Physics 3344

Lecture #1

Read Marion Ch. 1

<u>def</u> scalar - a quantity that does not change when the coordinate system is rotated (reflections later)
e.g. T, P, m, # oranges, ... , t, $|\vec{v}|$, Q, Φ

<u>demo</u> x to my right, y in front of me, standing at origin
T at student position, now turn, new T' is the same.

<u>def</u> vector - a quantity that changes like displacement ($\vec{r}$) under a rotation of coordinates.

<u>demo</u> student is at $\vec{r} = (x,y) = (0,3)$ meters, now turn, $\vec{r}' \neq \vec{r}$

e.g. other vectors: $\vec{x} = \vec{r}$ ($\vec{x}$ is Marion's notation)
$\vec{v} = \frac{d\vec{r}}{dt}$, $\vec{p}$, ... [not $\vec{L}$ angular momentum, see reflections]

<u>Notation</u> $\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{\text{Cartesian}}$ $\qquad \vec{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_\varphi \end{pmatrix}_{\text{spherical polar}}$

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{\text{column}}$ $\qquad \vec{v}^{T} = (v_1, v_2, v_3)$ row

v_i one index $\qquad$ # indices $\equiv$ rank

vectors are rank 1 objects ; scalars have rank 0.

$i = 1, \ldots, d$ where d = dimension of "space" ($d=3$ for now).

<u>Not</u> $\begin{pmatrix} 3 \text{ oranges} \\ 2 \text{ bananas} \\ 4 \text{ apples} \end{pmatrix}$ does not change like $\vec{r}$ under coordinate rotations $\Rightarrow$ not a vector.

• **def** rank n tensor – a quantity that changes like the exterior product of n position vectors

T_{ij} changes under rotations like $r_i r_j$ (or $v_i v_j$)
$\uparrow$ rank 2 $\quad i,j = 1,\ldots,3$ (d in general)

T_{ij} can be represented as a matrix, but not all matrices are rank 2 tensors. In particular, the transformation matrix that relates $\vec{r}'$ to $\vec{r}$ is not a tensor.

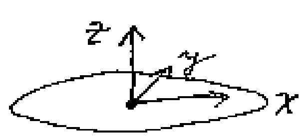
e.g. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 2×2 identity matrix is not a rank 2 tensor.

proof: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_1^2 & r_1 r_2 \\ r_2 r_1 & r_2^2 \end{pmatrix}$ $\quad r_1^2 = 1 \Rightarrow r_1 \neq 0$
$r_2^2 = 1 \Rightarrow r_2 \neq 0$
but $r_1 r_2 = 0$. Impossible.

e.g. rank 2 tensors: Θ_{ij} energy momentum tensor
I_{ij} moment of inertia tensor
(by the way, I = mass, $I_i \propto$ displacement vector of center of mass) $I_i = m\vec{R}_{cm}$
$\sim$ mean $\qquad \sim$ standard deviation
ϵ_{ij} dielectric tensor

T_{ijk} changes like $r_i r_j r_k$
$\uparrow$ rank 3

Hold it! I thought that the moment of inertia about the center of mass for a disk (for example) was $\frac{1}{2} mR^2$. How is this a tensor?



$I_{zz} = \frac{1}{2} m R^2 \qquad I_{xx} = I_{yy} \neq 0$

$I_{xy} = 0 \; ; \; I_{yz} = 0 \; ; \; I_{xz} = 0$

Examples:

scalar — rank 0 — no indices — does not transform under coordinate rotations

$$I = \int \rho \, dV = \text{mass} = m$$

I ← not standard notation; ρ ← density; dV ← volume

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vector — rank 1 — one index — transforms like x_i

$$I_i = \int x_i \rho \, dV = m (x_{cm})_i$$

$$\vec{I} = \int \vec{x} \rho \, dV = \int \vec{r} \rho \, dV = m \vec{x}_{cm}$$

$\vec{I}$ ← not standard notation; $\vec{x}_{cm}$ ← center of mass location

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rank 2 tensor — two indices — transforms like $x_i x_j$

$$I_{ij} = \int x_i x_j \rho \, dV$$

↑ standard notation for moment of inertia tensor