor

It takes <u>energy</u> to separate the charges on the plates of a capacitor.

Suppose that I have already transferred charge q from one plate to the other

And suppose that the potential difference (the voltage) is V.

$$V = \frac{q}{C}$$

more charge
⇒ higher voltage

How much work must be done on an infinitesimal piece of charge dq to move it from one plate to the other?

$$dW = V\,dq = \left(\frac{q}{C}\,dq\right)$$

The *total* work required to charge the capacitor from $q = 0$ to $q = Q$ is:

$$W = \int dW = \int_{q=0}^{Q} \left(\frac{q}{C}\,dq\right) = \boxed{\frac{Q^2}{2C}}$$

$$\frac{Q^2}{2C}$$

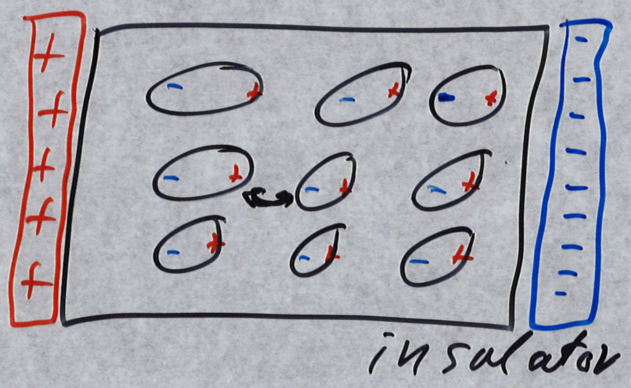
$Q = CV$

$$\boxed{W = \frac{(CV)^2}{2C} = \frac{1}{2} C V^2} = \frac{QV}{2}$$

energy stored in a capacitor

$$W = \frac{1}{2}\frac{Q^2}{C}$$

Dielectric κ (kappa)



Leiden Jar

or

Dissectible
Capacitor

1) How does it work?
2) Why doesn't the dielectric depolarize in your hands?