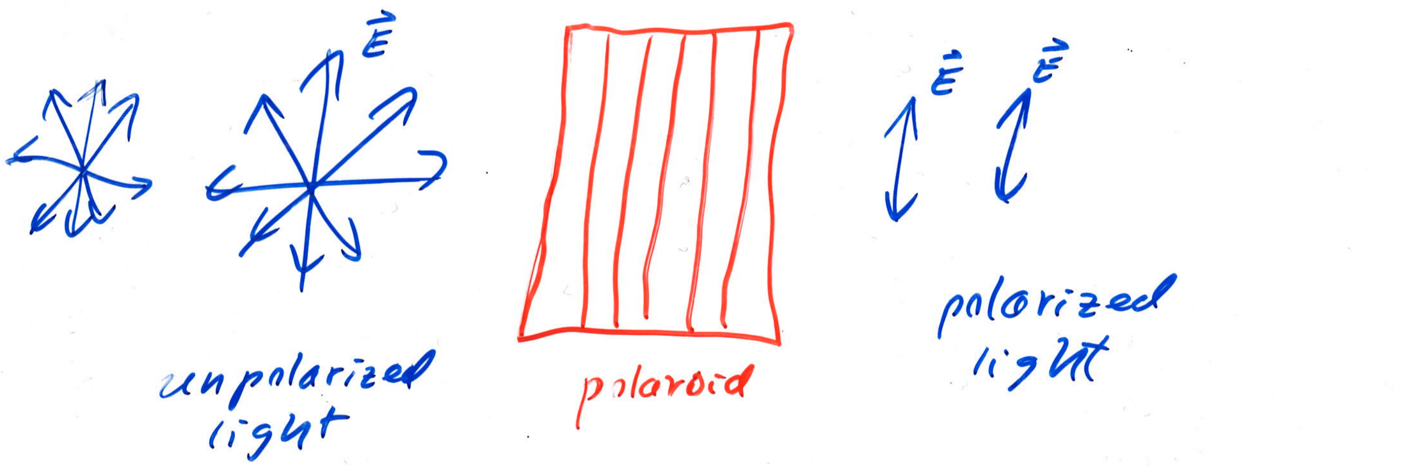
Polaroids

Light from the Sun, a lightbulb, a match, etc. is <u>unpolarized</u>, that is, it contains $\vec{E}$ fields pointing in all directions at random.

Think of a polaroid as a picket fence that only allows those $\vec{E}$ fields aligned along the pickets to pass through.



If the polaroid direction (picket direction) and the $\vec{E}$ field direction make an angle θ, then only

$$E_{max} \cos\theta \quad \text{gets through}$$

and the component of $\vec{E}$ that does get through is now polarized along the polaroid direction.

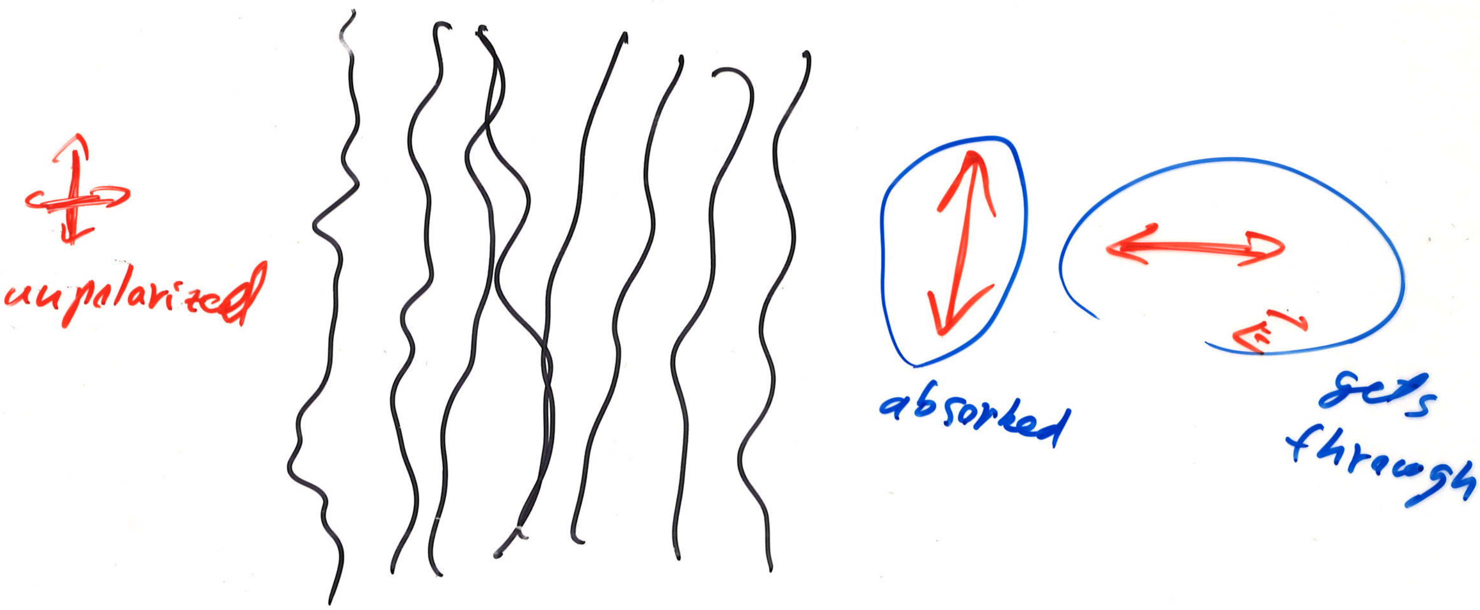
The **intensity** of radiation is proportional to $|\vec{E}|^2$

$$I_{max} = \frac{1}{c\mu_0} E_{rms}^2 = \frac{1}{c\mu_0} \frac{E_{max}^2}{2}$$

so the intensity (what your eye detects) that gets through the polaroid is

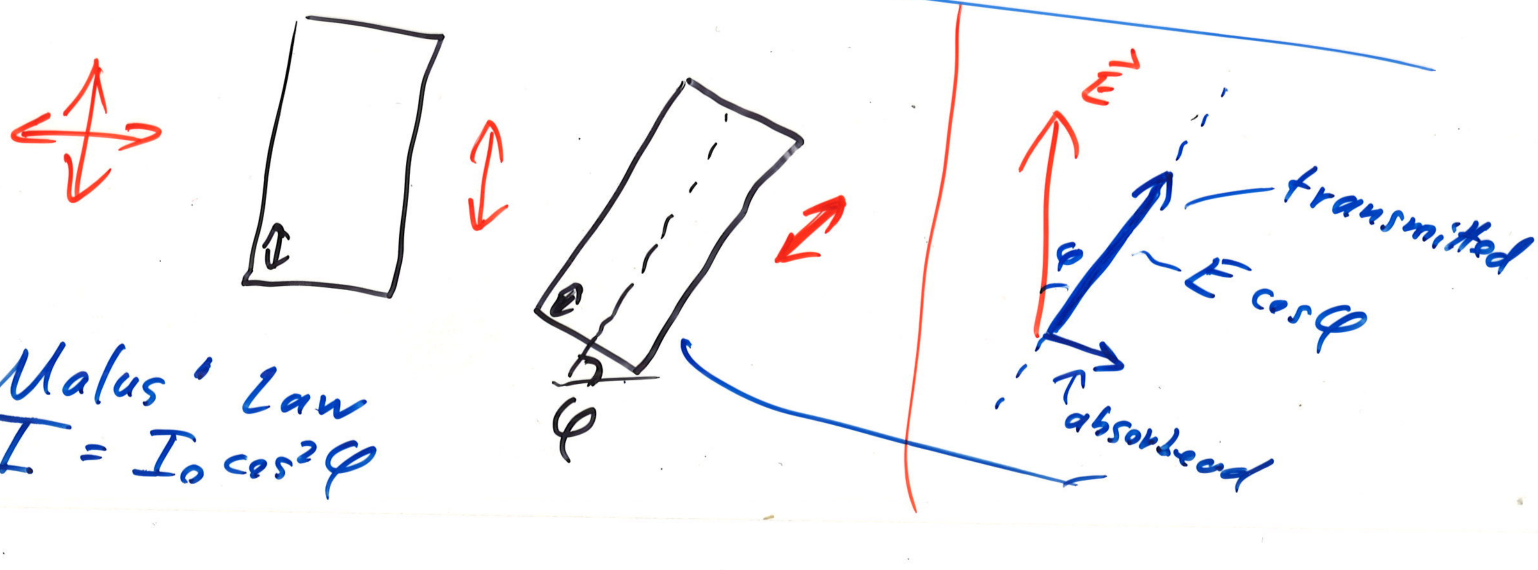
$$I_{max} \cos^2\theta$$

Polarization: direction of Electric field perpendicular to direction of travel



$$I_{unpolarized} \propto \vec{E}^2 = \vec{E} \cdot \vec{E} = E_x^2 + E_y^2$$

$$I_{polarized} \propto E_x^2 = \frac{1}{2} I_{unpolarized}$$



Malus' Law

$$I = I_0 \cos^2\varphi$$

One polaroid

What happens to the original intensity I_{max} after unpolarized light passes through <u>one</u> polaroid?

$$\frac{I_{max}}{2}$$

Crossed polaroids

What happens to I_{max} as unpolarized light is passed through 2 polaroids set at 90° to each other?

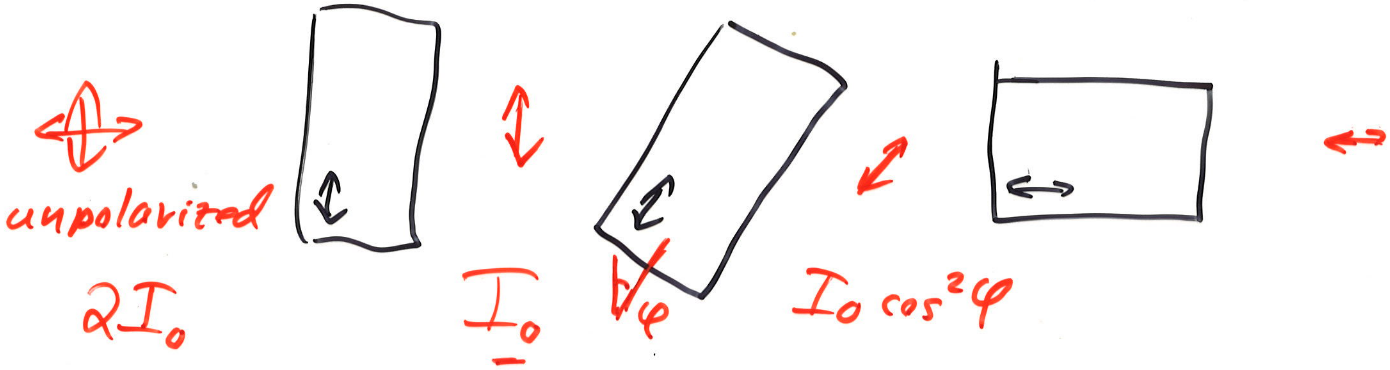
$$0$$

Stacked polaroids

A third sheet is inserted <u>between</u> 2 crossed polaroids 45° from each. I_{max}?

$$I_m \quad \updownarrow \quad \frac{I_{max}}{2} \quad \swarrow\!\!\nearrow \quad \left(\frac{I_{max}}{2}\right)\cos^2(45^\circ) = \frac{I_{max}}{4} \quad \leftrightarrow$$

$$\left(\frac{I_{max}}{4}\right)\cos^2(45^\circ) = \frac{I_{max}}{8}$$



$\vec{E} = E_x\hat{i} + E_y\hat{j}$

$I \propto \vec{E}^2 = \vec{E}\cdot\vec{E}$

$E_x^2 + E_y^2$

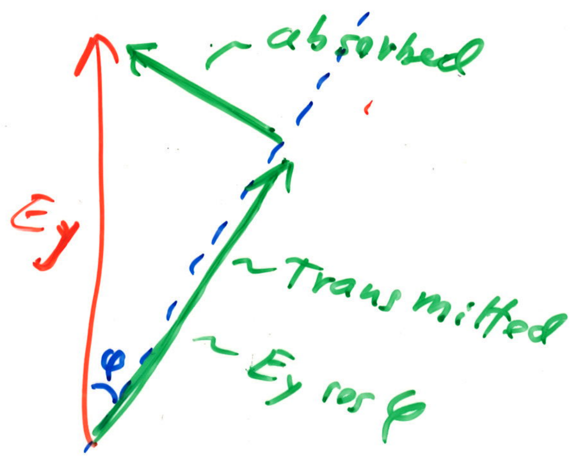
$\vec{E} = E_y\hat{j}$

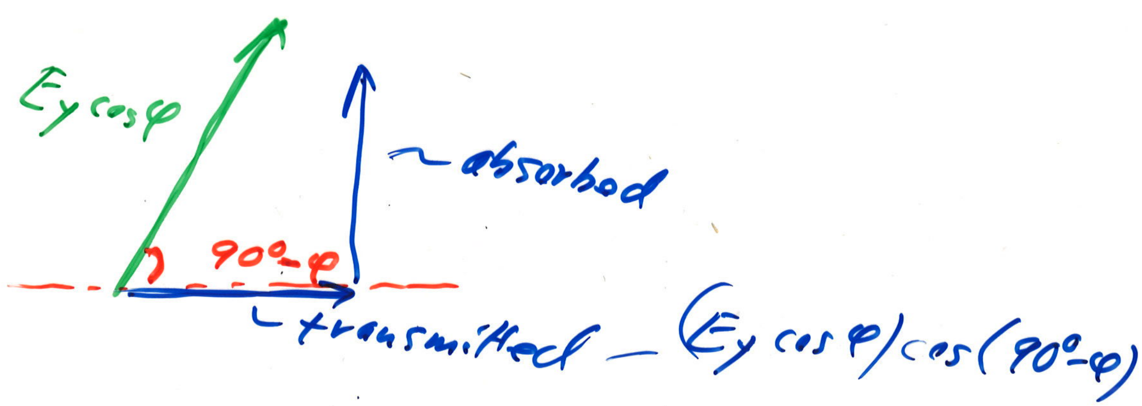
$I \propto \vec{E}^2 = E_y^2$

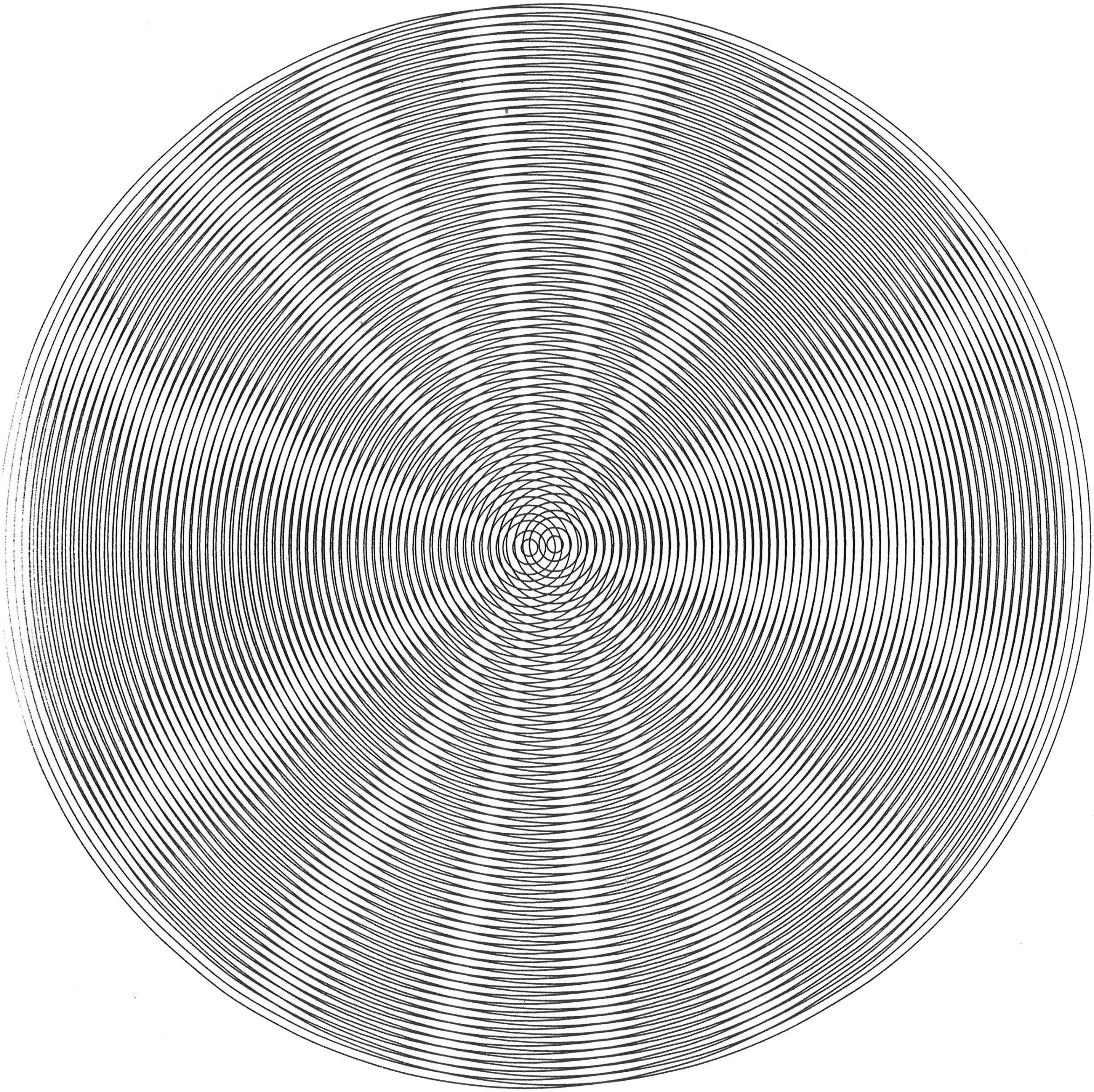
$I \propto E_y^2 \cos^2\varphi$

$$I = E_y^2\cos^2\varphi\,\cos^2(90^\circ-\varphi) = \begin{cases} 0, & \varphi = 0 \\ \max, & \varphi = 45^\circ \\ 0, & \varphi = 90^\circ \end{cases}$$

$I_{max} = \dfrac{I_0}{4}$



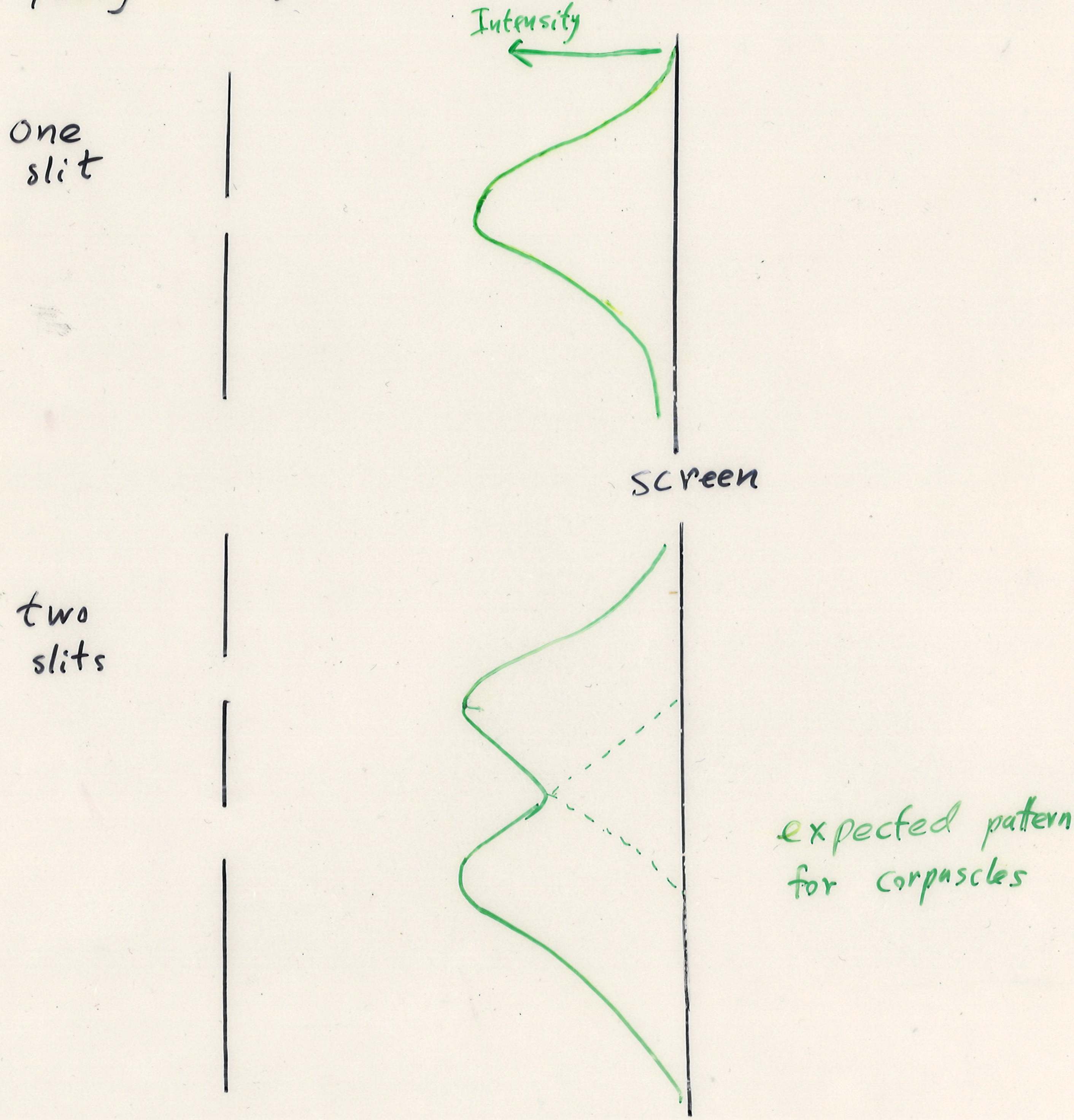




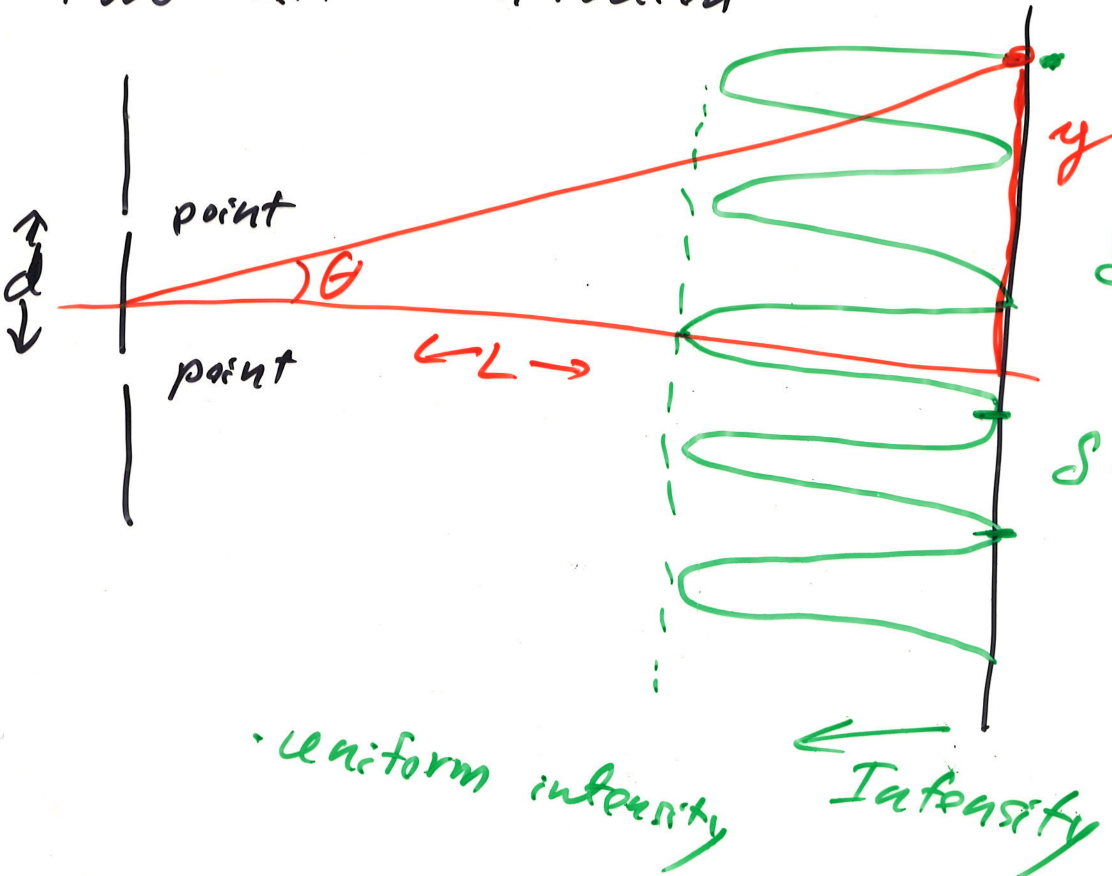
Interference

Newton : Light is made of particles or corpuscles.

Young : Light is made of waves.



Two slit diffraction



$\sin\theta \sim \frac{y}{L}$

$\delta = \begin{cases} m\lambda \\ (m+\frac{1}{2})\lambda \end{cases}$

$\delta = d\sin\theta$

path difference δ tells you where bright spot occur

Intensity function — requires

$E_{tot} = E_1 + E_2$

small angle approximation

$\theta \simeq \sin\theta \simeq \tan\theta$ for $\theta \lesssim 5°$

in radians

Path difference $\quad \delta = d\sin\theta$

Constructive Interference
Bright

$$\delta = d\sin\theta = m\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Destructive Interference
Dark

$$\delta = d\sin\theta = (m+\tfrac{1}{2})\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Small angle approximation

for $\theta \lesssim 0.1$ rad $(\approx 5°)$

$$\theta \approx \sin\theta \approx \tan\theta = \frac{opp}{adj} = \frac{y}{L}$$ (radian only!)

$$y = L\sin\theta = \begin{cases} Lm\lambda/d & \text{const. bright} \\ L(m+\tfrac{1}{2})\lambda/d & \text{destr. dark} \end{cases}$$

$$\underbrace{y}_{10cm} \ll \underbrace{L}_{10m} \qquad \underbrace{\lambda}_{700\,nm} \ll \underbrace{d}_{\sim 1\,mm}$$

m = order

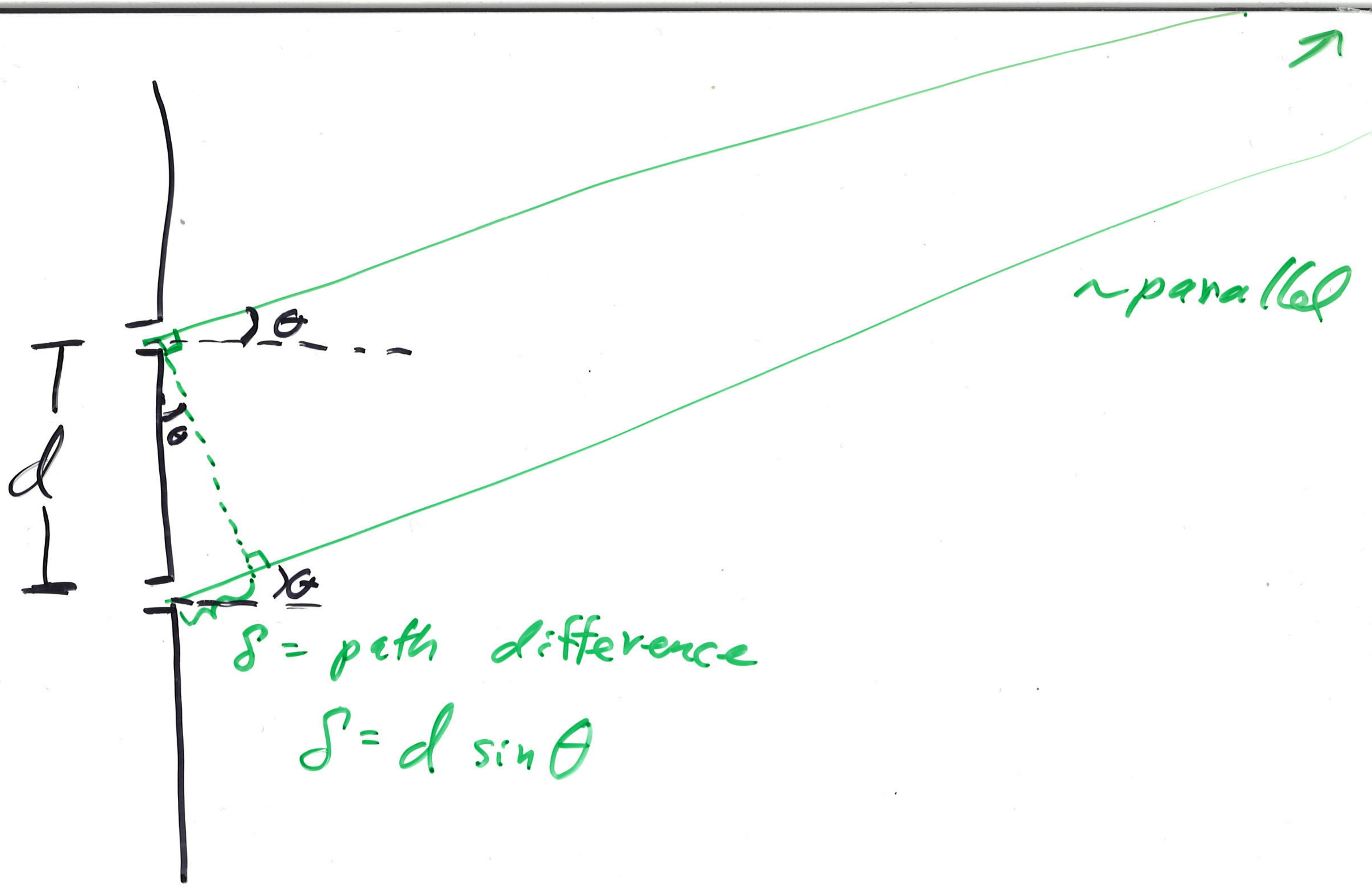
Phase Difference φ

when $\delta = 0$, $\varphi = 0$

when $\delta = \lambda$, $\varphi = 2\pi$

$$\frac{\delta}{\varphi} = \frac{\lambda}{2\pi} \Longrightarrow \varphi = \frac{2\pi\delta}{\lambda}$$

$$= \frac{2\pi \, d \sin\theta}{\lambda}$$



δ = path difference

$$\delta = d \sin\theta$$

Intensity

$$I = S_{AVG} = \frac{E_0^2}{2\mu_0 c}$$

$\wedge$ Poynting vector

$$E_{tot} = E_1 + E_2$$

$$= E_0 \sin(\omega t) + E_0 \sin(\omega t + \varphi)$$

$$= 2E_0 \cos\left(\frac{\varphi}{2}\right) \sin\left(\omega t + \frac{\varphi}{2}\right)$$

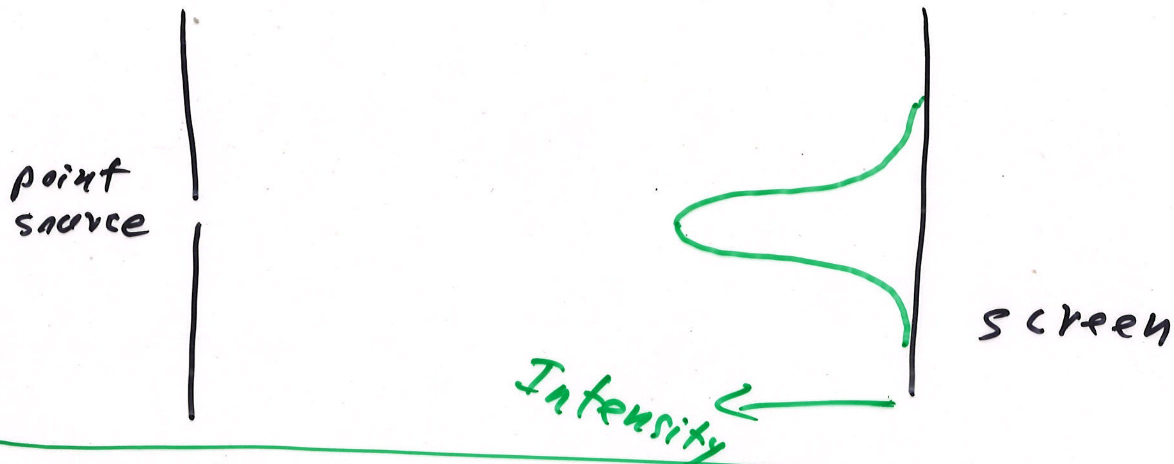
max is $2E_0$ - constr. - $\varphi = 0$, 2π, 4π, ...

min is 0 - destr. - $\varphi = 180° = \pi$ rad, 3π, 5π, ...

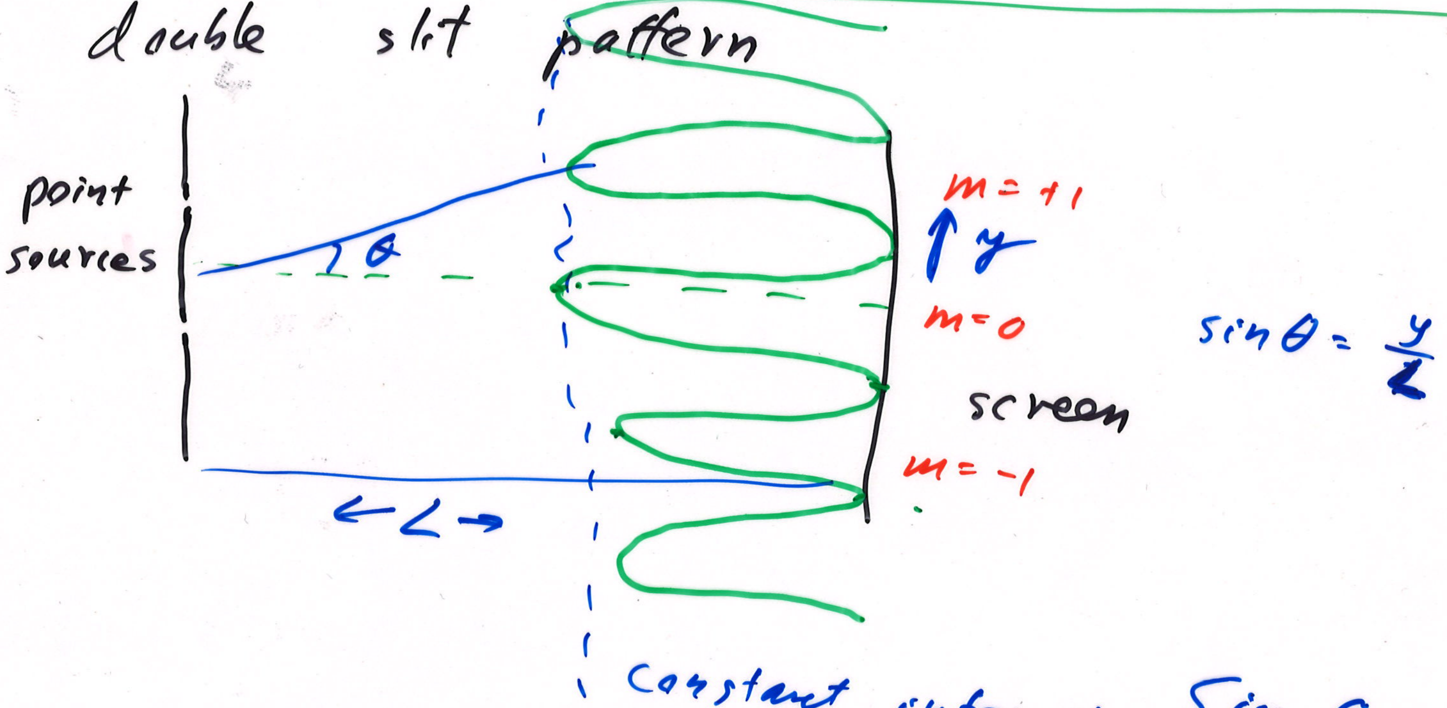
$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$= I_0 \cos^2\left(\frac{\pi d y}{L \lambda}\right)$$

Previously, single slit pattern



double slit pattern

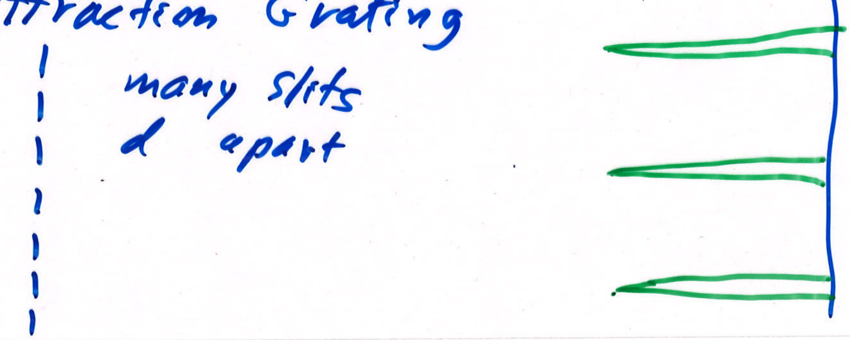


constant intensity $\begin{cases} \text{in } \theta \\ \text{in } y \end{cases}$

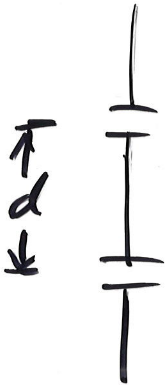
$$E_{tot} = E_1 + E_2 = E_0 \sin(\omega t) + E_0 \sin(\omega t + \varphi)$$

$$I \propto E_{tot}^2$$

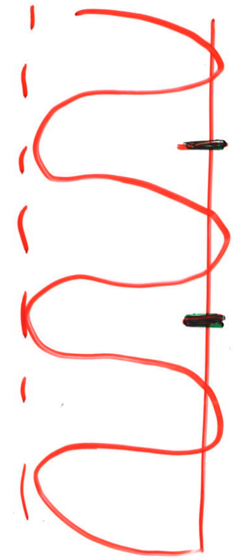
Diffraction Grating



$N=2$



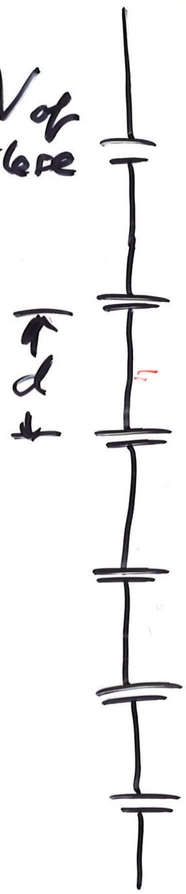
$\Delta y = \frac{L\lambda}{d}$

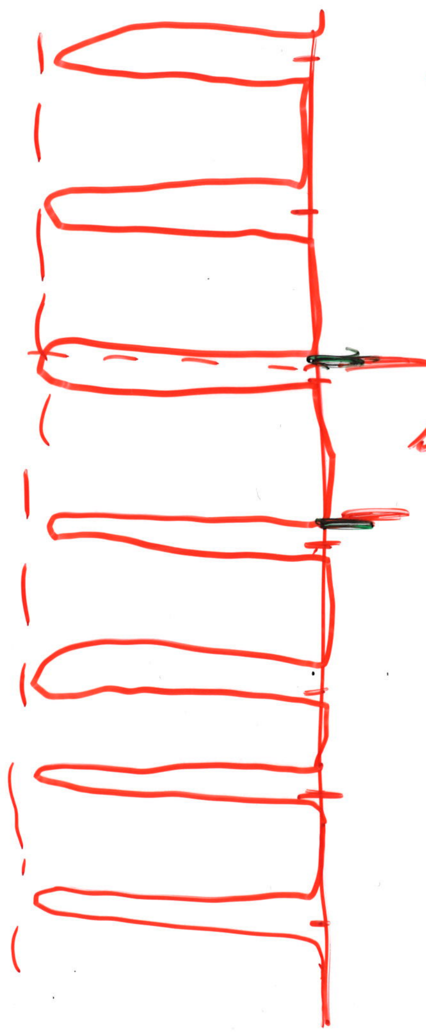


$\cos^2(\phi/2)$

All peaks equally high

N of these





peaks higher than $N=2$

$\Delta y = y_1 - y_0$

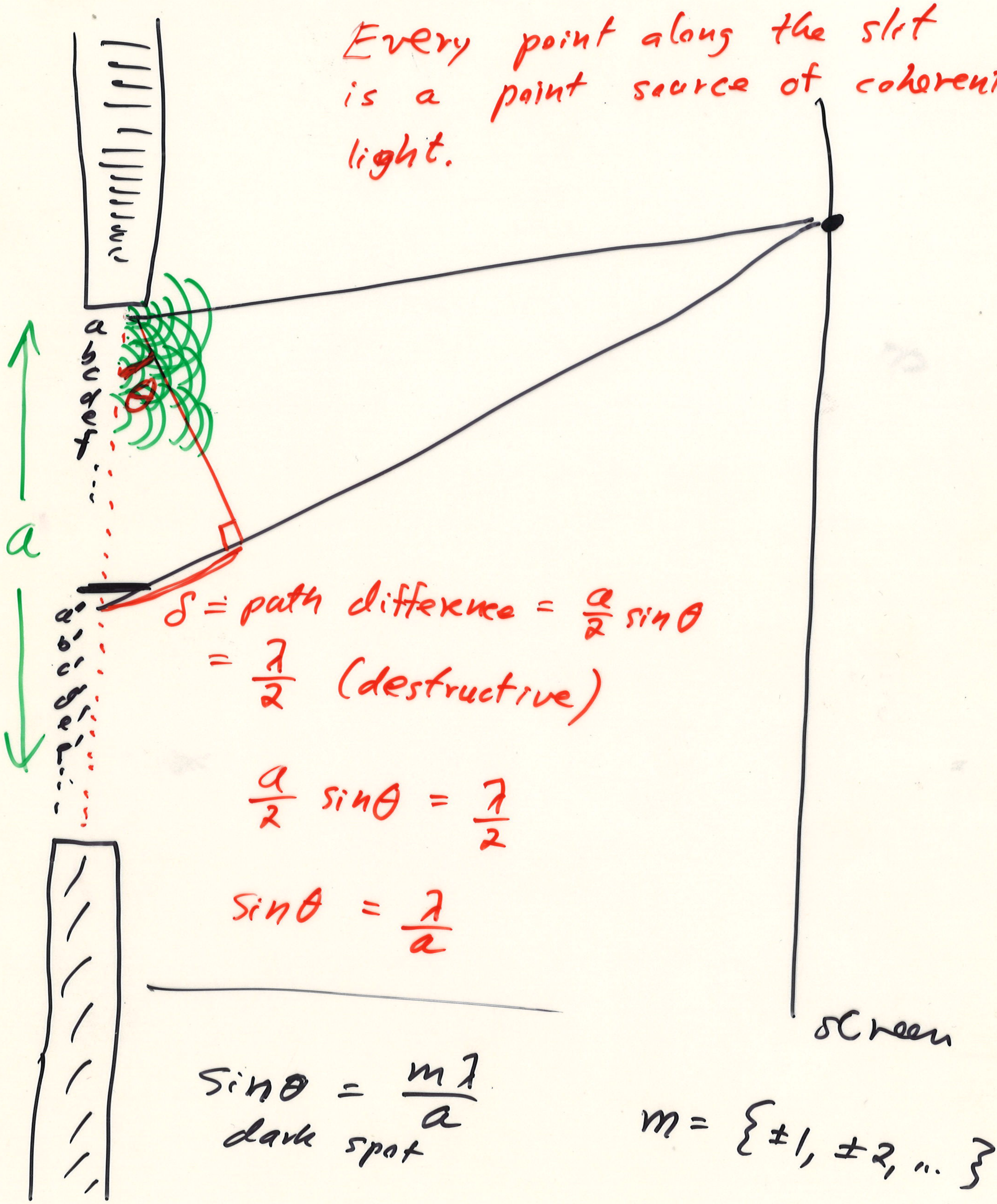
$= \frac{L\lambda}{d}$

$E_{tot} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6$

$I = E_{tot}^2$

Huygen's Principle

Every point along the slit is a point source of coherent light.



δ = path difference = $\frac{a}{2}\sin\theta$

$= \frac{\lambda}{2}$ (destructive)

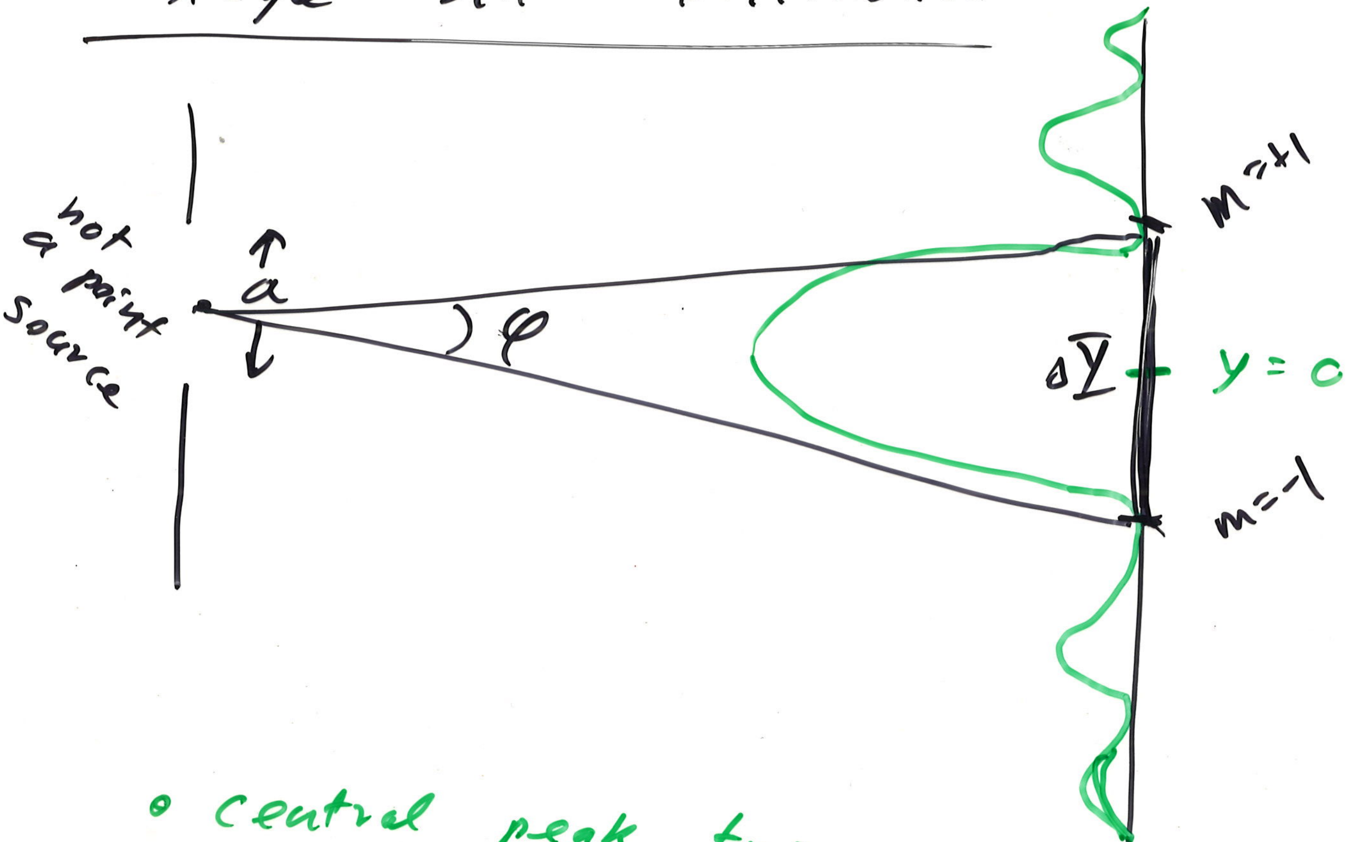
$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$\sin\theta = \frac{\lambda}{a}$$

$\sin\theta = \frac{m\lambda}{a}$ dark spot

$m = \{\pm 1, \pm 2, \ldots\}$

Single Slit Diffraction



- central peak twice as wide as others
- Intensity falls away from $y=0$

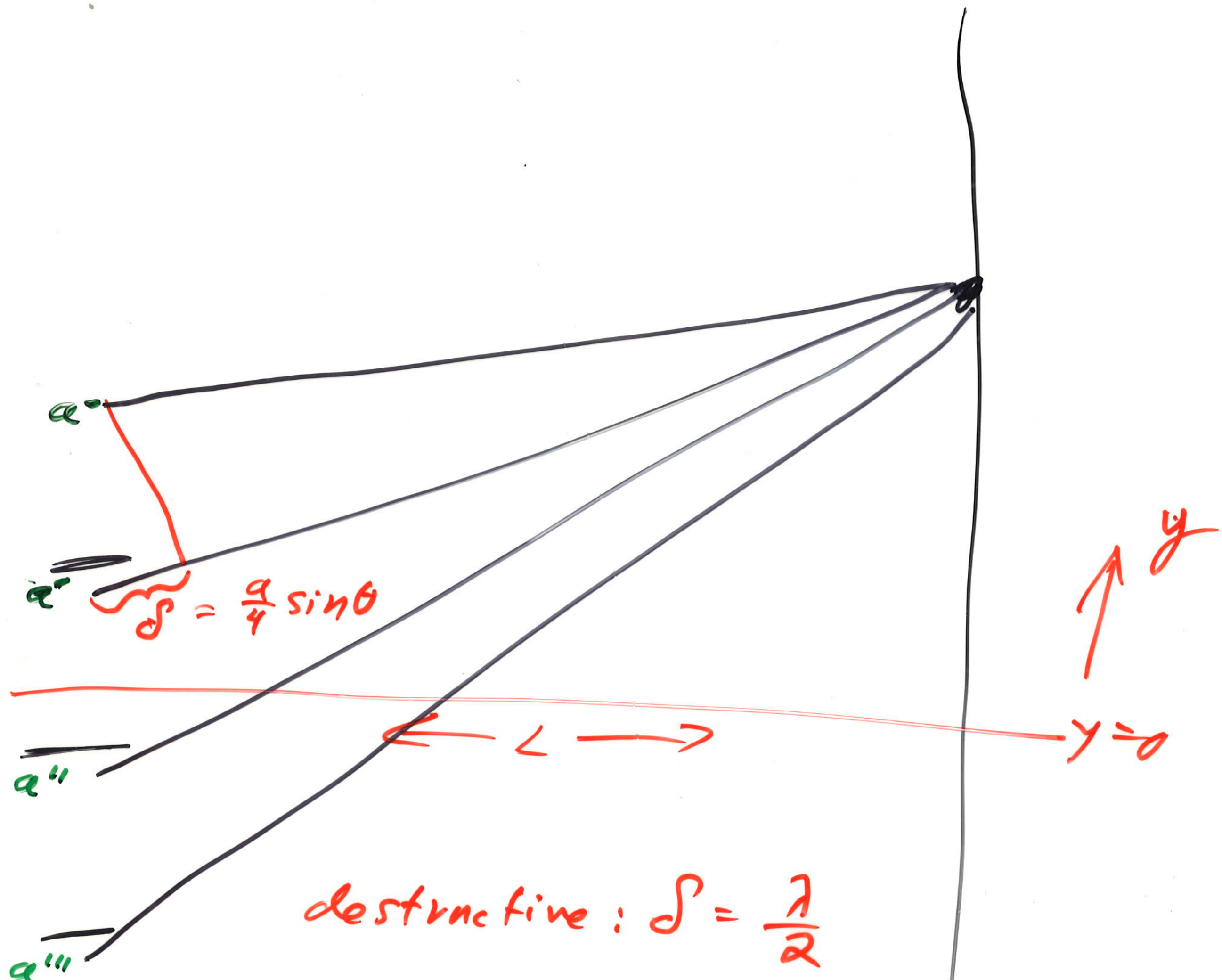
$$\varphi = 2 \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$\Delta \overline{Y} = 2\varphi$$

Single slit Diffraction pattern

Observations

- side peaks are not as bright as the central peak.
- central maximum is <u>twice</u> as wide as other maxima.



destructive: $\delta = \frac{\lambda}{2}$

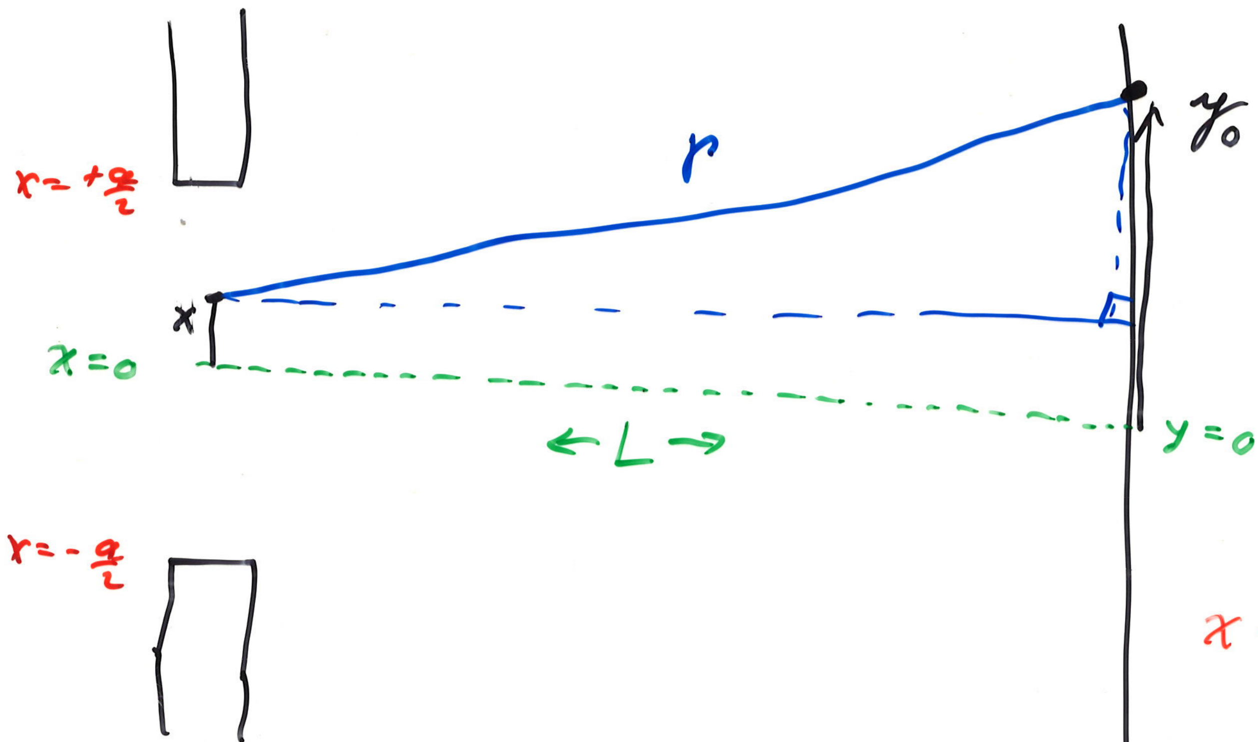
$$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$$

$$\sin\theta = \frac{2\lambda}{a}$$

In general
dark spots at

$$\sin\theta = \frac{m\lambda}{a} = \frac{y}{L}$$

$\boxed{m \neq 0}$



$x \ll y \ll R$

$$r^2 = (y_0 - x)^2 + L^2$$

$$= R^2 + x^2 - 2xy_0$$

$$= R^2\left(1 + \cancelto{0}{\frac{x^2}{R^2}} - \frac{2xy_0}{R^2}\right)$$

$$= R^2\left(1 - \frac{2x}{R}\sin\theta\right)$$

$$R^2 \equiv y_0^2 + L^2$$

$$\left(\frac{x}{R}\right)^2 \sim 0 \qquad \frac{y_0}{R} \sim \sin\theta$$

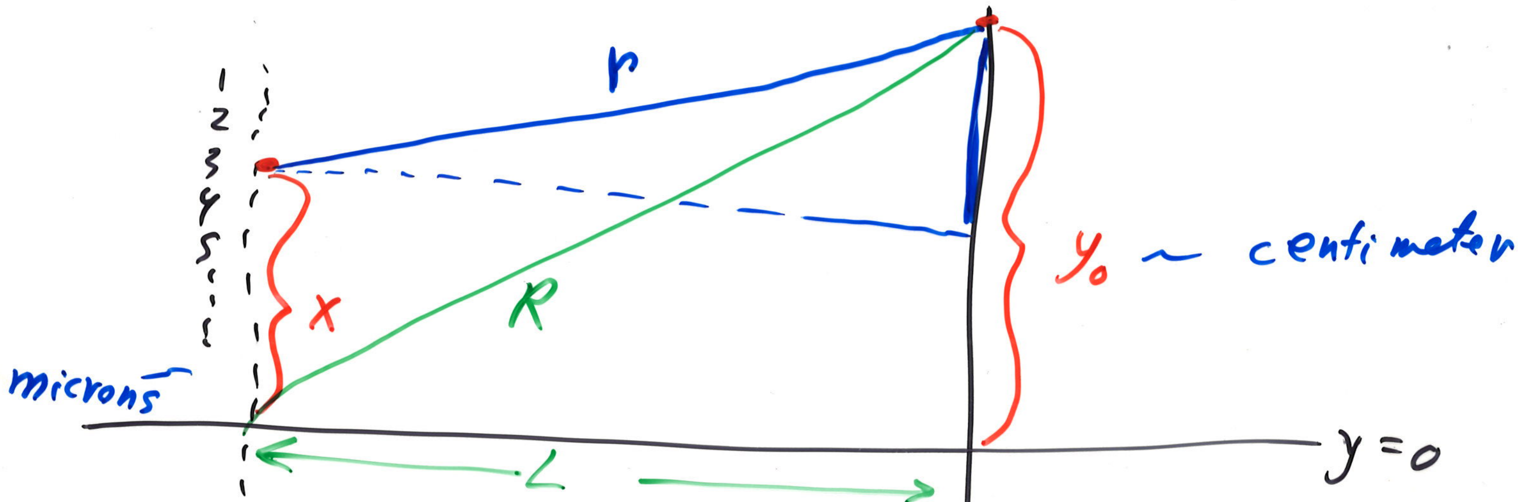
$$r = R\sqrt{1 - \frac{2x}{R}\sin\theta}$$

Binomial theorem $(1-z)^n \approx 1 - nz + \ldots.$ $\quad z \ll 1$

$$r \approx R\left(1 - \frac{x}{R}\sin\theta\right)$$

Intensity

$$E_{tot} = dE_1 + dE_2 + dE_3 + \ldots\ldots$$
$$= \sum_n dE_n \Rightarrow \int dE$$



$$r^2 = (y_0 - x)^2 + L^2$$
$$= y_0^2 + x^2 - 2xy_0 + L^2$$
$$= R^2 + x^2 + -2xy_0$$
$$= R^2\left(1 + \frac{x^2}{R^2} + \frac{-2xy_0}{R^2}\right)$$

$$R^2 = y_0^2 + L^2$$

$$y_0 \gg x$$

$$r = R\sqrt{1 + \cancel{\frac{x^2}{R^2}}^{\,0} - \frac{2xy_0}{R^2}}$$

$$\frac{Y_0}{R} \sim \sin\theta \sim \frac{Y_0}{L}$$

binomial

$$r \approx R\left(1 - \frac{x}{R}\sin\theta\right)$$

$$E_{tot} = dE_1 + dE_2 + \cdots = \sum_n dE_n = \int dE$$

at screen

$$dE = \frac{E_0}{a}dx \sin(kr - \omega t)$$

$K = \frac{2\pi}{\lambda}$

$$dE = \frac{E_0}{a}\sin(kR - \omega t - kx\sin\theta)\,dx$$

$$E_{tot} = \int dE = \int_{x=-\frac{a}{2}}^{+a/2} \frac{E_0}{a}\sin(kR - \omega t - kx\sin\theta)\,dx$$

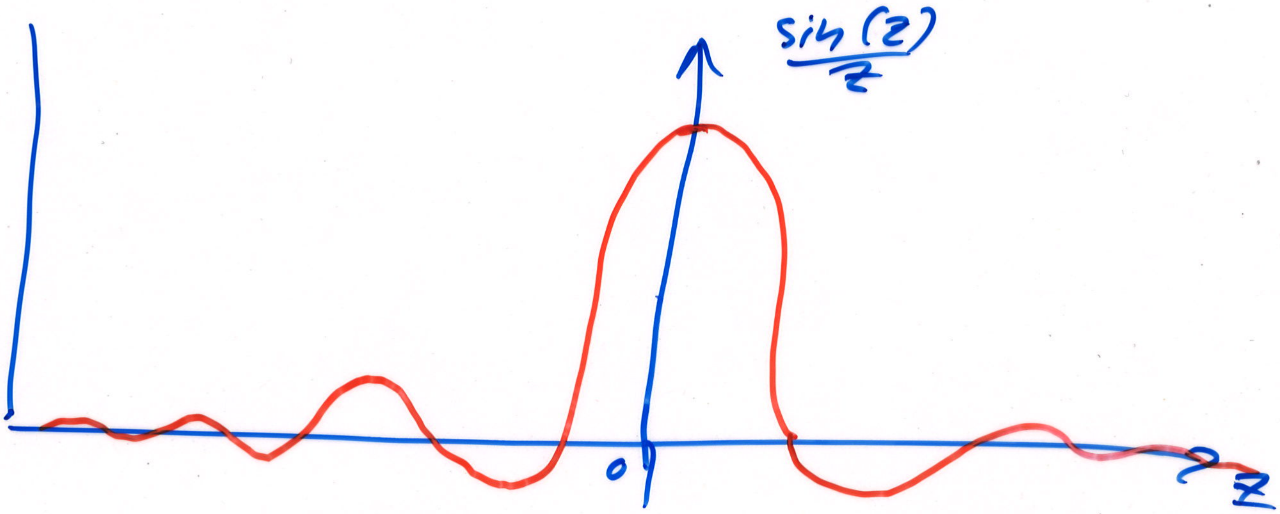
$$= \frac{E_0}{a}\left.\frac{-\cos(kR - \omega t - kx\sin\theta)}{-k\sin\theta}\right|_{x=-a/2}^{+a/2}$$

$$= \frac{E_0}{ak\sin\theta}\left[\cos\left(kR - \omega t - \frac{ka}{2}\sin\theta\right) - \cos\left(kR - \omega t + \frac{ka}{2}\sin\theta\right)\right]$$

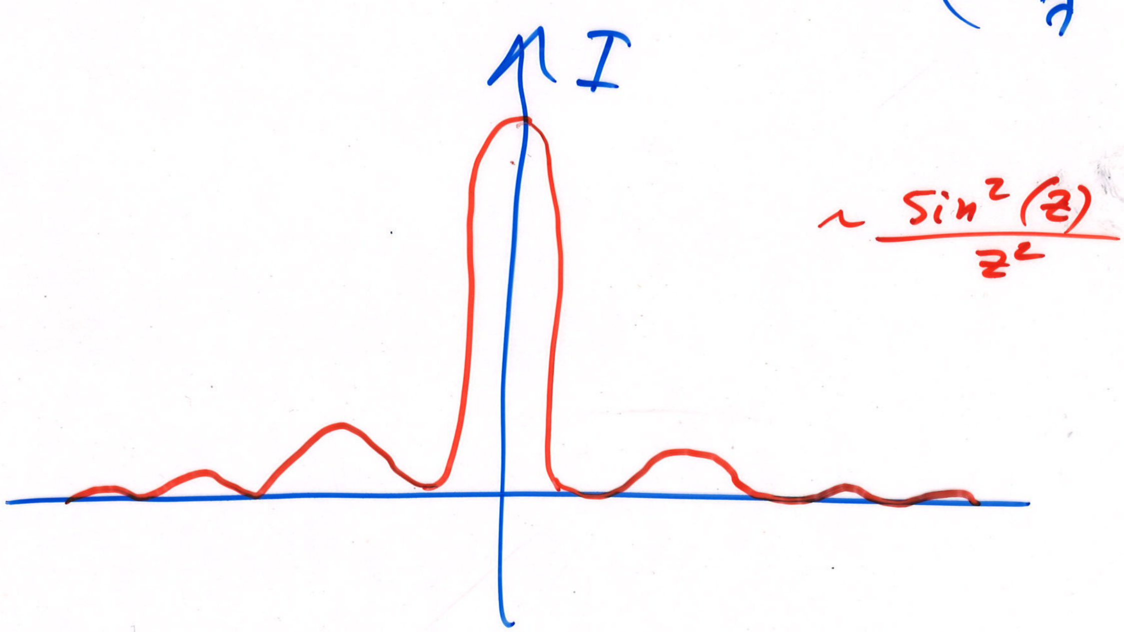
$$= \frac{E_0}{ak\sin\theta}\,2\sin(kR - \omega t)\sin\left(\frac{ka\sin\theta}{2}\right)$$

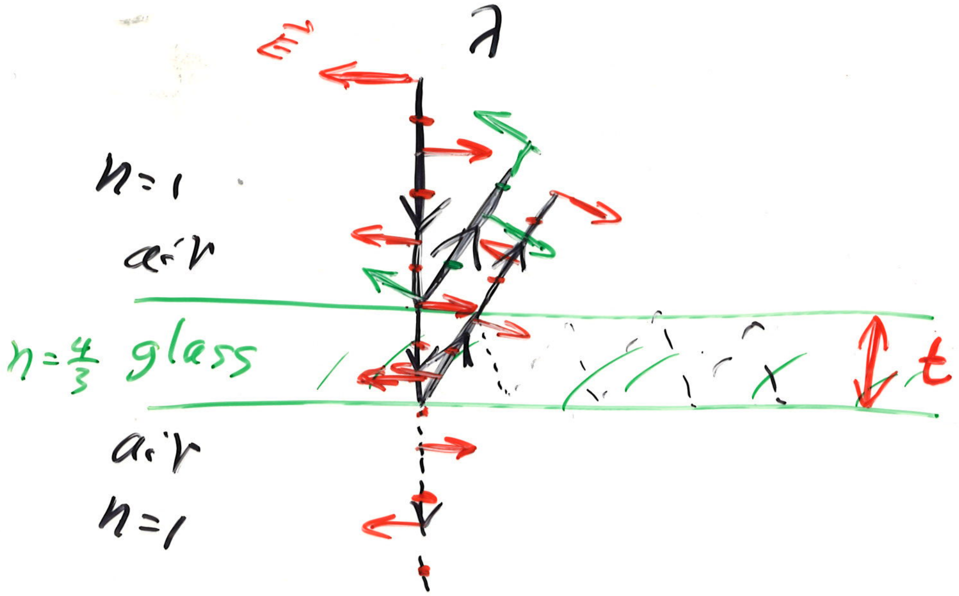
$$= \left[E_0\sin(kR - \omega t)\right]\frac{\sin\left(\frac{ka\sin\theta}{2}\right)}{\frac{ka\sin\theta}{2}}$$

$$E_{tot} = E_0 \sin(kR-\omega t) \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}$$



Intensity: $I \propto \vec{E}^2 \propto \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$





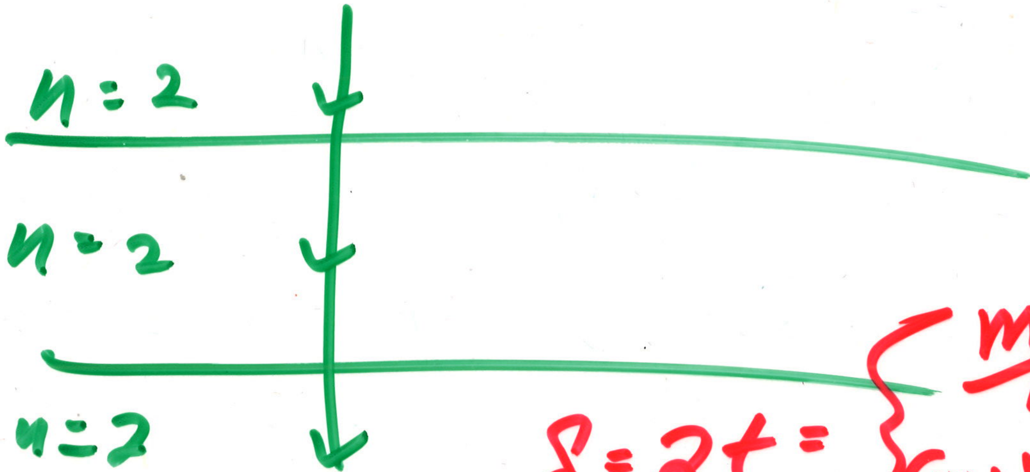
normal incidence $\theta = 0$

from small n to large n, there is a phase shift of 180° upon reflection

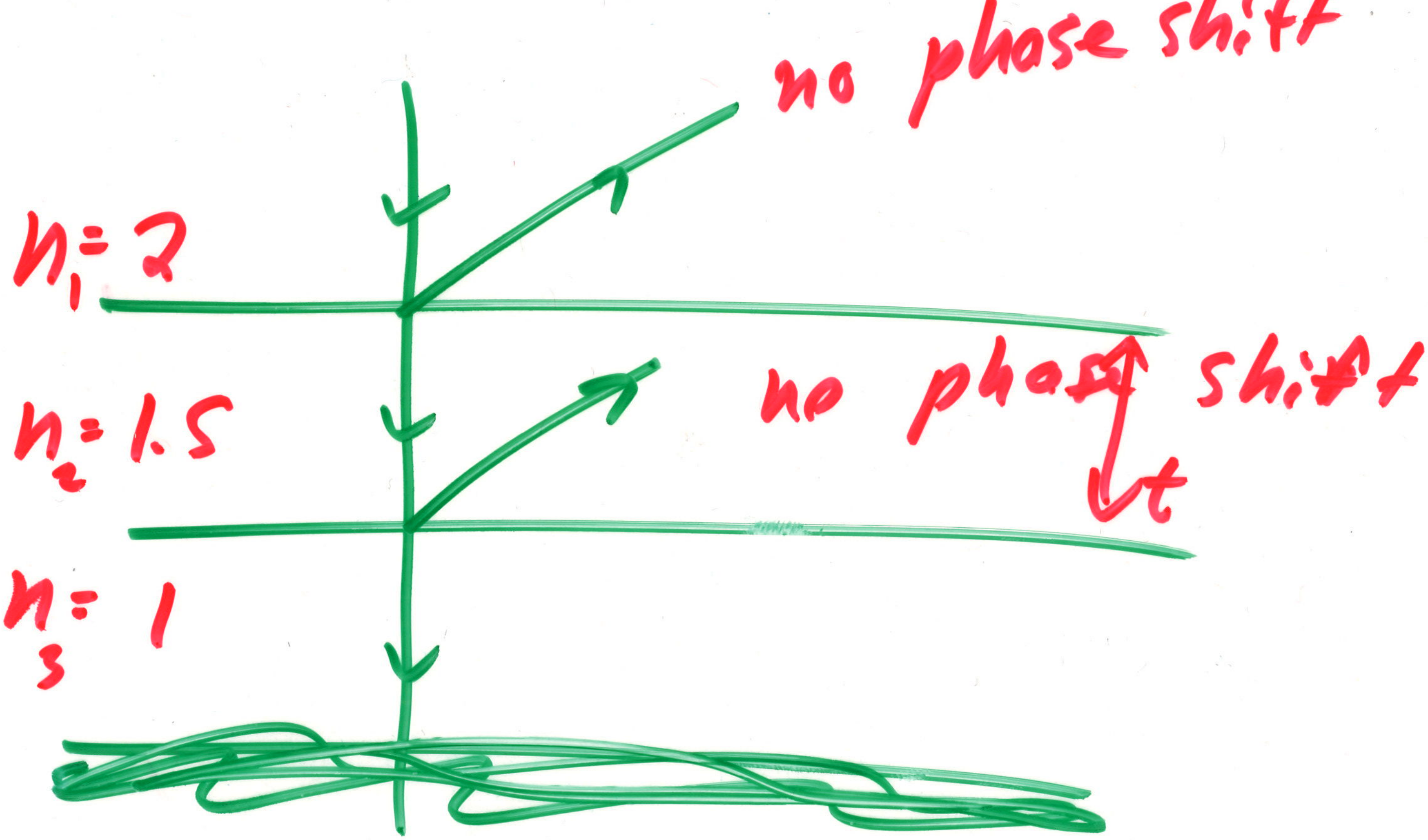
from large n to small n, there is no phase shift, upon reflection

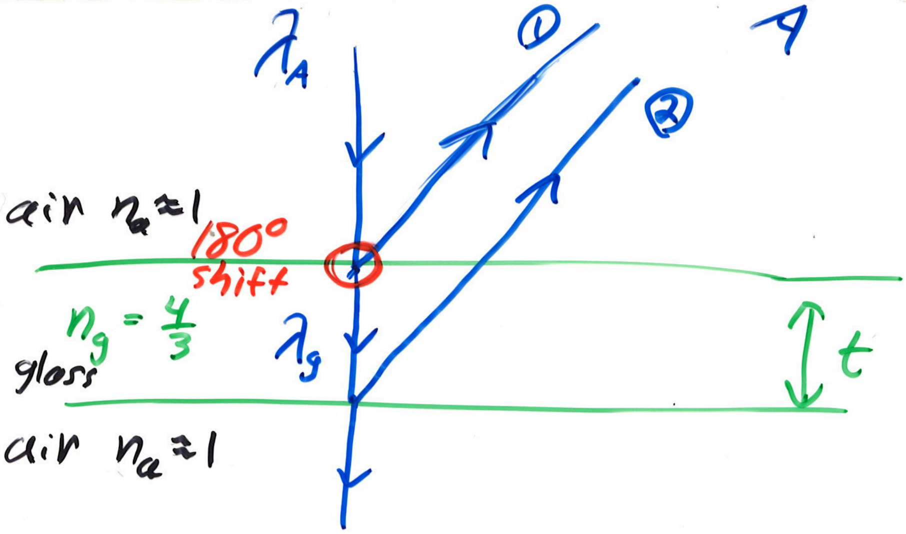
No phase shift upon transmission.

$$\delta = 2t = \begin{cases} (m+\frac{1}{2})\frac{\lambda}{n} & \text{constructive} \\ m\frac{\lambda}{n} & \text{destructive} \end{cases}$$



$$\delta = 2t = \begin{cases} \dfrac{m\lambda}{n_2} & \text{con} \\ \left(m+\frac{1}{2}\right)\lambda / n_2 & \text{des} \end{cases}$$





normal incidence

$\theta_i = 0$

$\delta = 2t$

When the ray travels from small-n medium to a large-n medium, there is a 180° phase shift upon reflection.

When the ray travels from large-n to small-n, there is no phase shift

$$2t = \delta = \begin{cases} m\lambda_g & \text{destructive} \\ (m+\frac{1}{2})\lambda_g & \text{constructive} \end{cases}$$

$$2t = \begin{cases} m\lambda/n_{glass} & \text{dest} \\ (m+\frac{1}{2})\lambda/n_{glass} & \text{const.} \end{cases}$$