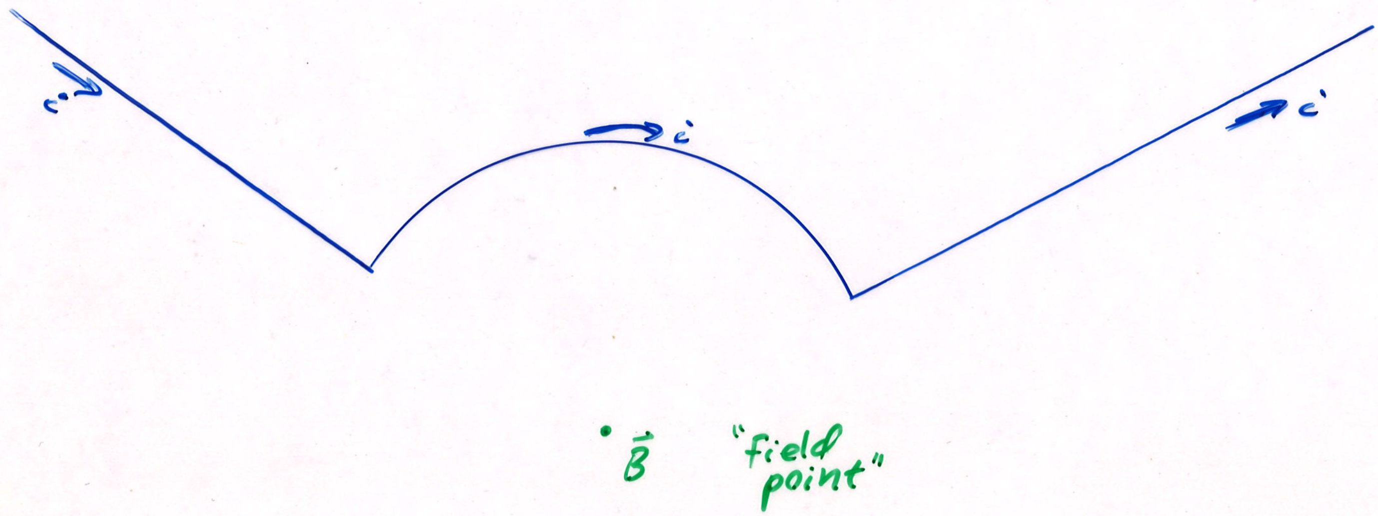
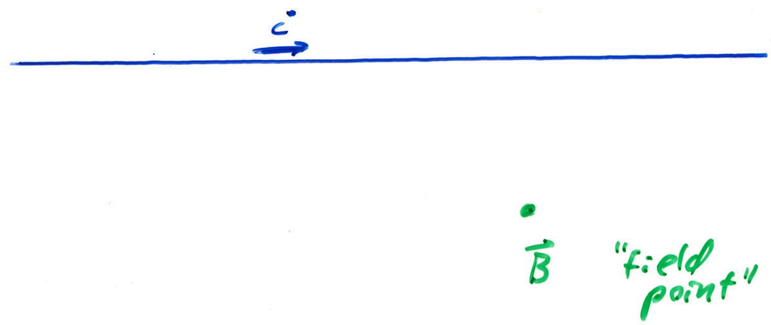
Biot-Savart Examples

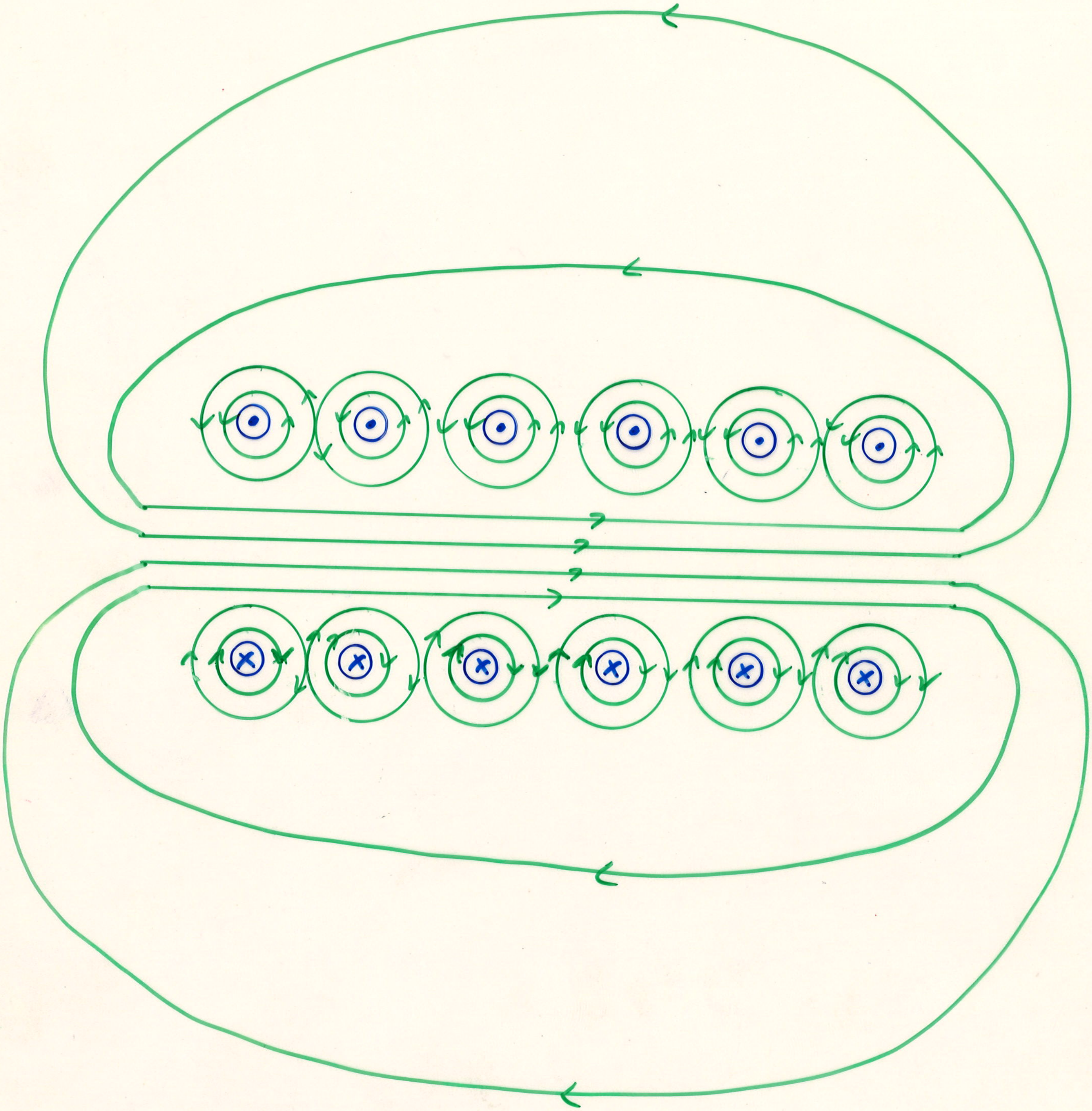
Ex Circular Arc



$\underline{\text{Ex}}$: Magnetic field due to a <u>finite</u> length of current-carrying wire.



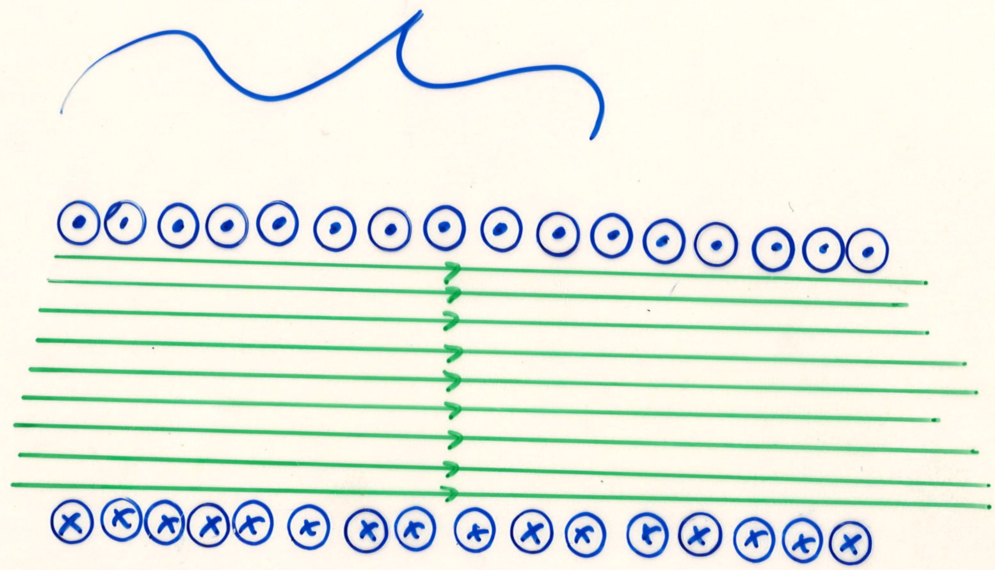
The Solenoid



$\vec{B}$ field lines are closed loops.

The Solenoid

n turns per unit length
(100 wires per inch)



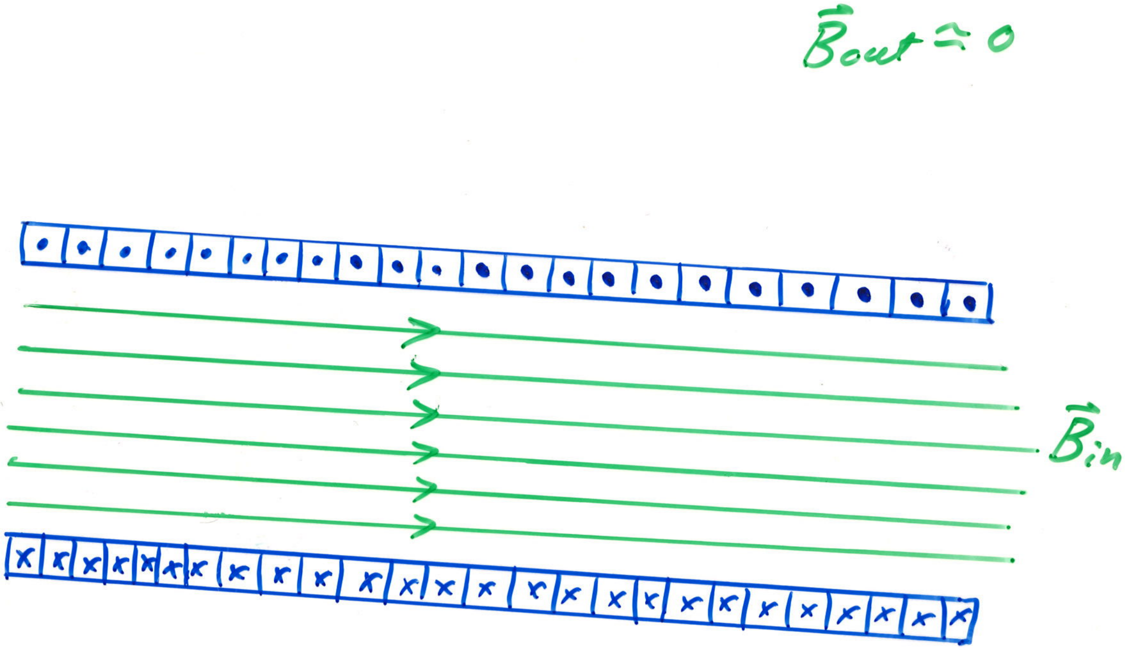
If the solenoid is very long compared to its radius and if the coils are closely spaced then:

$\vec{B}_{inside} \approx$ constant

$\vec{B}_{outside} \approx 0$

Well, not really, but the $\vec{B}$ field is <u>much</u> less dense outside.

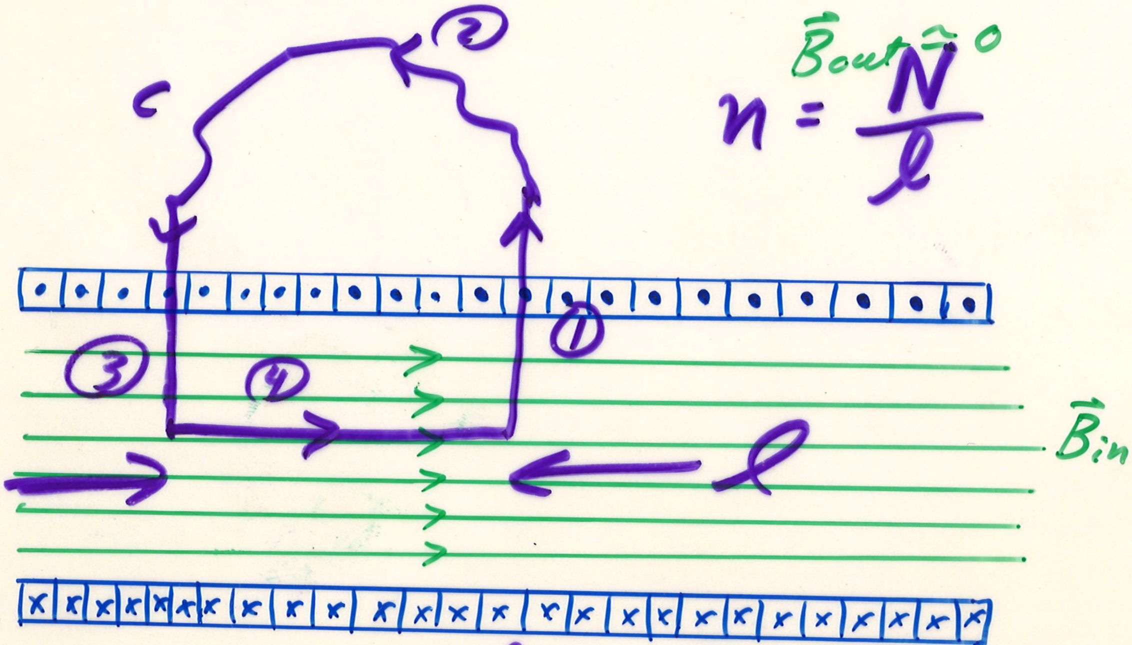
Magnetic field inside a solenoid by Ampere's Law:



Magnetic field inside a solenoid by Ampere's Law: n turns of wire per unit length

$$\vec{B}_{out} \simeq 0$$

$$n = \frac{N}{l}$$



Ampere's Law $\oint_c \vec{B}\cdot d\vec{l} = I_{enc}\mu_0 =$

$$= \underset{1}{\int} \underset{\vec{B}\perp d\vec{l}}{\cancelto{0}{\vec{B}\cdot d\vec{l}}} + \underset{2}{\int} \underset{B=0}{\cancelto{0}{\vec{B}\cdot d\vec{l}}} + \underset{3}{\int} \underset{\vec{B}\perp d\vec{l}}{\cancelto{0}{\vec{B}\cdot d\vec{l}}} + \underset{4}{\int} \vec{B}\cdot d\vec{l}$$

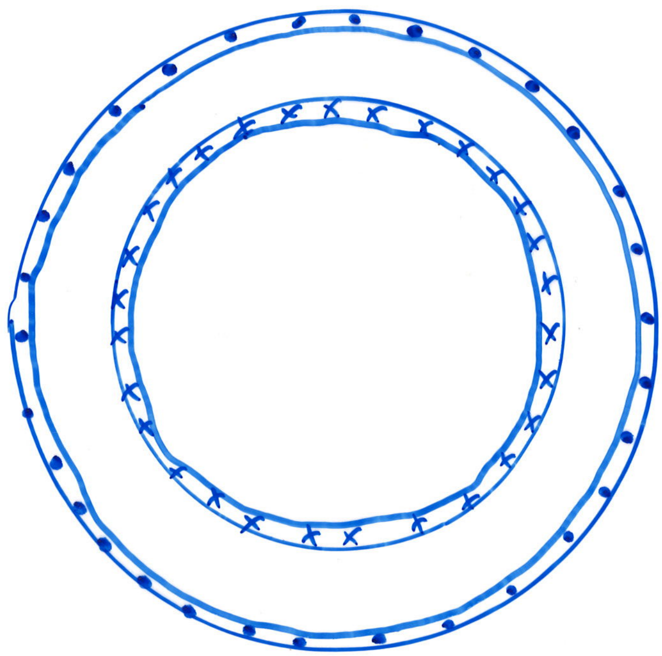
$$= \int_4 B\,dl = B\int_4 dl = B\cancel{l} = \mu_0 I (n\cancel{l})$$

$$B = \begin{cases} \mu_0 n I, & \text{inside} \\ 0, & \text{outside} \end{cases}$$

The Toroid

Instead of making the solenoid infinitely long to get a very small $\vec{B}$ field outside, one can attach the open ends to each other to make a doughnut shape.

Total of
N turns



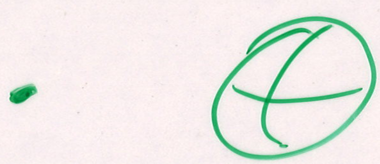
Gauss' Law

infinitesimal area
vector that points out

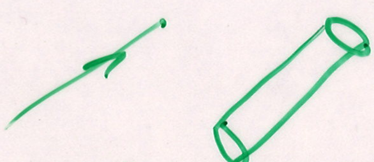
$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

closed surface

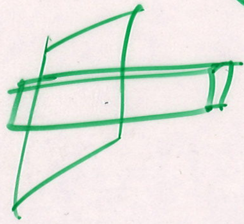
evaluated
on the closed
surface S



spherical Gaussian
surface



cylinder



pillbox

Gauss' Law for Magnetism

$$\oint\!\!\!\oint_S \vec{B} \cdot d\vec{A} = 0$$

Ampere's Law

infinitesimal line element points along the curve C

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enc}$$

closed ~~curve~~ curve

"Amperean Loops"

evaluated on closed curve C

current within the closed curve C

Useful in cases of high symmetry

The Laws

of Electricity & Magnetism so far

Gauss' Law: $$\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$ by S

closed surface

Ampere's law: $$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0\, i_{enc}$$ by C

closed curve

Faraday's law:

$$-\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A} = \oint_C \vec{E}\cdot d\vec{\ell}$$

open surface bounded by C

closed curve

Faraday's Law

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{A}$$

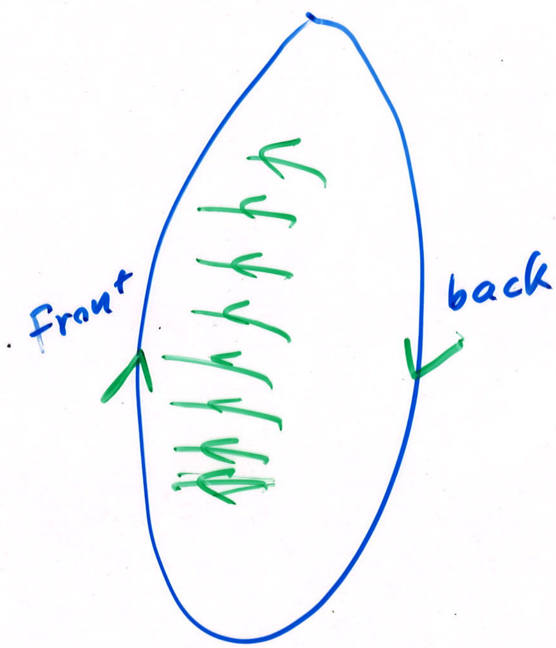
$$\mathcal{E} = -\frac{d}{dt} \Phi_B$$

e.m.f. = electro-motive force
(voltage)

$$\Delta V_{A \to B} = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

Lenz's Law

An induced current in a closed conducting loop will create a magnetic field that opposes the change in the external magnetic field.

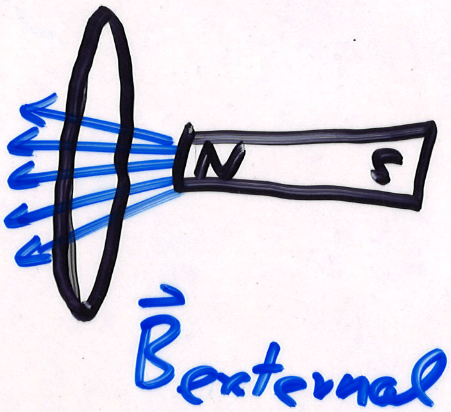


Lenz's Law

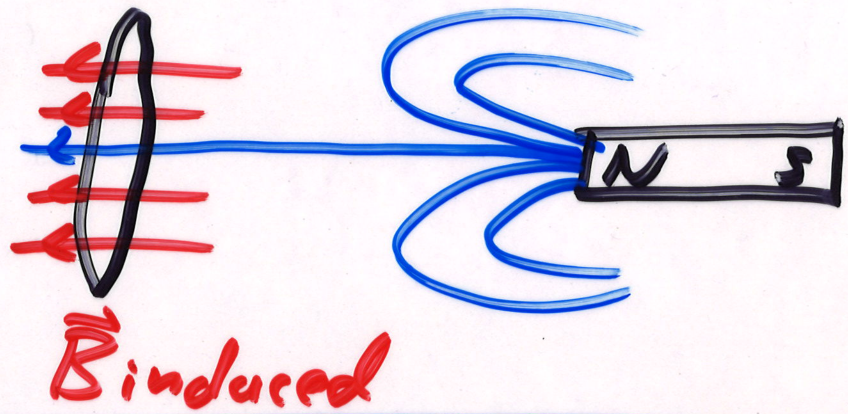
"Lenz's law tries to maintain the status quo."

$I_{ind} = \begin{cases} \text{up in front} \\ \text{down in back} \end{cases}$

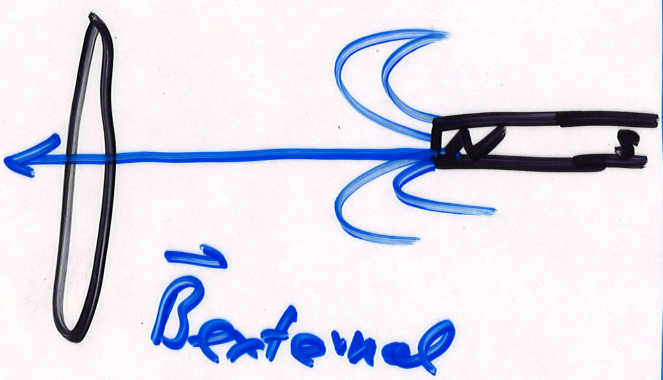
Before



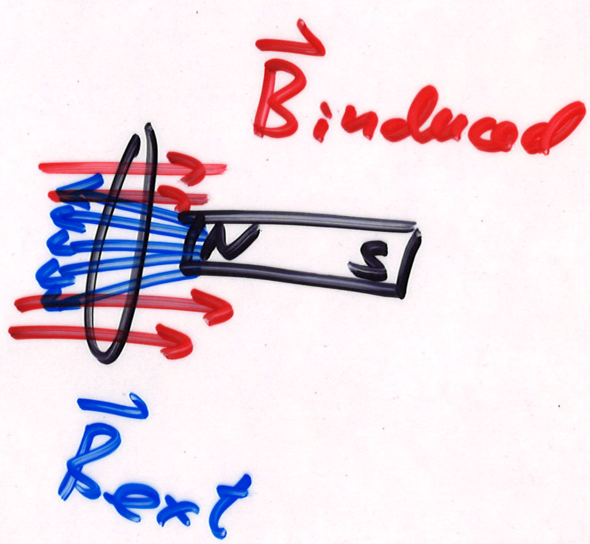
After



Before

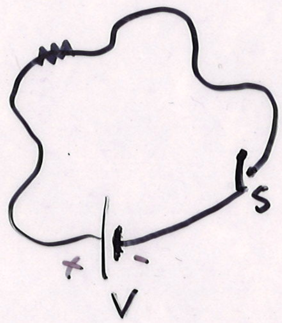


After



$I_{ind} = \begin{cases} \text{up in back} \\ \text{down in front} \end{cases}$

Self Inductanse (L)



Suppose that you want to start current flowing in a circuit.

When you close the switch, the external voltage V will begin to push charges around the loop, a current i, time dependent.

But this current will produce a magnetic field, $\vec{B}$. Some of the $\vec{B}$ lines will penetrate the loop giving a magnetic flux Φ_B. Because the current changes in time, so does the flux.

The changing magnetic flux gives rise to a back e.m.f. $\mathcal{E}'$ by Faraday's Law

$$\mathcal{E}' = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_{\substack{\text{Area} \\ \text{of loop}}} \vec{B}\cdot d\vec{A}$$

This is a little push against the battery voltage.

Since $\Phi_B \propto B$ and $B \propto i$

we have

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \propto \frac{di}{dt}$$

the constant of proportionality is called the (self) inductance

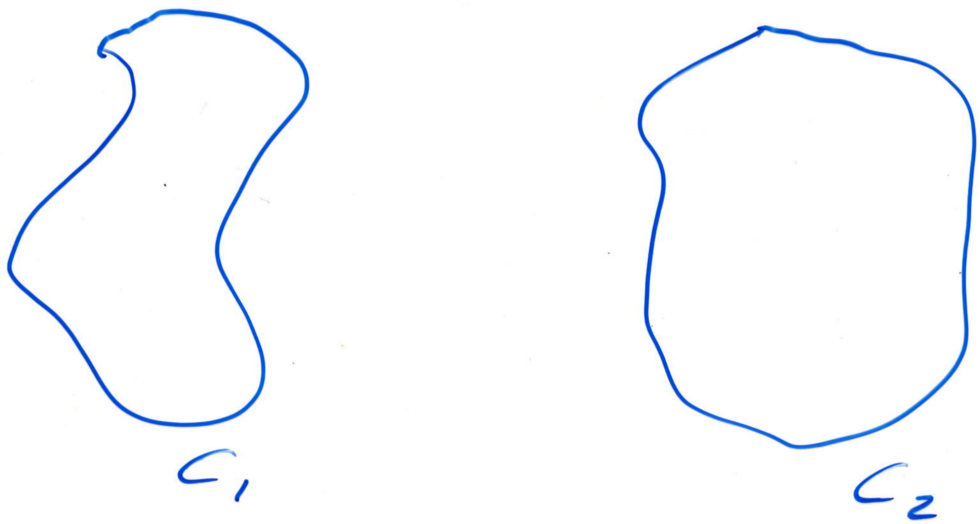
$$\mathcal{E} = -L\frac{di}{dt}$$

$Q = CV$

The MKS unit of inductance is the henry (H).

$$H = \frac{V \cdot s}{A}$$

Mutual Inductance (M)



changing the current in loop 1
induces an e.m.f. in loop 2

$$\mathcal{E}_2 = - M \frac{di_1}{dt}$$

changing the current in loop 2
induces an e.m.f. in loop 1

$$\mathcal{E}_1 = - M \frac{di_2}{dt}$$