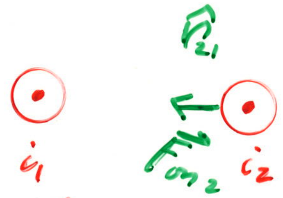
From Experiment:

The force due to current i_1 on i_2 in a wire of length ℓ is

$$\vec{F}_{\text{of } 1 \text{ on } 2} = \left(\frac{\mu_0}{4\pi}\right) \frac{2\,\ell\, i_1\, i_2 \cos\theta}{r_{12}}\, \hat{r}_{21}$$

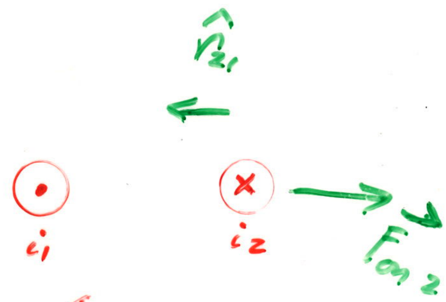
θ is the angle between i_1 and i_2.

$\hat{r}_{21}$ is a unit vector from i_2 to i_1.



$\theta = 0$

$\cos\theta = 1$



$\theta = 180°$

$\cos\theta = -1$

<u>attractive</u> if i_1 and i_2 are <u>parallel</u>.

<u>repulsive</u> if i_1 and i_2 are <u>antiparallel</u>.

<u>zero</u> if i_1 and i_2 are <u>perpendicular</u>.

The constant

$$\mu_0 \equiv 4\pi \times 10^{-7}\ T \cdot m/A$$

is called the <u>permeability of free space.</u>

and "T" stands for "tesla",

the MKS unit of magnetic field $\vec{B}$.

$$1\,T = 1\,\frac{N}{A \cdot m}$$

The electric field unit does not have a special name. The MKS unit of $\vec{E}$ is

$$\frac{V}{m} = \frac{N}{C}$$

$$\boxed{1\,V = \frac{1\,J}{C}}$$

From the last chapter, the force on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is:

$$\vec{F}_m = q\,\vec{v} \times \vec{B}$$

Can we reconcile this with:

$$\vec{F}_{\text{of 1 on 2}} = \frac{\mu_0}{4\pi} \frac{2\ell\, i_1 i_2 \cos\theta}{r_2} \hat{r}_{21} \quad ?$$

- if current i_2 flows for time T, then charge $q_2 = i_2 T$ has passed by.

- if the charge q_2 moves with speed v, then it flows a distance $\ell = vT$.

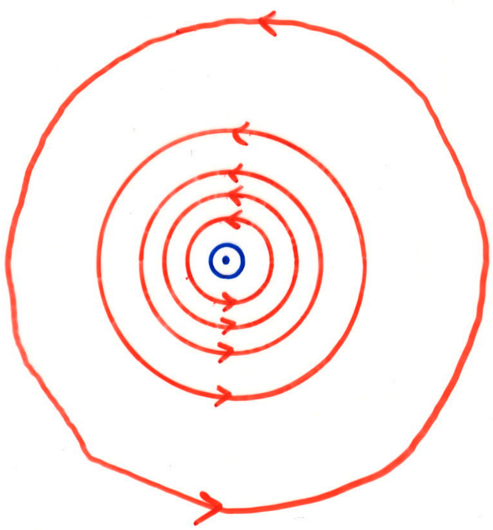
$$\vec{F}_{\text{of 1 on 2}} = \frac{\mu_0}{4\pi} \frac{2\,(v\cancel{T})\, i_1 \left(\frac{q_2}{\cancel{T}}\right) \cos\theta}{r_{12}} \hat{r}_{21}$$

So the magnetic field due to current i_1 in a straight wire is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2 i_1}{r} \qquad \text{(magnitude)}$$

How about direction ?

To recover the experimental laws of attraction and repulsion for parallel and antiparallel currents, the $\vec{B}$ field must look like:



- more dense (stronger $\vec{B}$ field) close to the wire
- right hand rule
- lines of $\vec{B}$ never end (no magnetic charges - monopoles)

$$\vec{F}_{\text{of 1 on 2}} = i_2 \vec{\ell}_2 \times \vec{B}_1 \qquad \vec{F}_{\text{of 1 on 2}} = q_2 \vec{E}_1$$

$$\vec{F}_{\text{of 2 on 1}} = i_1 \vec{\ell}_1 \times \vec{B}_2 \qquad \vec{F}_{\text{of 2 on 1}} = q_1 \vec{E}_2$$

What if the wire is <u>not</u> straight?

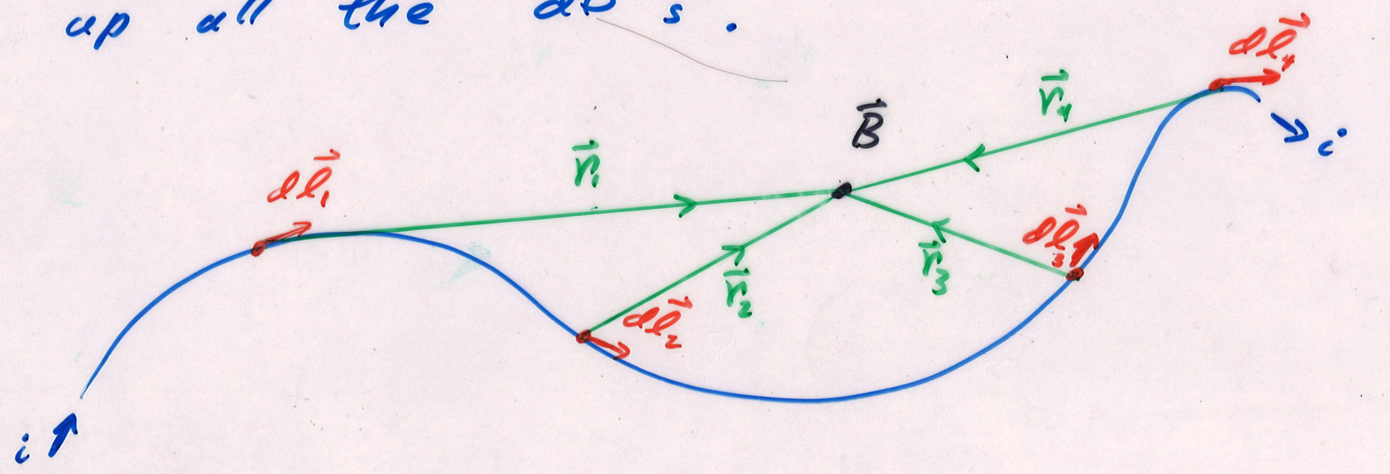
Then the piece of the magnetic field due to an infinitesimal chunk of wire (of length $d\ell$) is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell} \times \vec{r}}{r^3}$$

$\vec{r}$ points <u>from</u> the current element <u>to</u> the field point.

(Biot-Savart Law)

To get the total $\vec{B}$ field, simply integrate along the wire and add up all the $d\vec{B}$'s.



What if the wire is <u>not</u> straight?

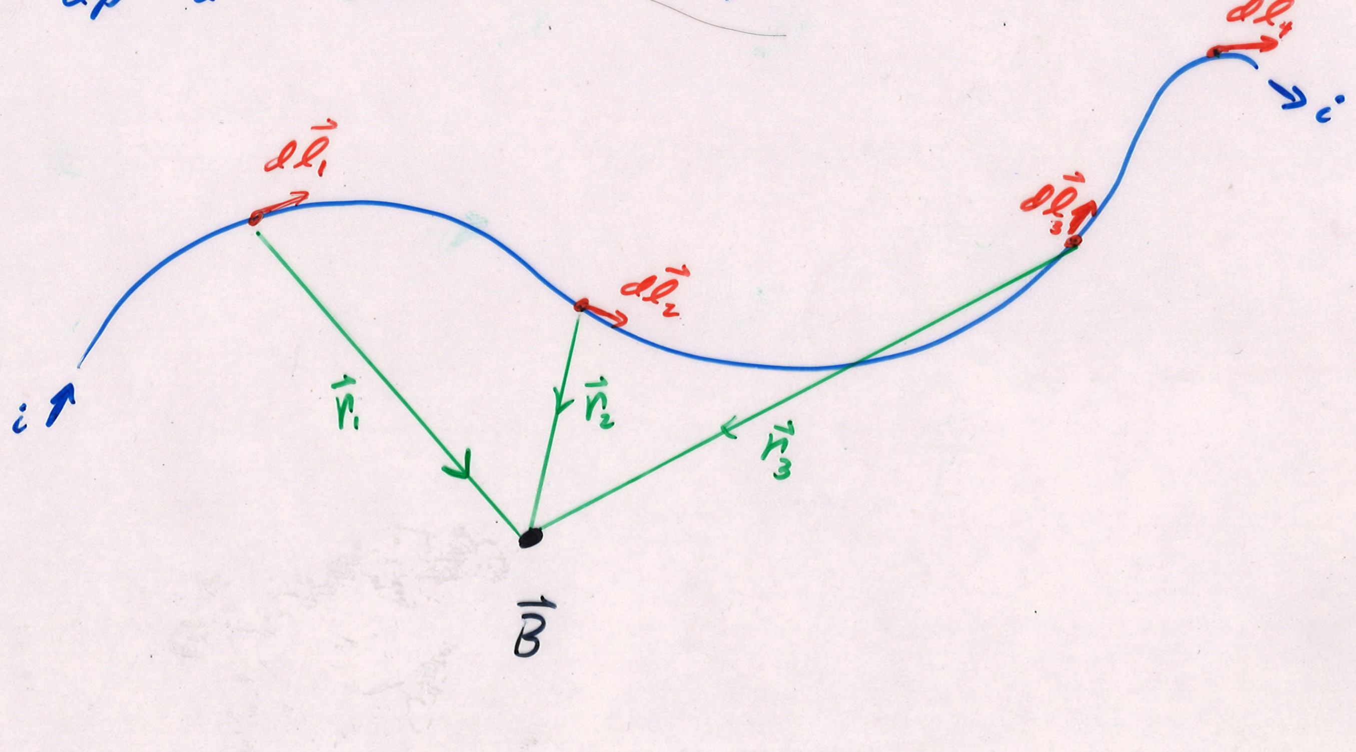
Then the piece of the magnetic field due to an infinitesimal chunk of wire (of length dl) is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i \, d\vec{l} \times \vec{r}}{r^3}$$

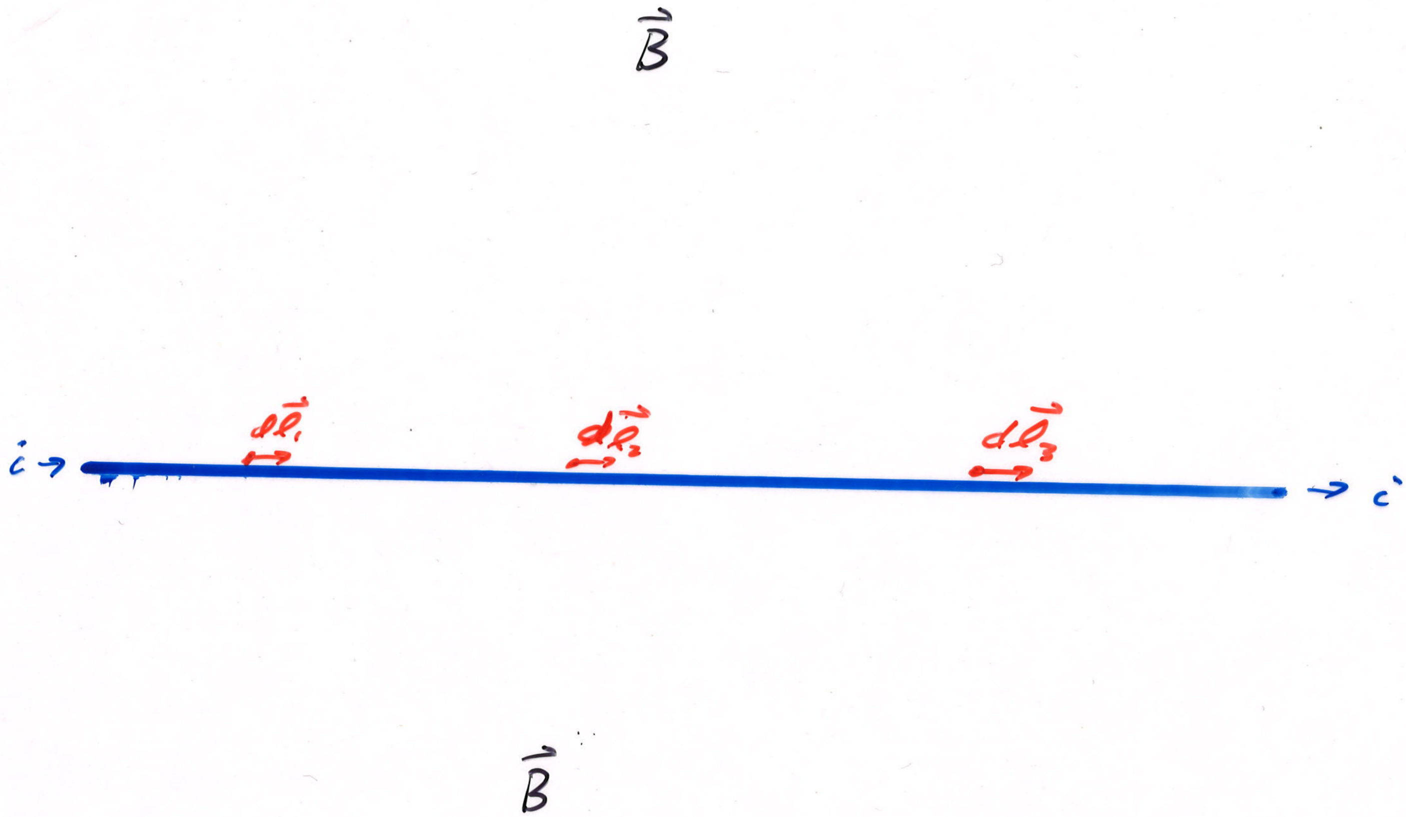
$\vec{r}$ points <u>from</u> the current element <u>to</u> the field point.

(Biot-Savart Law)

To get the total $\vec{B}$ field, simply integrate along the wire and add up all the $d\vec{B}$'s.



Does this work for a straight wire?



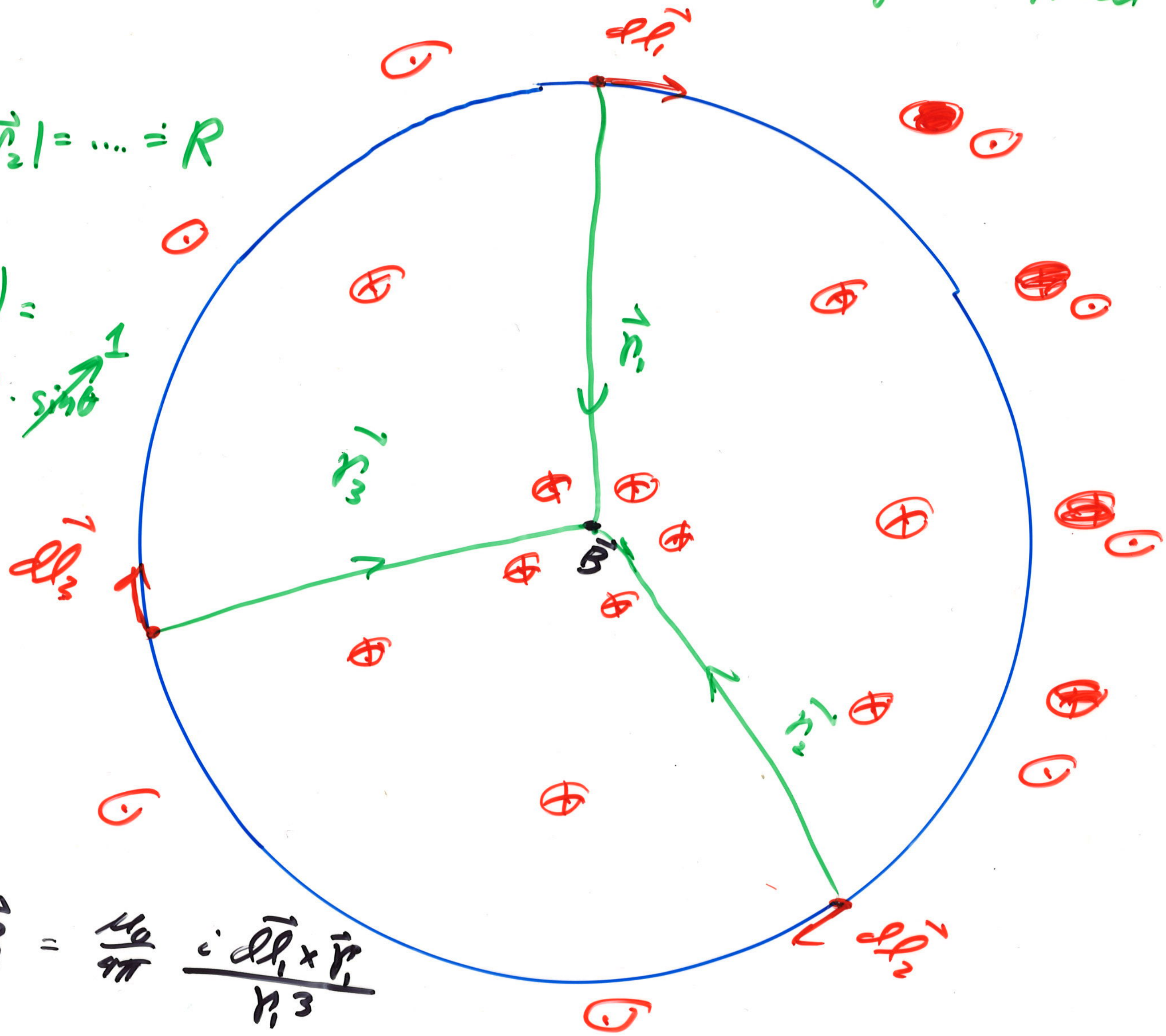
See HRW pages 850-1 for a proof.

$\vec{B}$ field at center of circle of current

$|\vec{r}_1| = |\vec{r}_2| = \dots = R$

$|d\vec{l} \times \vec{R}| =$

$dl \cdot R \cdot \cancel{\sin\theta}^{\,1}$



$$d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l}_1 \times \vec{r}_1}{r_1^{\,3}}$$

$$|\vec{B}| = \left|\int d\vec{B}\right| = \int \frac{\mu_0}{4\pi} \frac{i\, dl \cdot R}{R^3}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R^2}\left(\int dl\right) = \frac{\mu_0}{4\pi} \frac{i}{R^2}(2\pi R) = \boxed{\frac{\mu_0}{4\pi} \frac{2i\pi}{R}}$$

direction is $\boxed{\text{in}}$

Mechanics $\vec{F} = m\vec{a}$

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

Gauss' Law (for electricity)

$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

outward normal vector

evaluated on S

permittivity of free space.

closed surface

S = gaussian surface

Gauss' Law (for magnetism)

$$\oint\!\!\!\oint_S \vec{B}\cdot d\vec{A} = \mu_0 \cancel{Q_{mag}}^{\,0} = 0$$

$\vec{B}$ lines are closed loops

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Electricity

useful with a lot of symmetry

Coulomb's

$$\vec{F}_{12} = \frac{k q_1 q_2}{(r_{12})^2} \hat{r}_{12}$$

$$\oint\!\!\!\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

Gauss' Law

point charges

symmetry

Magnetism

Ampere's Law

$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Biot-Savart

Ampere's Law

infinitesimal line element tangent to the curve C

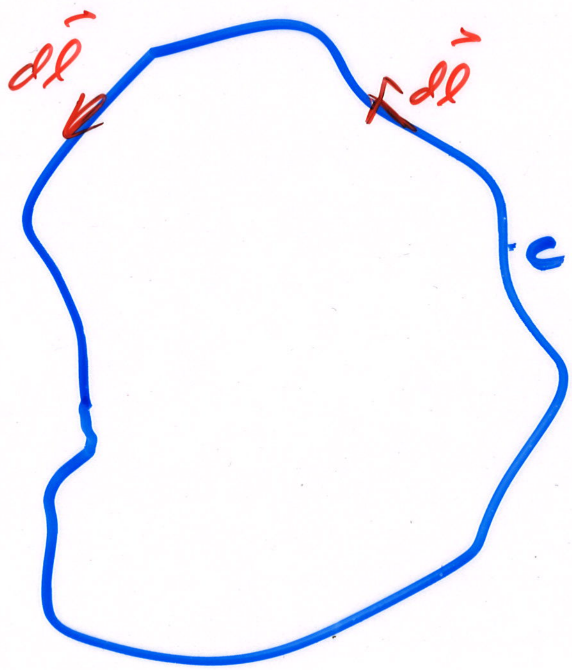
$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0\, i_{enc}$$

closed curve C

Amperean Loop.

evaluated on C

current captured by C



Ampere's Law

infinitesimal line element
tangent to the curve C

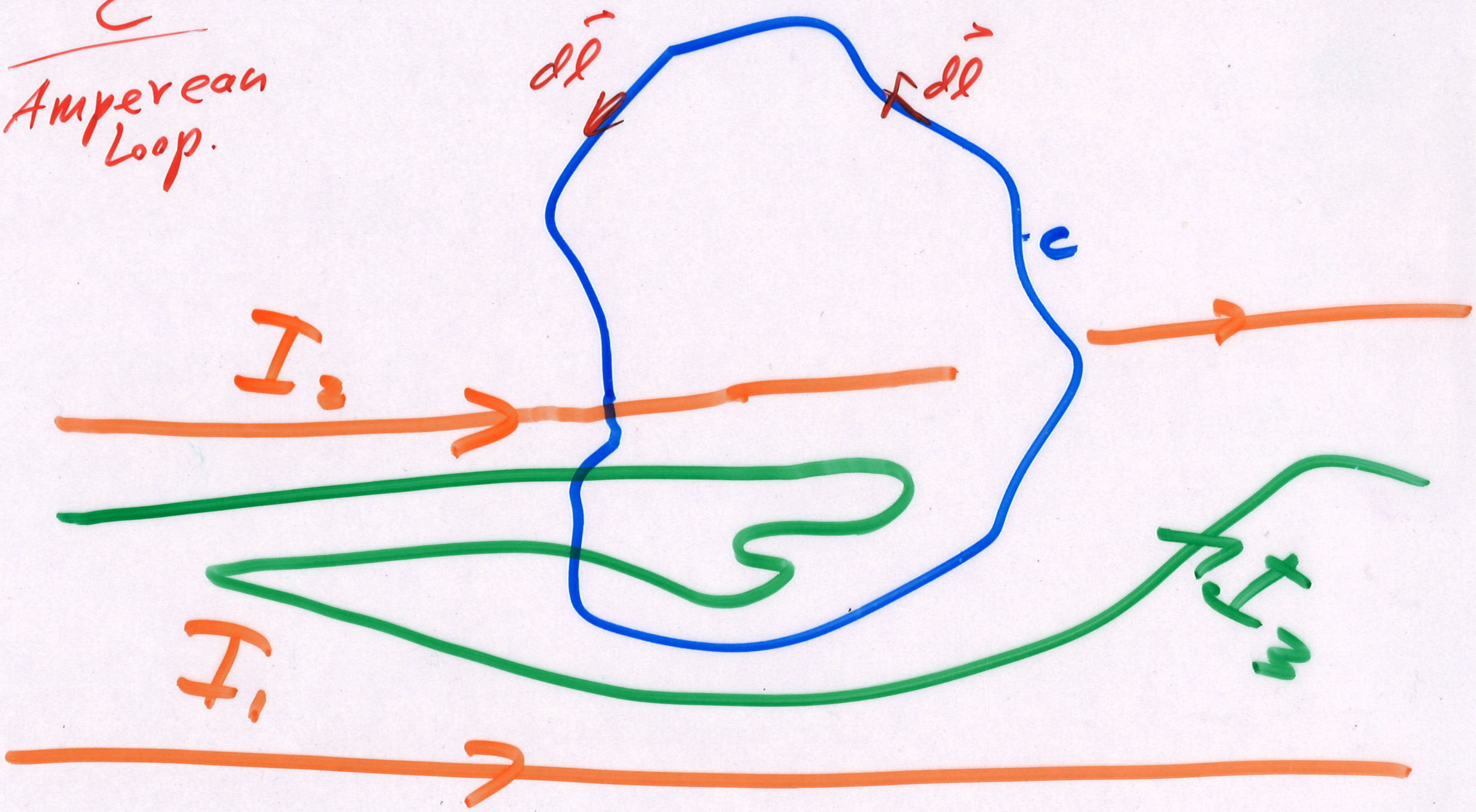
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enc}$$

evaluated
on C

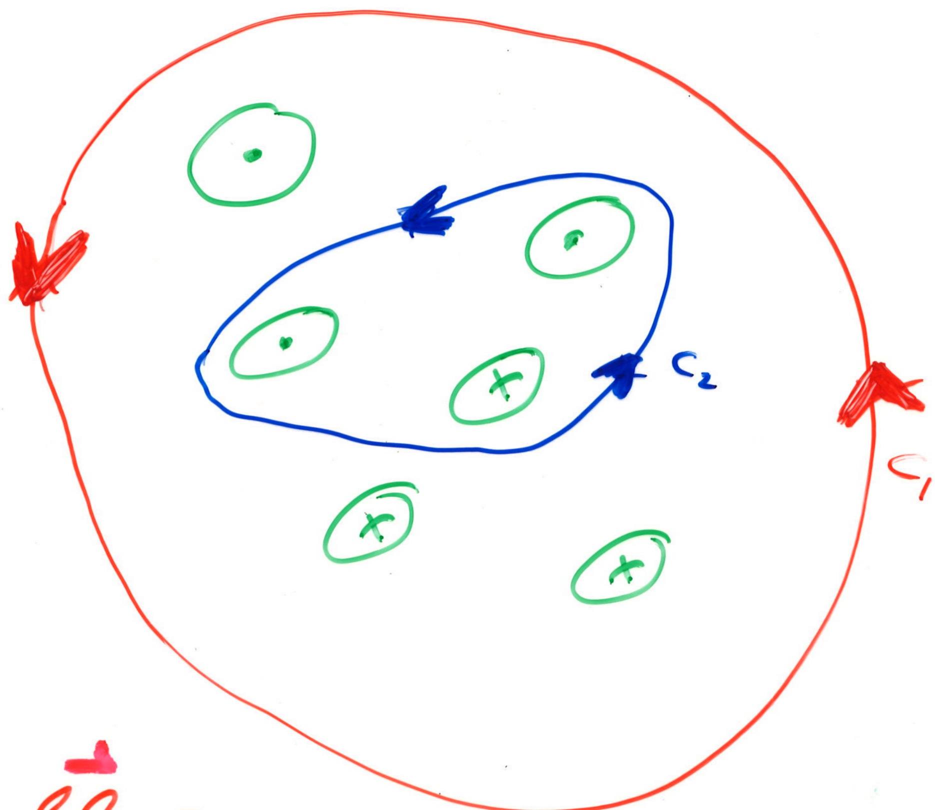
current captured
by C

closed
curve
C

Amperean
Loop.



Ex



$$\oint_{C_1} \vec{B} \cdot d\vec{l} = 0$$

$$\oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 (I + I - I)$$

If the direction of C_2 is reversed then $i_{enc} = -I = (-I - I + I)$

Ex.

A straight wire of radius R carries a current I distributed uniformly over its cross-sectional area. Find the magnetic field everywhere.