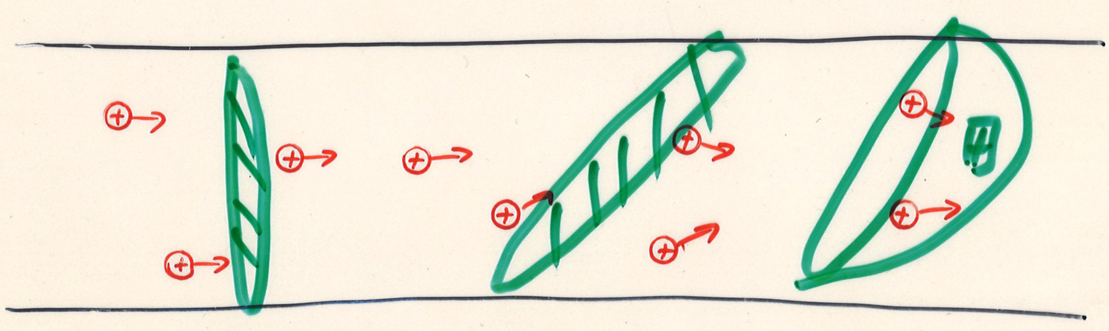
Current & Resistance

Until now, we have been studying Electrostatics (charges at rest).

<u>Current</u>: the rate at which charge moves past a hypothetical plane.

$$\boxed{I(t) = \frac{dq}{dt}}$$



$$q = \int_{t_i}^{t_f} I(t)\, dt$$

MKS Unit

The unit of current is the ampere (A).
This is one of the fundamental set

$$\{\underset{L}{\text{meter}},\ \underset{M}{\text{kilogram}},\ \underset{T}{\text{second}},\ \underset{\text{current}}{\text{ampere}}\}$$

$1A = 1\frac{C}{s}$

Steady State

The current is <u>not</u> a function of time — it is constant.

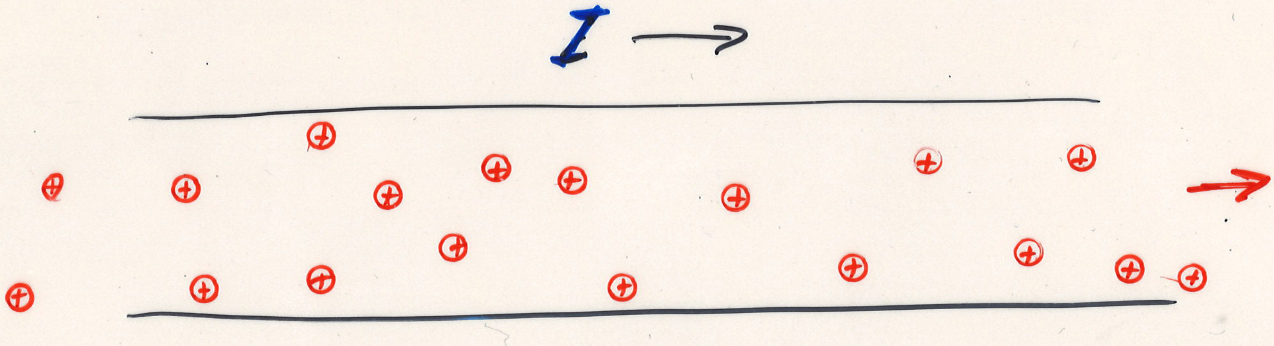
$$\boxed{I = \text{constant}}$$

Under steady state conditions, charge cannot "pile up" in the wire.

Direction of Current

Current (I) is a <u>scalar</u>, but there is an associated <u>direction</u>.

Current is defined by convention to flow in the direction that <u>positive</u> charges would move even if the moving charges are negative!



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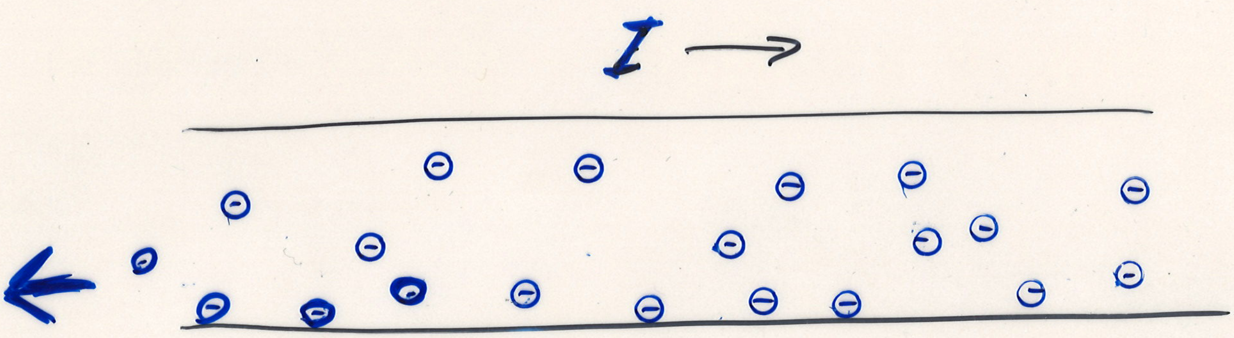
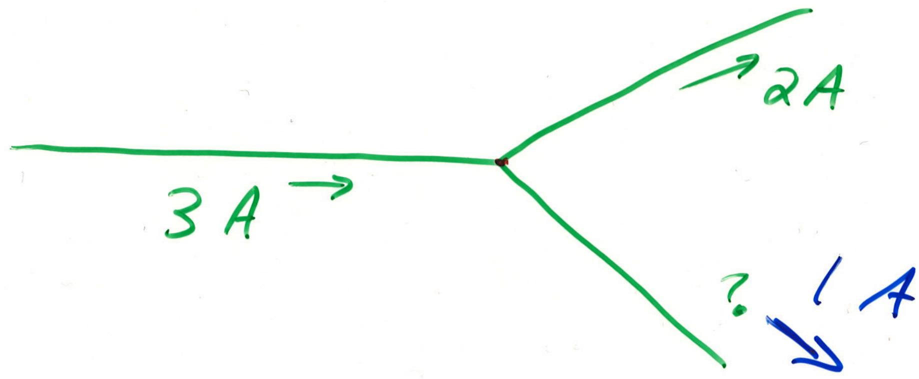
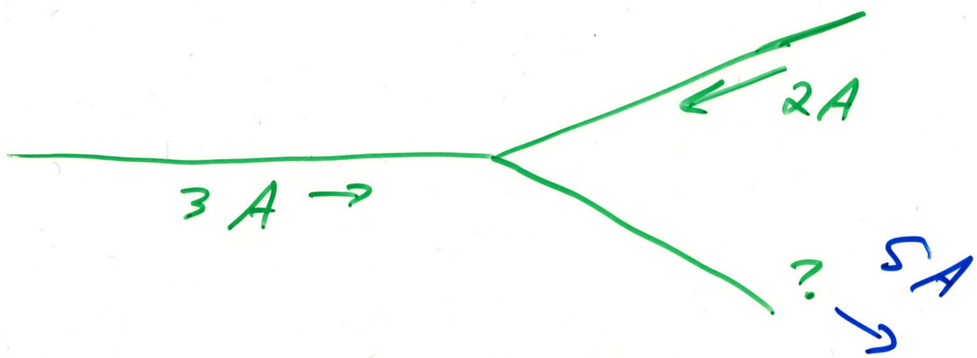


current in = current out

Ex



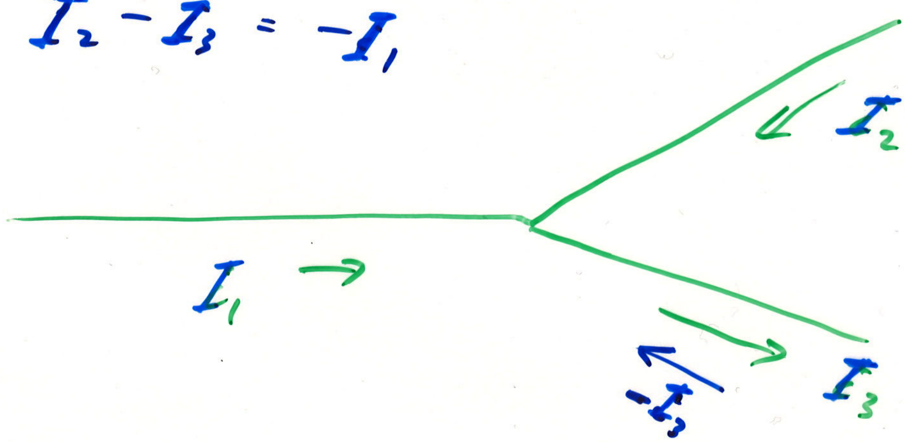
$3A + 2A + (-5A) = 0$ ✓



$I_1 + I_2 = I_3$

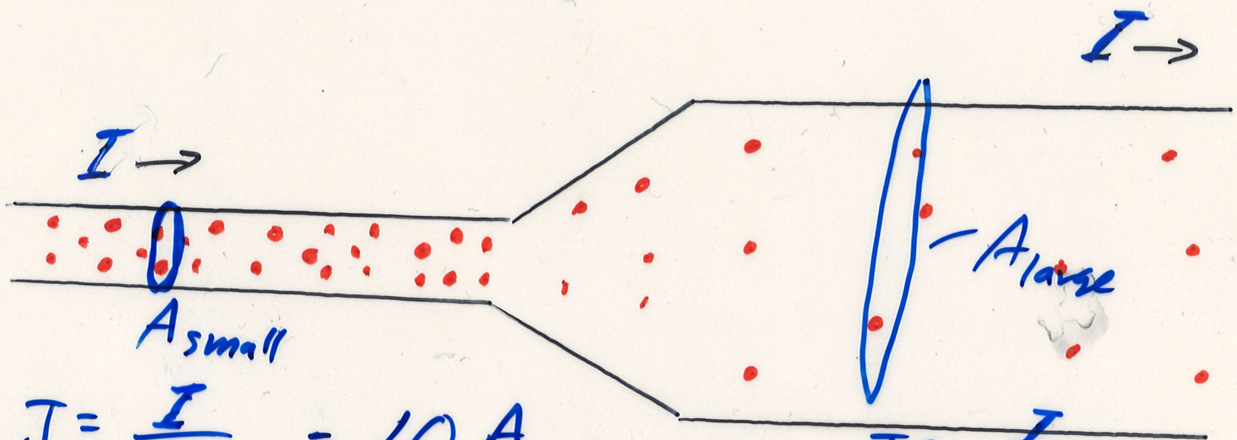
$I_2 - I_3 = -I_1$

$I_1 + I_2 + (-I_3) = 0$



Steady state current conservation is a consequence of charge conservation.

Current Density



$$J = \frac{I}{A_{small}} = 10 \frac{A}{m^2} \qquad J = \frac{I}{A_{large}} = 1 \frac{A}{m^2}$$

Current Density : $J \equiv \dfrac{I}{Area}$

(magnitude)

$\vec{J}$ is a <u>vector</u> quantity.

The direction of $\vec{J}$ is the same as that of the electric field $\vec{E}$, regardless of the sign of the charge carriers.

Whoa!

What electric field???

Something must cause the moving charges to move: An electric field in the conductor.

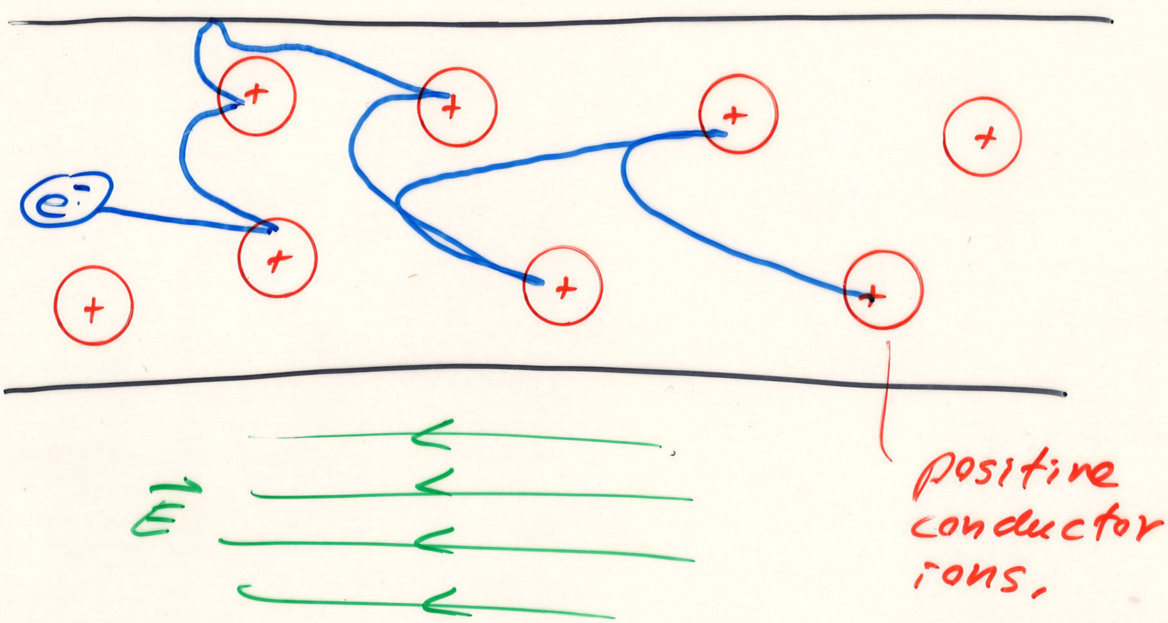
I thought $\vec{E} = 0$ inside a conductor.

This is true for electro<u>statics</u>.

Now we are considering charges in motion: electro<u>dynamics</u>.

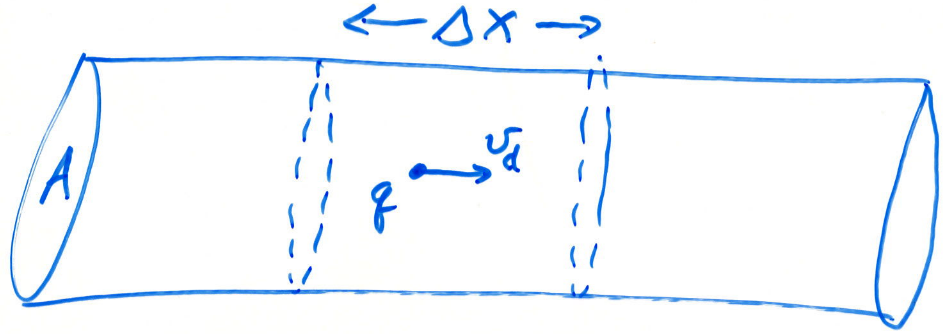
Doesn't an electric field cause charges to <u>accelerate</u>, so the current (I) will not be a <u>constant</u> but will increase with time?

An electric field would cause **free** charges to accelerate. In a conductor (like a wire), the charges accelerate for a **very** short time (10^{-14} seconds) then collide with atoms in the conductor, scatter, and accelerate again, ...



Electric field supplied by battery

These are massive and fixed in a crystal lattice.



$$\Delta x = v_d \Delta t \qquad V = A \Delta x$$

n = number of mobile charge carriers (conduction electrons, not positive metal ions) per unit volume.

ΔQ = (number of mobile charge carriers in slice of width Δx) · (charge of one carrier)

$$\Delta Q = nVq = nA\Delta x q = nAv_d \Delta t q$$

Current $I = \frac{\Delta Q}{\Delta t} = nAv_d q$

Current Density

$$J = \frac{I}{A} = n v_d q \qquad \vec{J} = n \vec{v}_d q$$

In <u>some</u> materials, the current density in proportional to the applied electric field.

$$\vec{J} = \sigma \vec{E}$$

The constant of proportionality is called

the conductivity : σ $\boxed{\frac{1}{\sigma} \equiv \rho}$

resistivity

$\vec{J} = \sigma \vec{E}$ is one way to express Ohm's Law.

$$E = \frac{\Delta V}{\Delta \ell} = \frac{V}{\ell} \Rightarrow V = E\ell = \frac{J\ell}{\sigma} = \frac{I\ell}{A\sigma} = IR$$

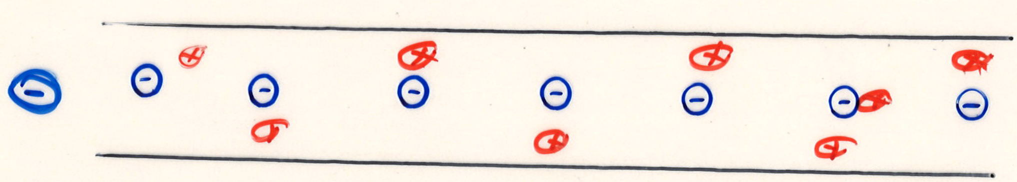
$$R = \left(\frac{\ell}{A\sigma}\right) = \frac{\rho \ell}{A}$$

The result of the scattering and acceleration is that electrons move with a constant average velocity called the "drift velocity."

$$\text{Typically}, \ |\vec{v}_{drift}| = 10 \frac{cm}{hour}$$

A snail could race an electron and win!

So why doesn't it take a week to turn the lights on?



The speed of the "push" is almost the speed of light.

Resistance

It is an experimentally observed fact that for most (not all!) conductors the current is directly proportional to the potential difference across the conductor.

$$V = I R$$

Ohm's Law

R = constant

The constant of proportionality is called the resistance.

MKS Unit

$$1 \text{ ohm } (\Omega) \equiv 1 \frac{V}{A} \left(\frac{\text{volt}}{\text{ampere}}\right)$$

$$V = \frac{J}{C} = \frac{\text{joule}}{\text{coulomb}}$$

Ohm's Law $\vec{J} = \sigma \vec{E}$

$V = IR$ ← definition of resistance.

V is proportional to R ✗

I is inversely proportional to R ✗

V is proportional to I ✓

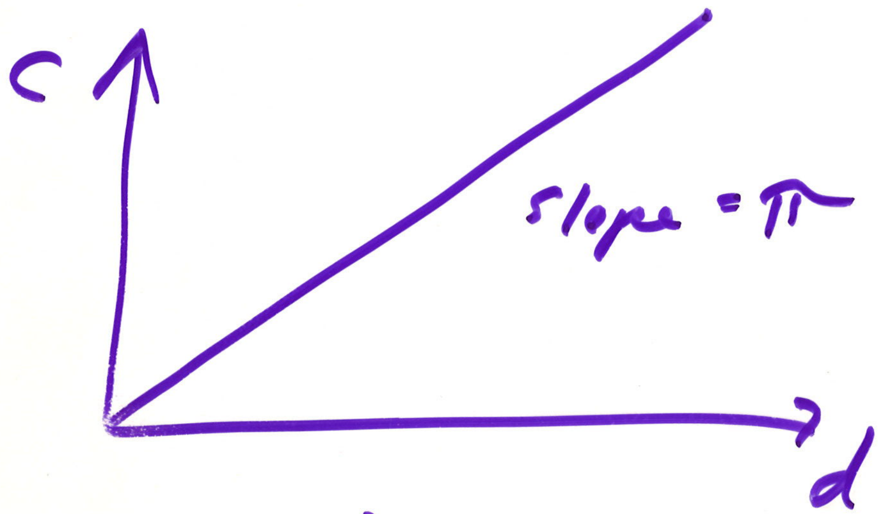
R is a constant

↑ Ohmic

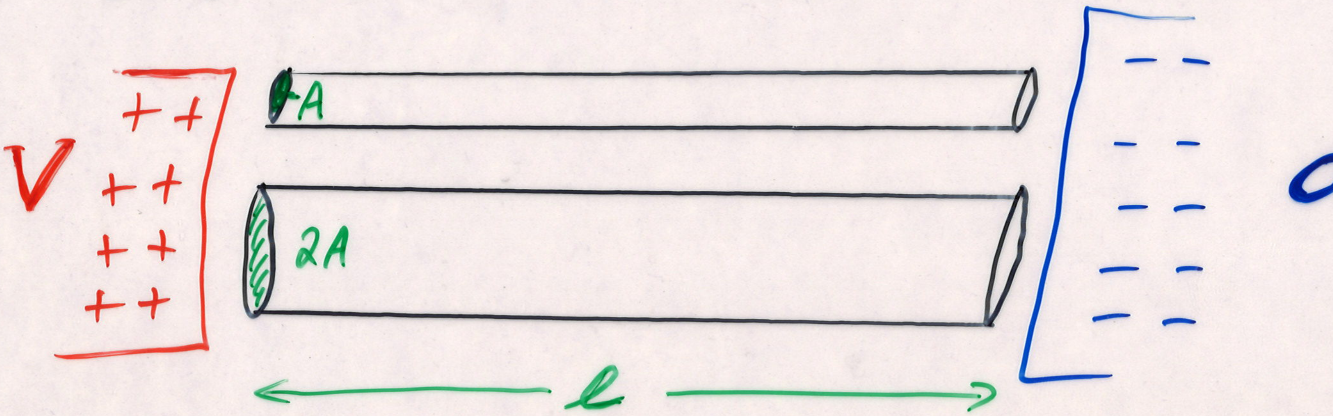
$$C = \pi d$$

C is proportional to d

C is not proportional to π



I can decrease the resistance of a conductor by <u>increasing</u> its cross-sectional area.



<u>Warning</u>: increasing the radius of a cylindrical wire by a factor of 2 increases the cross-sectional area by a factor of 4. ($A = \pi r^2$)

I can also decrease the resistance of a conductor by <u>decreasing</u> its length.

The same voltage V applied over a shorter distance gives rise to a larger electric field: $E = \frac{V}{\ell}$

While individual conductors are characterized by their resistance, the material from which the conductor is made is characterized by its resistivity (ρ).

$$R = \rho \frac{\ell}{A}$$

$$\rho_{copper} = 1.69 \times 10^{-8} \ \Omega \cdot m$$

$$\rho_{iron} = 9.68 \times 10^{-8} \ \Omega \cdot m$$

Power

Dissipated in electric circuits

Resistance is a "lossy" effect, like friction. Electric potential energy (in the battery or capacitor) and kinetic energy (of the moving charges) is converted into <u>heat</u> energy.

$$dU = dq\, V = (I\, dt)\, V$$

$$\frac{dU}{dt} = \boxed{P = IV}$$

The MKS unit of power is the watt (W)

$1\,W = 1\,\frac{J}{s} = 1\,V\cdot A$

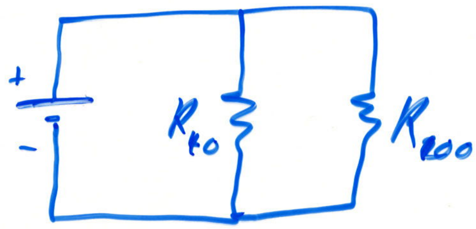
Using Ohm's Law: $V = IR$

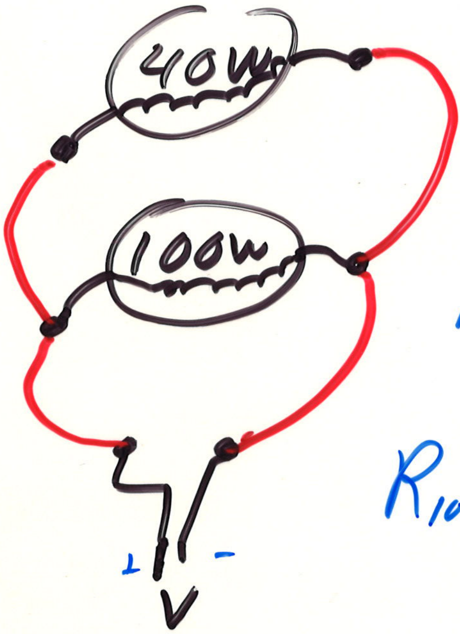
$$\boxed{P = I^2 R}$$

$$\boxed{P = \frac{V^2}{R}}$$

Consider a 40W and a 100W lightbulbs in parallel:

$$V = IR$$
$$P = IV$$
$$P = I^2R$$
$$P = \frac{V^2}{R} \checkmark$$





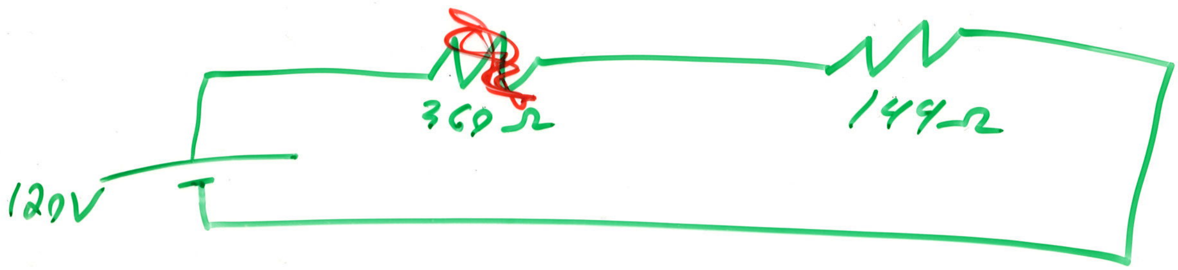
$$R_{40} = \frac{V^2}{P_{40}} = \frac{(120V)^2}{40W} = 360\,\Omega$$

$$R_{100} = \frac{V^2}{P_{100}} = \frac{(120V)^2}{100W} = 144\,\Omega$$

$$P_{40} = \frac{V^2}{R_{40}} = 40W$$

$$P_{100} = \frac{V^2}{R_{100}} = 100W$$

Consider a 40W and a 100W lightbulb in <u>series</u>:



same current I (meaning of series)

Find I:

$$V = I(R_{equiv}) = I(360\Omega + 144\Omega)$$

$$I = \frac{120V}{360\Omega + 144\Omega} = 0.238A$$

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brighter

$$P_{40} = I^2 R_{40} = (0.238A)^2(360\Omega) = 20.4W$$

$$P_{100} = I^2 R_{100} = (0.238A)^2(144\Omega) = 8.2W$$