

Due: 31 August 2006

**Read Marion (& Thornton) Chapters 1 and 2.**

1. Use MKS units throughout and give the answers both analytically and numerically. From the following four fundamental constants (and only these)

$$\begin{aligned} G &= 6.67259 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} & c &= 2.99792458 \times 10^8 \text{ m/s} \\ \hbar &= 1.05457266 \times 10^{-34} \text{ J}\cdot\text{s} & k_B &= 1.380658 \times 10^{-23} \text{ J/K} \end{aligned}$$

using dimensional analysis, construct quantities with the dimensions of:

- (a) length
  - (b) time
  - (c) velocity
  - (d) mass
  - (e) energy
  - (f) temperature
2. (a) In the matrix equation  $\underline{\underline{D}} = \underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}}$ , if  $\underline{\underline{D}}$  is a  $2 \times 2$  matrix,  $\underline{\underline{A}}$  is a  $2 \times 5$  matrix, and  $\underline{\underline{C}}$  is a  $2 \times 2$  matrix, what are the dimensions of  $\underline{\underline{B}}$ ? How many rows and how many columns?
- (b) Write the matrix equation  $\underline{\underline{D}} = \underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}}$  in index notation. Include limits on the sums.
3. If  $\mathbf{x}$  is the column vector  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ , what is the result of the following matrix multiplications?
- (a)  $\mathbf{x} \mathbf{x}^T$
  - (b)  $\mathbf{x}^T \mathbf{x}$
4. If two lines have direction cosines  $(\alpha, \beta, \gamma)$  and  $(\alpha', \beta', \gamma')$ , show that the angle  $\theta$  between these lines satisfies:

$$\cos \theta = \cos \alpha \cos \alpha' + \cos \beta \cos \beta' + \cos \gamma \cos \gamma'$$

5. Rotate the vector  $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$  about the axis  $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$  by  $30^\circ$  counterclockwise as seen by looking down the axis from the tip to the origin.

(a) Find the  $3 \times 3$  rotation matrix  $\underline{\lambda}$ .

(b) Find the image of  $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$  under the transformation.

(c) Verify that the axis does not change direction under the rotation.  $\underline{\lambda} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = ?$

(d) Find  $\det(\underline{\lambda})$

(e) What is the image of the moment of inertia tensor below under the transformation?  
(Hint: remember to use two  $\lambda$ 's.)

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$