$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{3f} + \frac{1}{d_i} = \frac{1}{f}$$

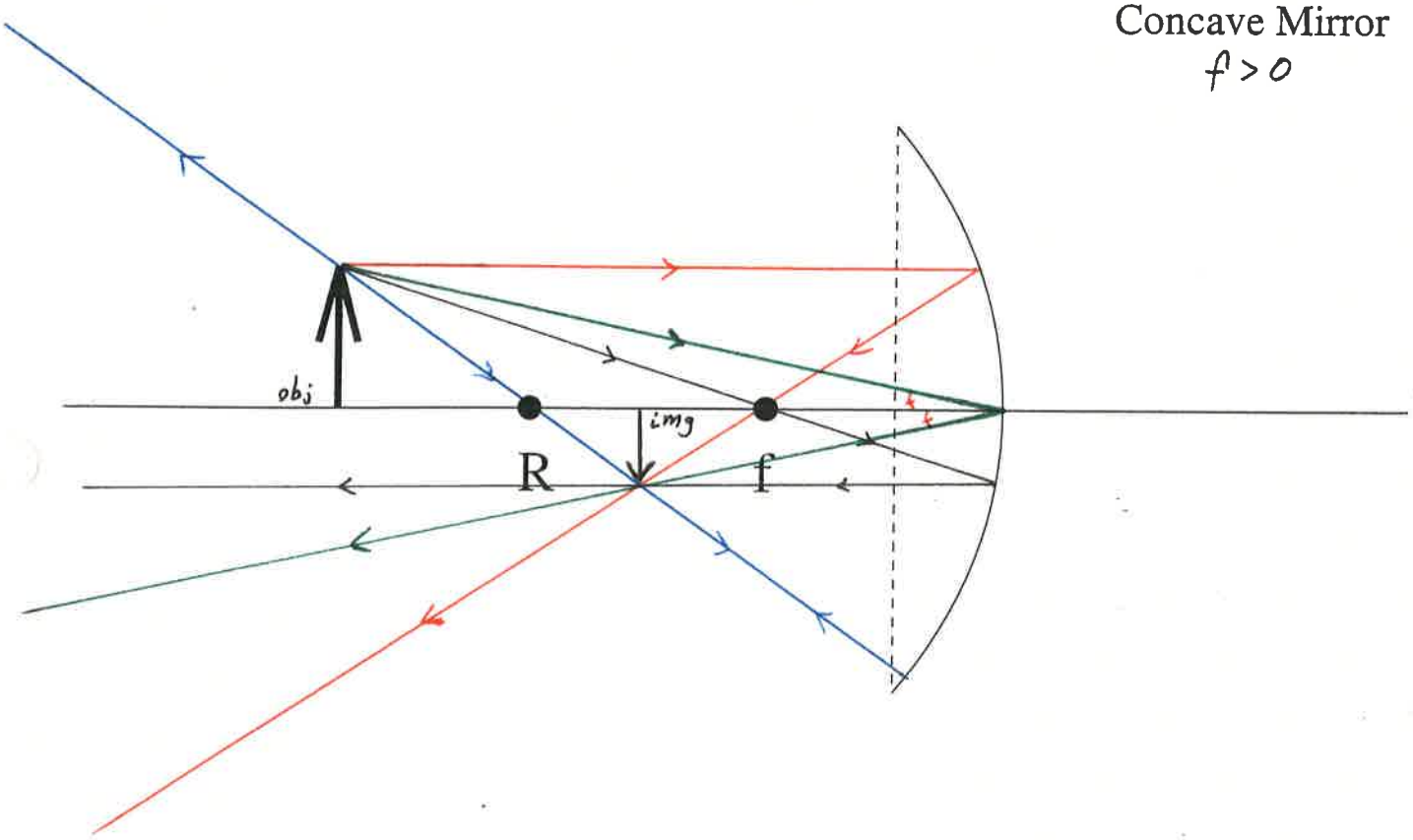
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{3f} = \frac{3}{3f} - \frac{1}{3f} = \frac{2}{3f}$$

$$d_i = \frac{3}{2}f$$

$$M = -\frac{d_i}{d_o} = -\frac{\frac{3}{2}f}{3f} = -\frac{1}{2}$$

parallel ray
focal ray
chief ray
verter ray

Concave Mirror
$f > 0$



Object

$d_o = 3f > R$

Image

real $(d_i > 0)$
inverted $(M < 0)$
reduced $(|M| < 1)$

As $d_o \to \infty$, $d_i \to f$ and $M \to 0$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{2f} + \frac{1}{d_i} = \frac{1}{f}$$

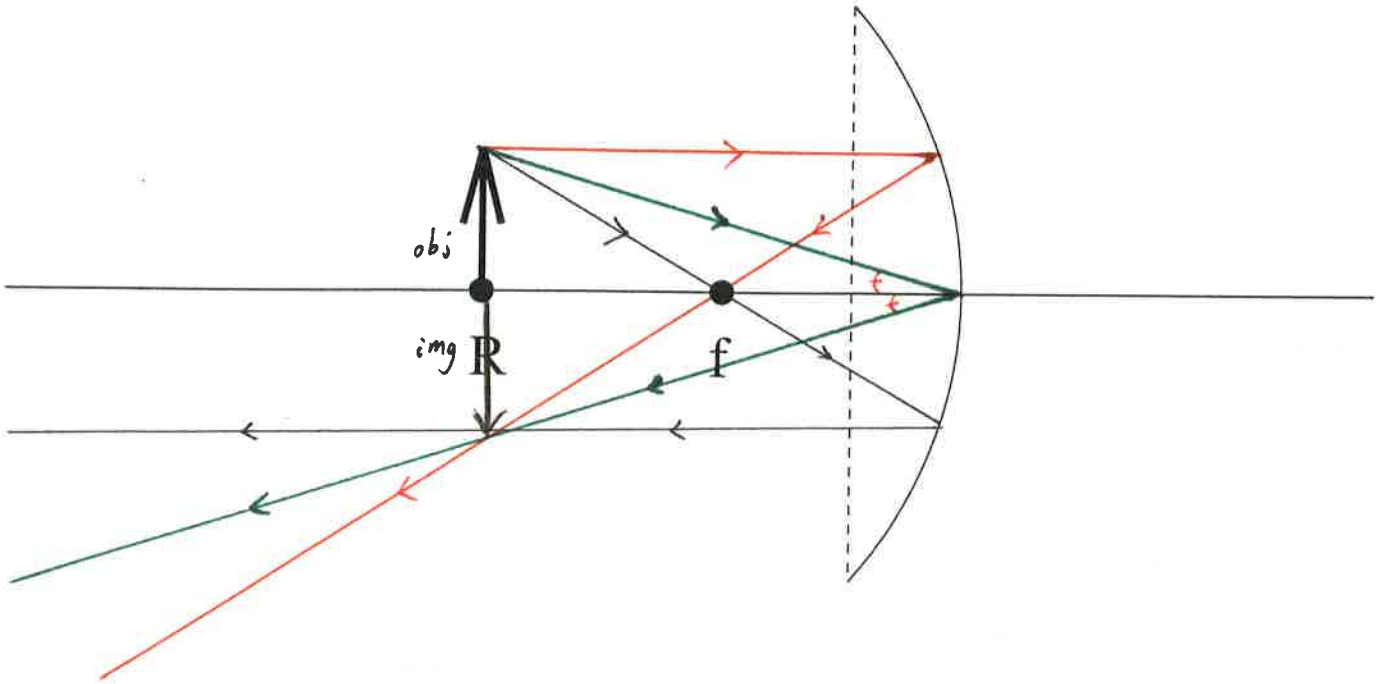
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{2f} = \frac{2}{2f} - \frac{1}{2f} = \frac{1}{2f}$$

$$d_i = 2f$$

$$M = -\frac{d_i}{d_o} = -\frac{2f}{2f} = -1$$

parallel ray
focal ray
no chief ray
vertex ray

Concave Mirror
$f > 0$



Object

$d_o = 2f = R$

Image

real $(d_i > 0)$
inverted $(M < 0)$
same size $(|M| = 1)$

$d_o = 2f$ is the dividing line between reduced $(d_o > 2f)$ and enlarged $(d_o < 2f)$ images.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{\frac{3}{2}f} + \frac{1}{d_i} = \frac{1}{f}$$

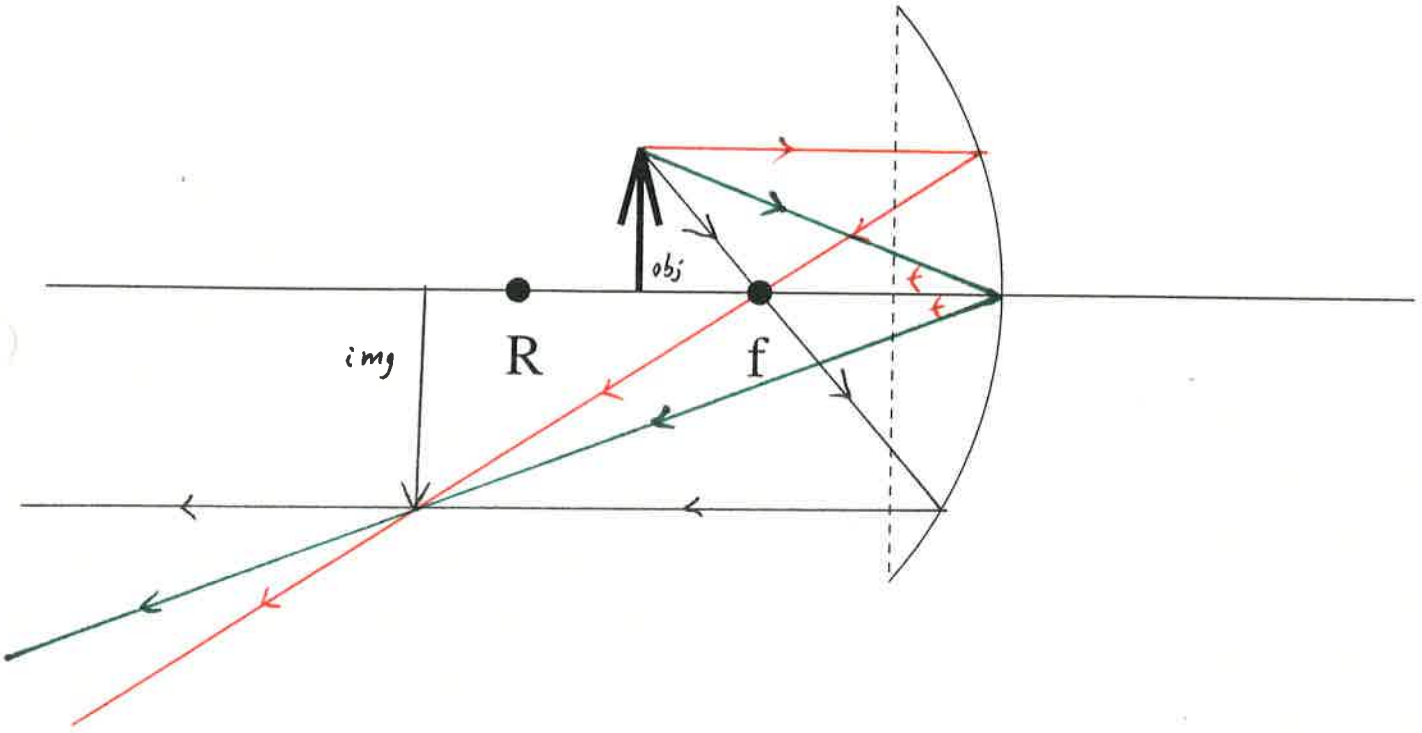
$$\frac{1}{d_i} = \frac{1}{f} - \frac{2}{3f} = \frac{3}{3f} - \frac{2}{3f} = \frac{1}{3f}$$

$$d_i = 3f$$

$$M = -\frac{d_i}{d_o} = -\frac{3f}{\frac{3}{2}f} = -2$$

parallel ray
focal ray
vertex ray

Concave Mirror
$f > 0$



Object

$d_o = \frac{3}{2}f$

Image

real $(d_i > 0)$

inverted $(M < 0)$

enlarged $(|M| > 1)$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{f} + \frac{1}{d_i} = \frac{1}{f}$$

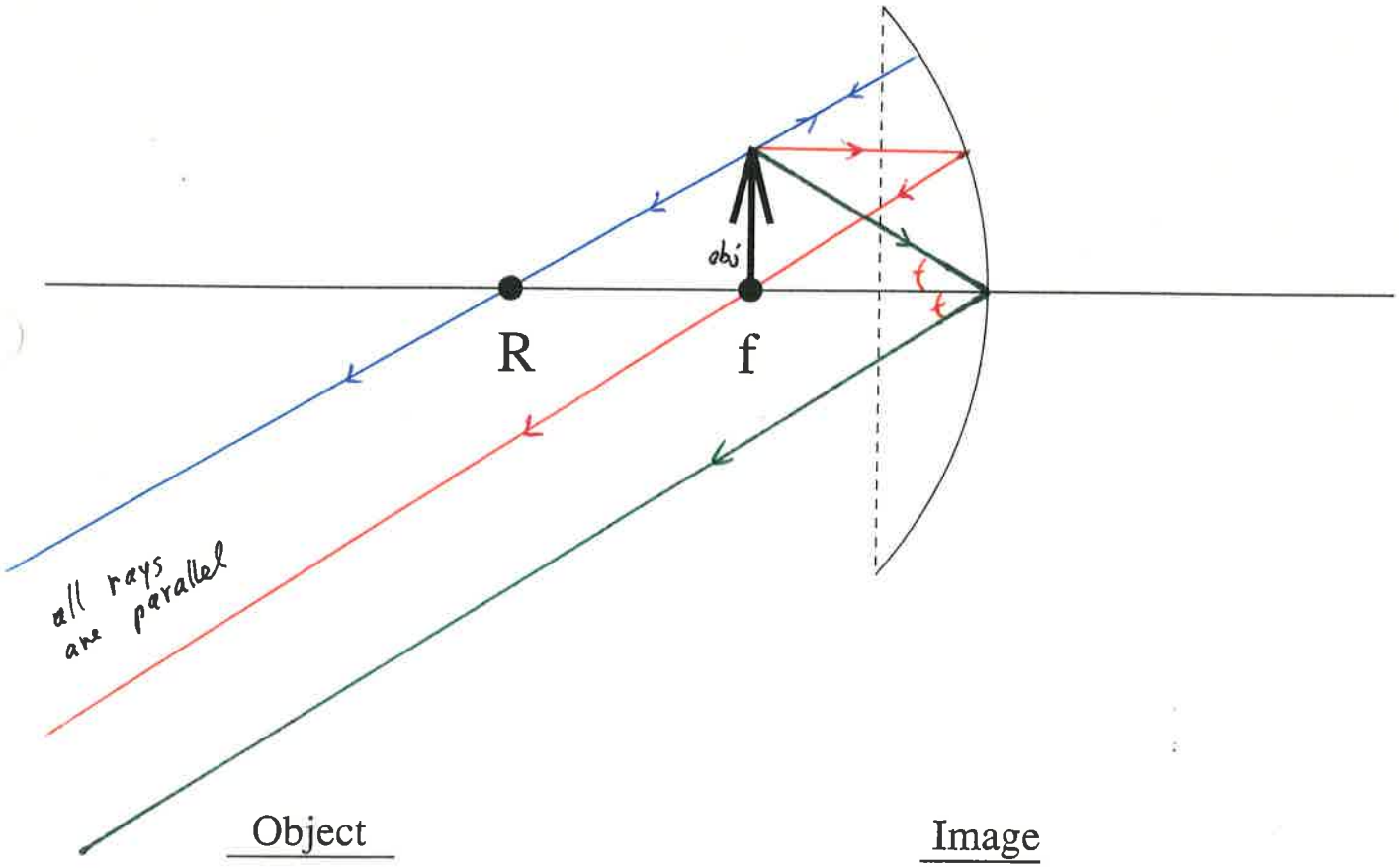
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{f} = 0$$

$$d_i \to \infty$$

$$M = -\frac{d_i}{d_o} \longrightarrow \infty$$

parallel ray
no focal ray
chief ray
vertex ray

Concave Mirror
$f > 0$



Object

$d_o = f$

Image

d_i at infinity
$|M| \to \infty$

$d_o = f$ is the dividing line between real ($d_o > f$) and virtual ($d_o < f$) images.

$d_o = f$ is also the dividing line between inverted ($d_o > f$) and upright ($d_o < f$) images.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{f/2} + \frac{1}{d_i} = \frac{1}{f}$$

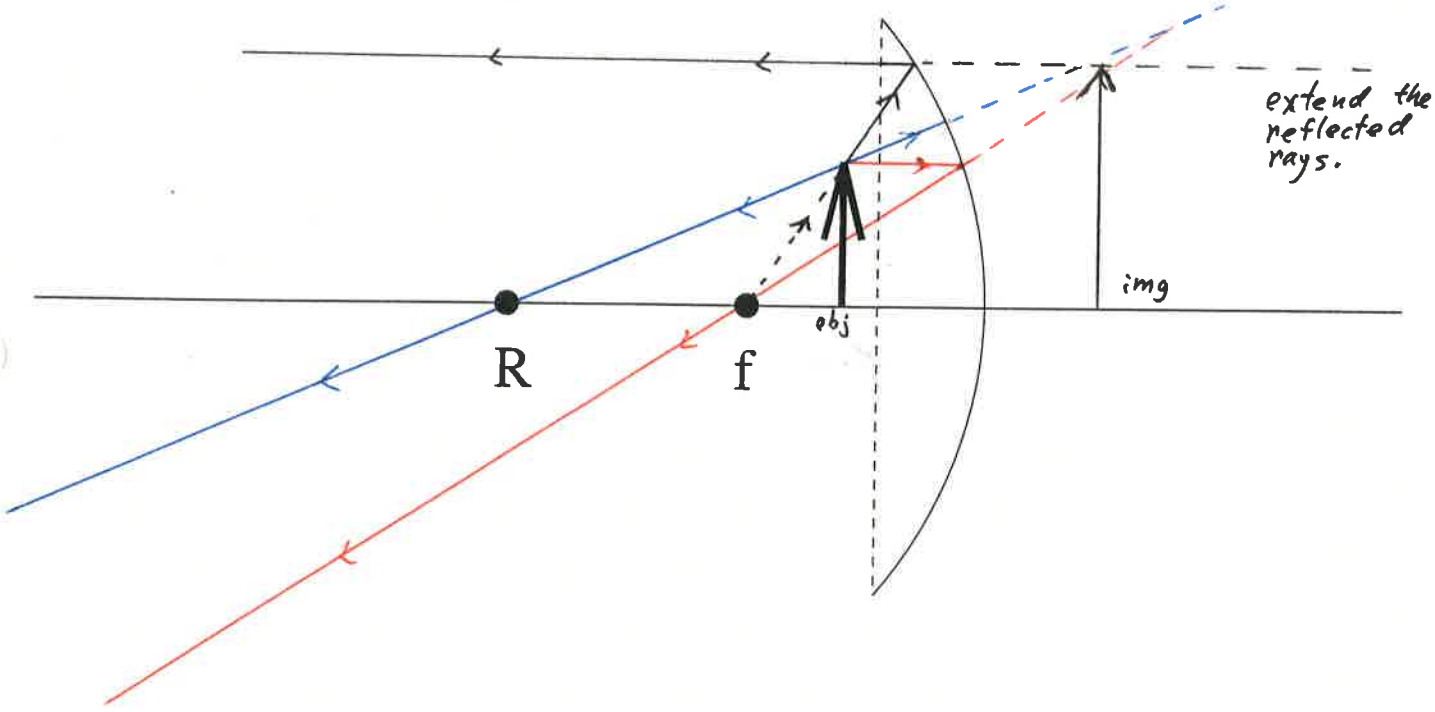
$$\frac{1}{d_i} = \frac{1}{f} - \frac{2}{f} = -\frac{1}{f}$$

$$d_i = -f$$

$$M = -\frac{d_i}{d_o} = -\frac{(-f)}{f/2} = +2$$

parallel ray
focal ray
chief ray

Concave Mirror
$f > 0$



Object

$d_o = \frac{f}{2} < f$

Image

virtual $(d_i < 0)$
upright $(M > 0)$
enlarged $(|M| > 1)$

As $d_o \to 0$, $d_i \to 0$ and $M \to +1$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

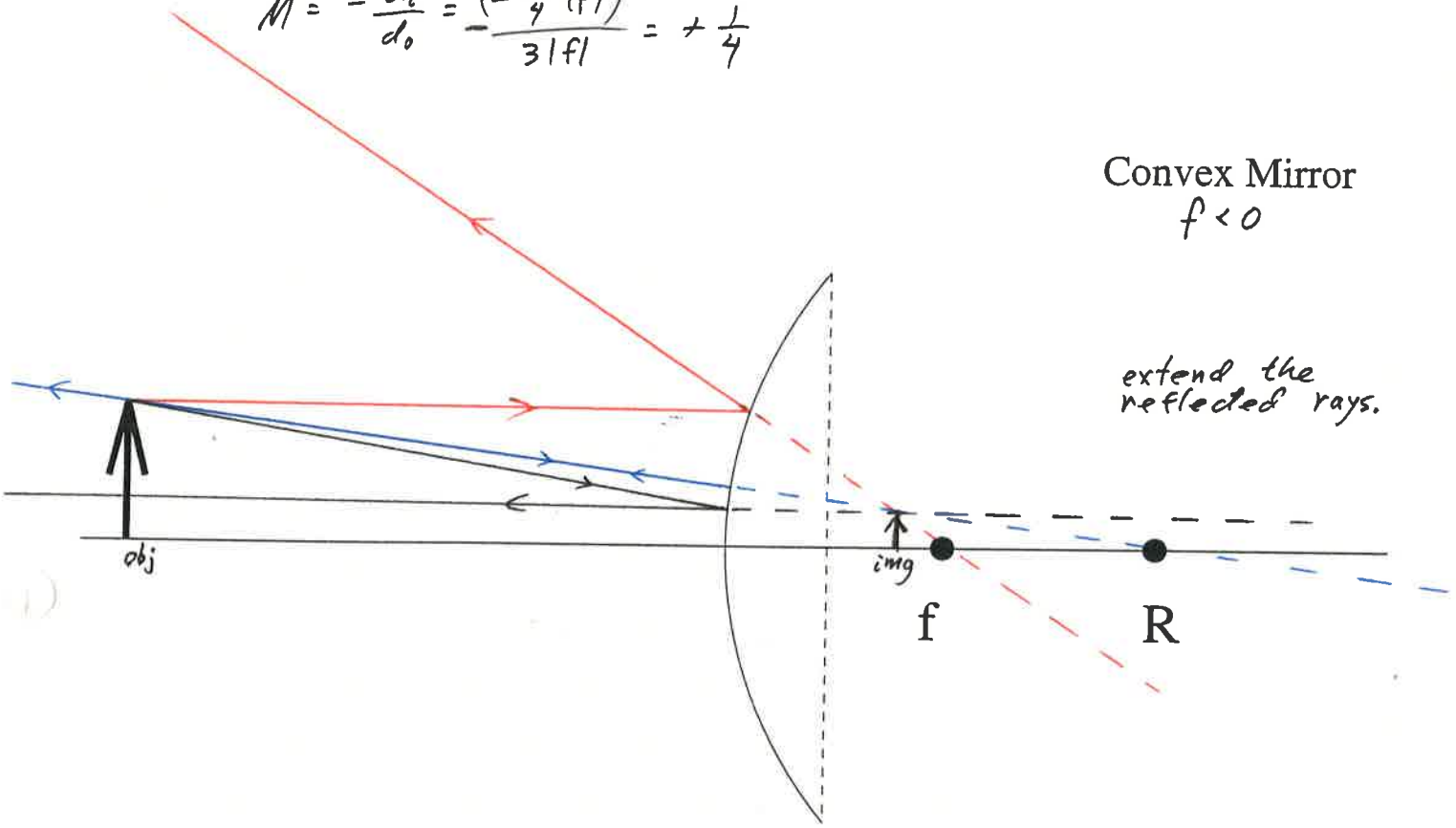
$$\frac{1}{3|f|} + \frac{1}{d_i} = -\frac{1}{|f|}$$

$$\frac{1}{d_i} = -\frac{1}{|f|} - \frac{1}{3|f|} = \frac{-3}{3|f|} - \frac{1}{3|f|} = \frac{-4}{3|f|}$$

$$d_i = -\frac{3}{4}|f|$$

$$M = -\frac{d_i}{d_o} = \frac{-\left(-\frac{3}{4}|f|\right)}{3|f|} = +\frac{1}{4}$$

parallel ray
focal ray
chief ray



Object

Any $d_o > 0$

e.g. $d_o = 3|f|$

Image

virtual ($d_i < 0$)
upright ($M > 0$)
reduced ($|M| < 1$)

As $d_o \to \infty$, $d_i \to -|f|$ and $M \to 0$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{|f|} + \frac{1}{d_i} = -\frac{1}{|f|}$$

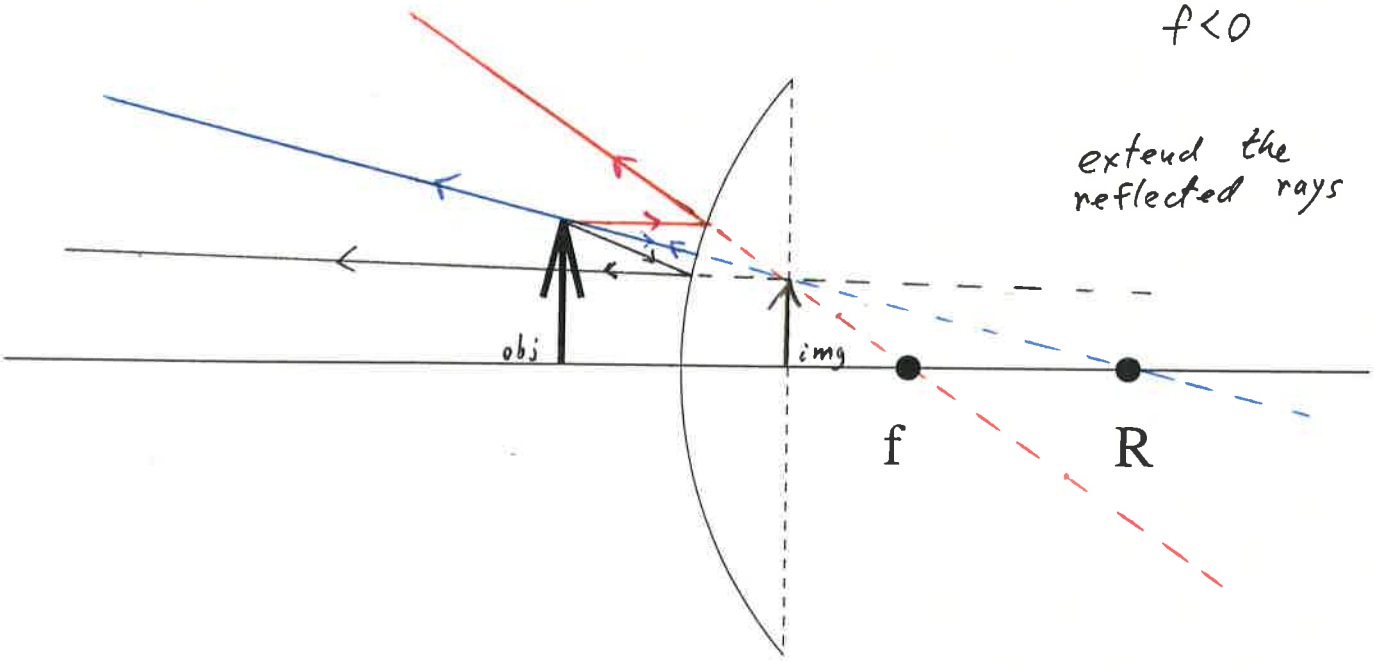
$$\frac{1}{d_i} = -\frac{1}{|f|} - \frac{1}{|f|} = \frac{-2}{|f|}$$

$$d_i = -\frac{1}{2}|f|$$

$$M = -\frac{d_i}{d_o} = -\frac{(-\frac{1}{2}|f|)}{|f|} = +\frac{1}{2}$$

parallel ray
focal ray
chief ray

Convex Mirror
$f < 0$



extend the reflected rays

Object

Any $d_o > 0$

e.g. $d_o = |f|$

Image

virtual ($d_i < 0$)
upright ($M > 0$)
reduced ($|M| < 1$)

As $d_o \to 0$, $d_i \to 0$ and $M \to +1$

d_i always between 0 and $-f$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{3f} + \frac{1}{d_i} = \frac{1}{f}$$

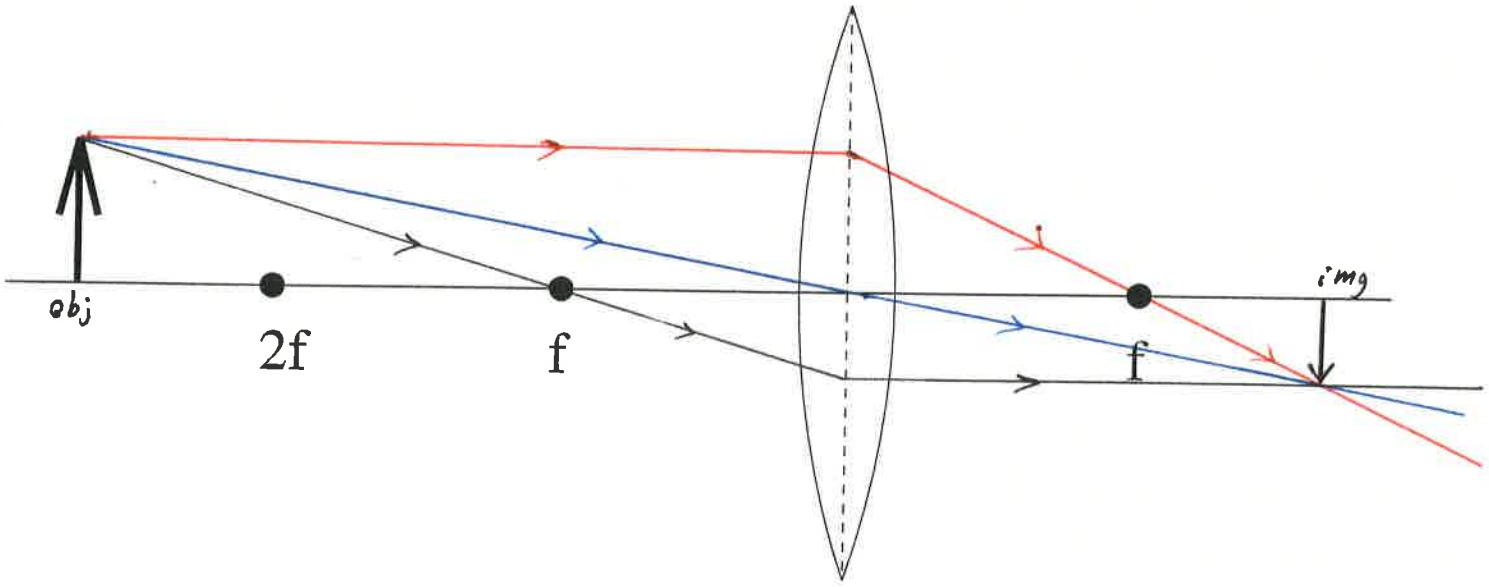
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{3f} = \frac{3}{3f} - \frac{1}{3f} = \frac{2}{3f}$$

$$d_i = \frac{3}{2} f$$

$$M = -\frac{d_i}{d_o} = -\frac{\frac{3}{2}f}{3f} = -\frac{1}{2}$$

parallel ray
focal ray
central ray

Converging Lens
$f > 0$



Object

$d_o = 3f > 2f$

Image

real $(d_i > 0)$
inverted $(M < 0)$
reduced $(|M| < 1)$

As $d_o \to \infty$, $d_i \to f$ and $M \to 0$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{2f} + \frac{1}{d_i} = \frac{1}{f}$$

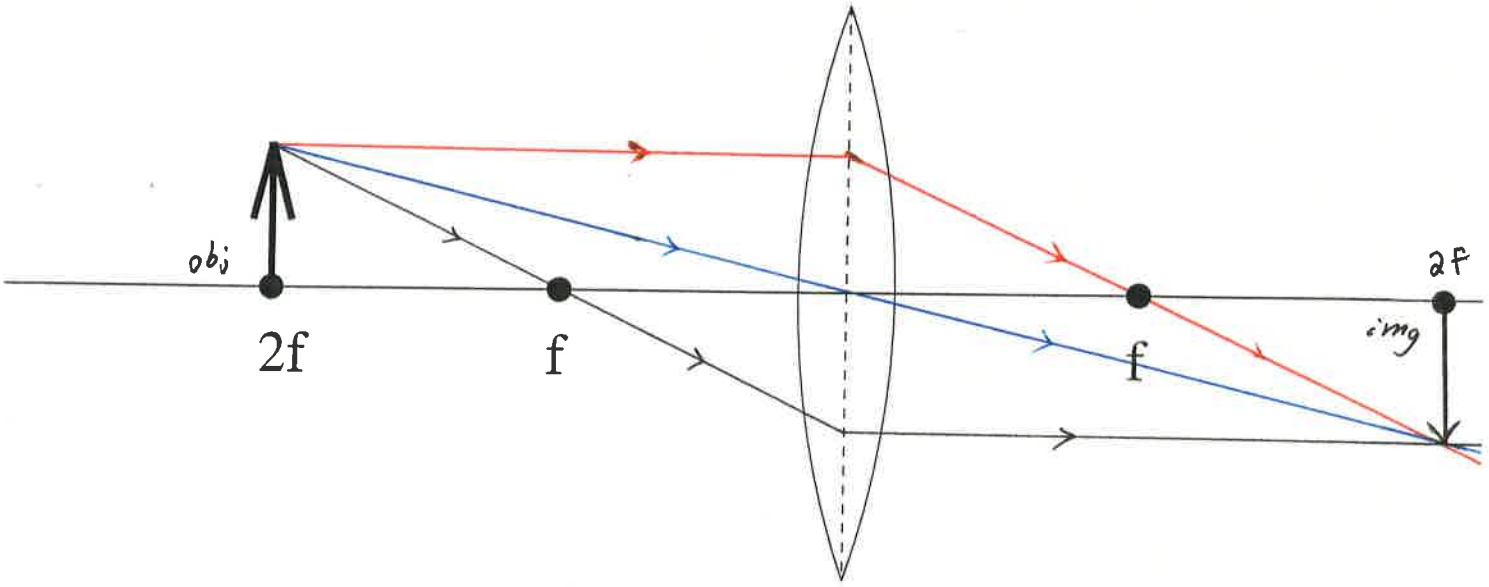
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{2f} = \frac{2}{2f} - \frac{1}{2f} = \frac{1}{2f}$$

$$d_i = 2f$$

$$M = -\frac{d_i}{d_o} = -\frac{2f}{2f} = -1$$

parallel ray
focal ray
central ray

Converging Lens
$f > 0$



Object

$d_o = 2f$

Image

real ($d_i > 0$)
inverted ($M < 0$)
same size ($|M| = 1$)

$d_o = 2f$ is the dividing line between reduced ($d_o > 2f$) and enlarged ($d_o < 2f$) images.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{\frac{3f}{2}} + \frac{1}{d_i} = \frac{1}{f}$$

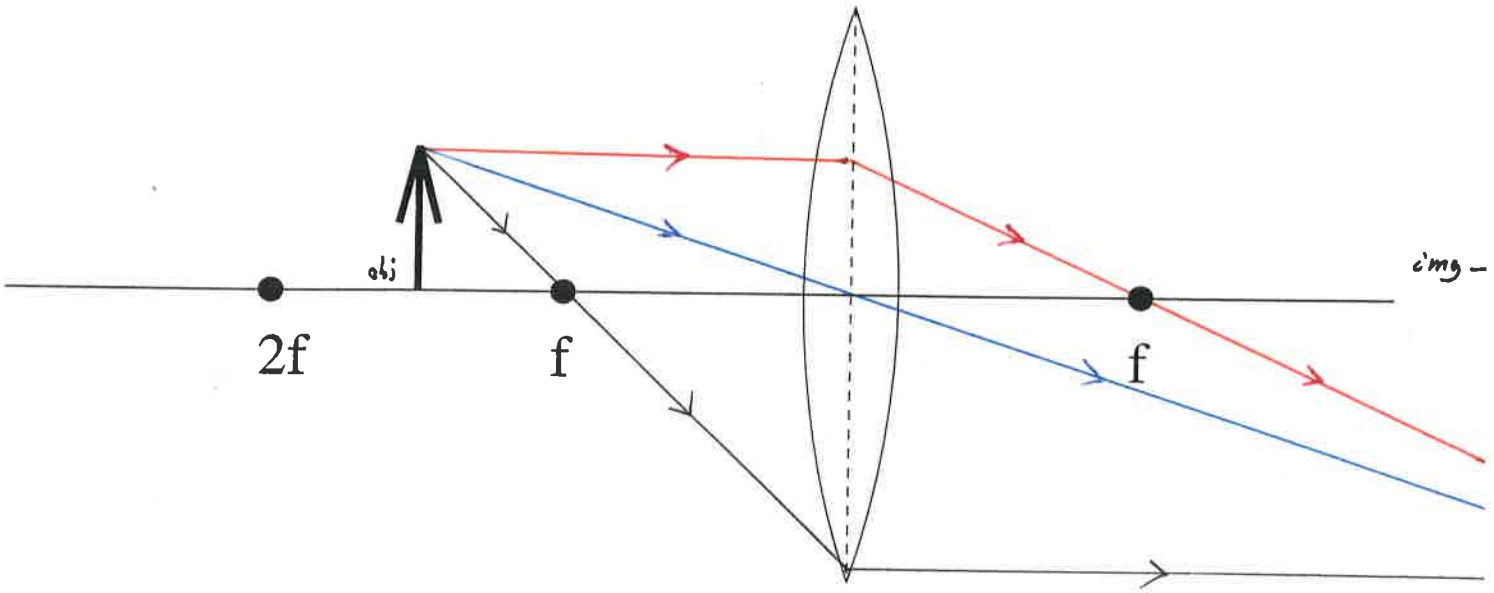
$$\frac{1}{d_i} = \frac{1}{f} - \frac{2}{3f} = \frac{3}{3f} - \frac{2}{3f} = \frac{1}{3f}$$

$$d_i = 3f$$

$$M = -\frac{d_i}{d_o} = -\frac{3f}{3f_2} = -2$$

parallel ray
focal ray
central ray

Converging Lens
$f > 0$



Object

$d_o = \frac{3}{2}f$

Image

real $(d_i > 0)$
inverted $(M < 0)$
enlarged $(|M| > 1)$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{f} + \frac{1}{d_i} = \frac{1}{f}$$

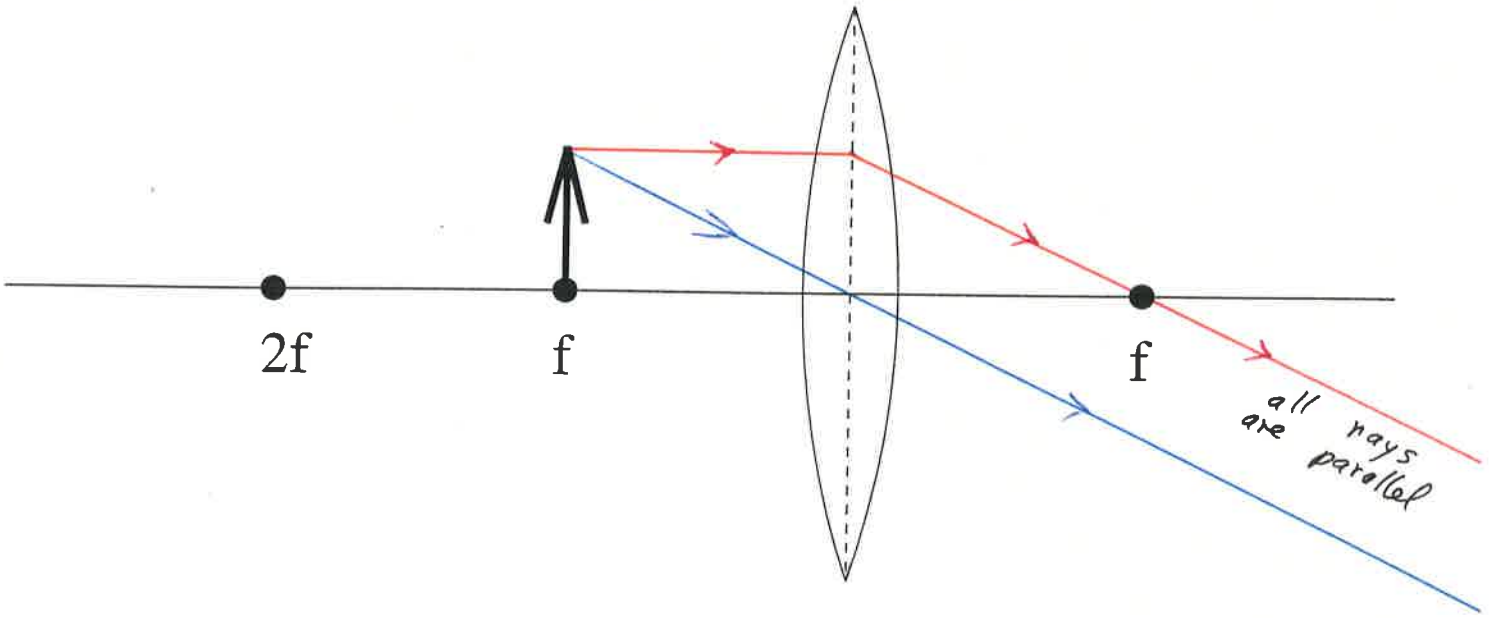
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{f} = 0$$

$$d_i \to \infty$$

$$M = -\frac{d_i}{d_o} \to \infty$$

parallel ray
no focal ray
central ray

Converging Lens
$f > 0$



Object	Image
$d_o = f$	d_i at infinity $\lvert M \rvert \to \infty$

$d_o = f$ is the dividing line between real ($d_o > f$) and virtual ($d_o < f$) images.
$d_o = f$ is also the dividing line between inverted ($d_o > f$) and upright ($d_o < f$) images.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

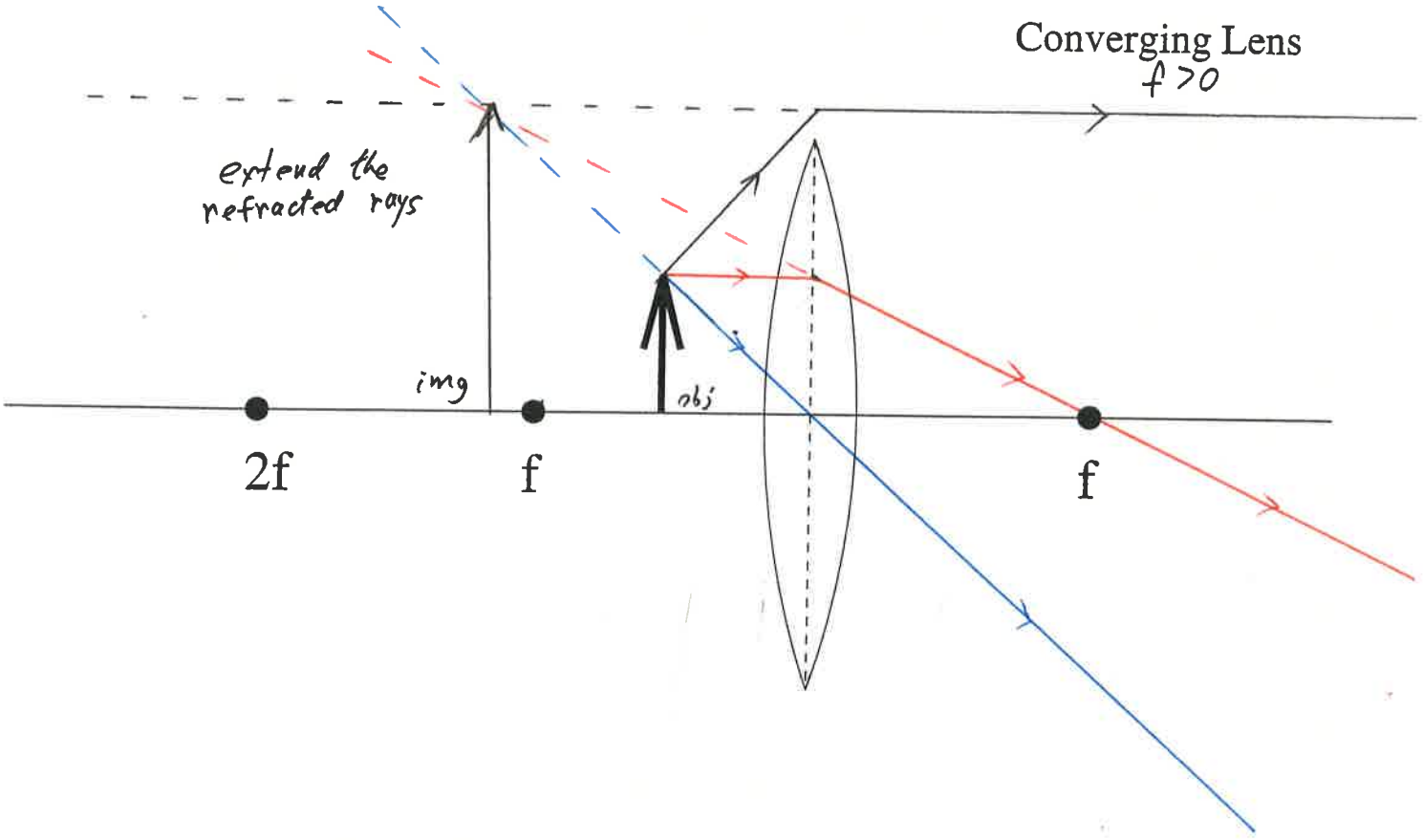
$$\frac{1}{f/2} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{2}{f} = -\frac{1}{f}$$

$$d_i = -f$$

$$M = -\frac{d_i}{d_o} = -\frac{(-f)}{f/2} = +2$$

parallel ray
focal ray
central ray



Object

$d_o = \frac{f}{2}$

Image

virtual ($d_i < 0$)
upright ($M > 0$)
enlarged ($|M| > 1$)

As $d_o \to 0$, $d_i \to 0$ and $M \to +1$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{3|f|} + \frac{1}{d_i} = \frac{-1}{|f|}$$

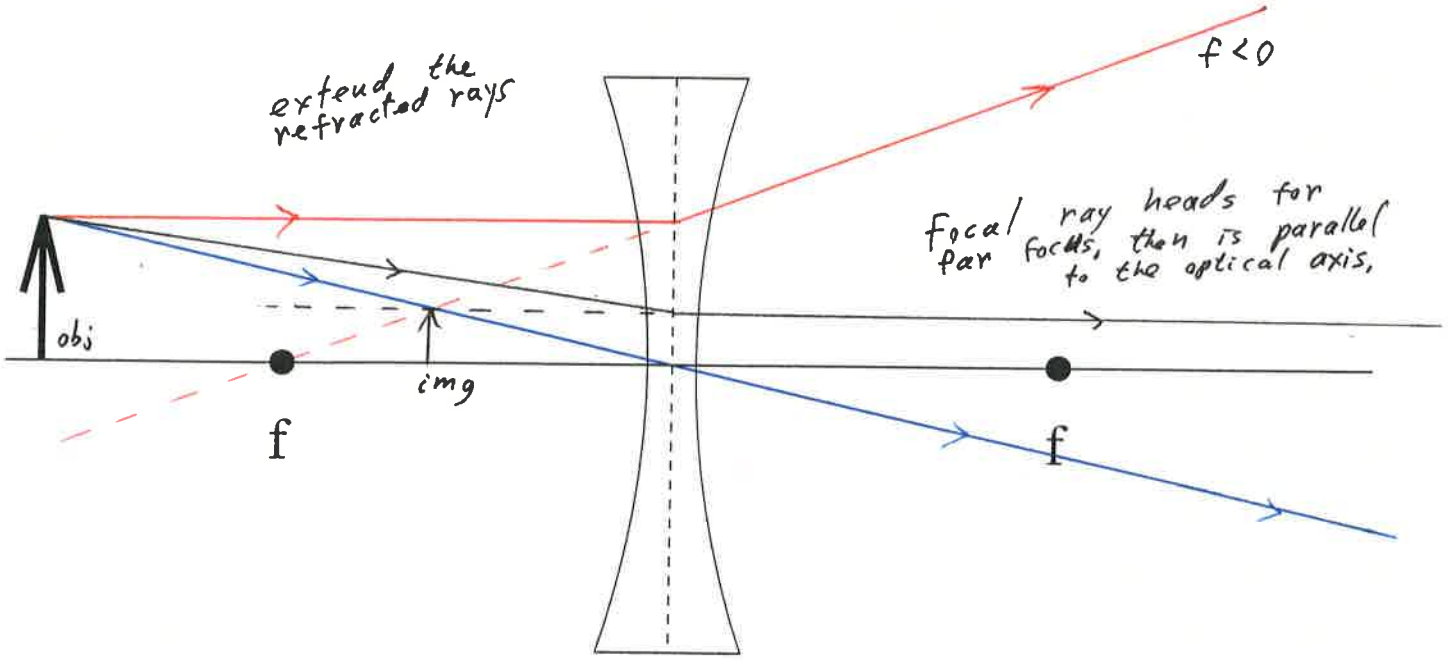
$$\frac{1}{d_i} = -\frac{1}{|f|} - \frac{1}{3|f|} = \frac{-3}{3|f|} - \frac{1}{3|f|} = \frac{-4}{3|f|}$$

$$d_i = -\frac{3}{4}|f|$$

$$M = -\frac{d_i}{d_o} = -\frac{\left(-\frac{3}{4}|f|\right)}{3|f|} = +\frac{1}{4}$$

parallel ray
focal ray
central ray

Diverging Lens



Object

Any $d_o > 0$

e.g. $d_o = 3|f|$

Image

virtual ($d_i < 0$)
upright ($M > 0$)
reduced ($|M| < 1$)

As $d_o \to \infty$, $d_i \to -|f|$ and $M \to 0$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{|f|/2} + \frac{1}{d_i} = \frac{-1}{|f|}$$

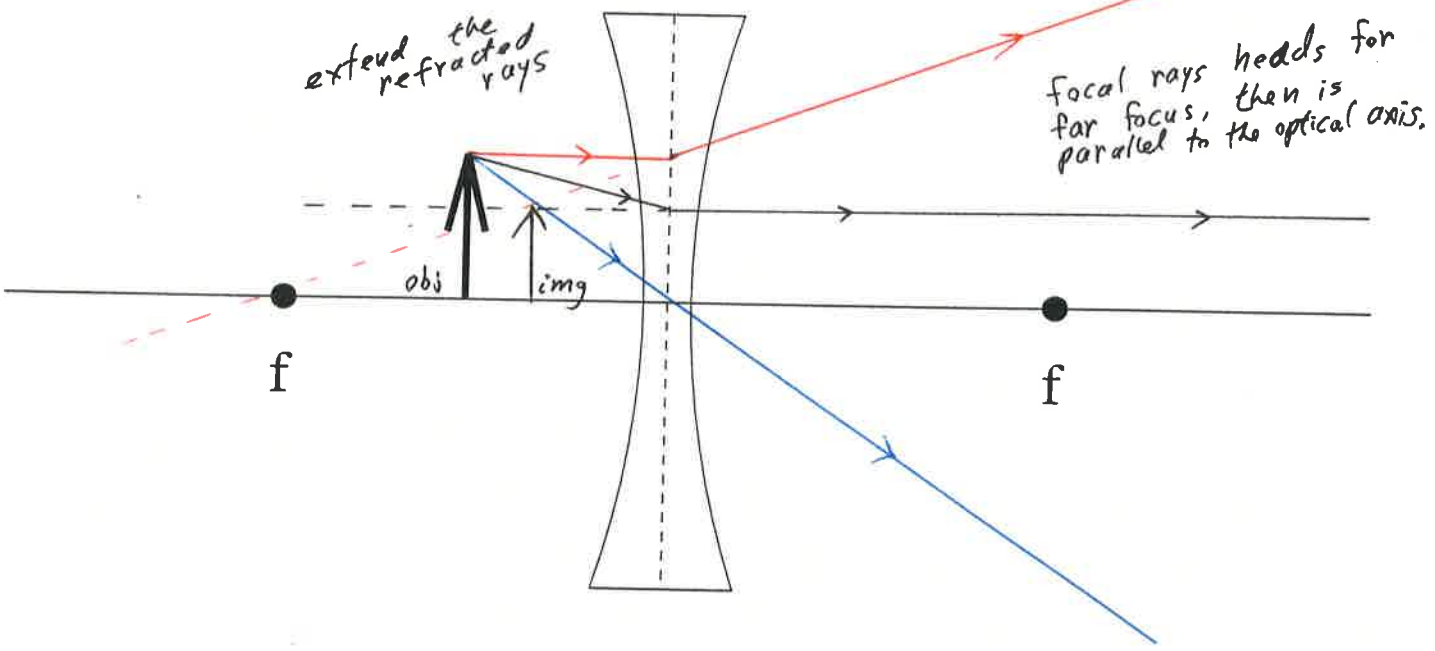
$$\frac{1}{d_i} = -\frac{1}{|f|} - \frac{2}{|f|} = -\frac{3}{|f|}$$

$$d_i = -\frac{1}{3}|f|$$

$$M = -\frac{d_i}{d_o} = -\frac{(-\frac{1}{3}|f|)}{\frac{1}{2}|f|} = +\frac{2}{3}$$

parallel ray
focal ray
central ray

Diverging Lens
$f < 0$



Object

Any $d_o > 0$

e.g. $d_o = \frac{|f|}{2}$

Image

virtual ($d_i < 0$)
upright ($M > 0$)
reduced ($|M| < 1$)

As $d_o \to 0$, $d_i \to 0$ and $M \to +1$