

Due: 14 September 2006

1. If the scalar function  $\Phi(x, y, z) = x^2y \sin(z) + xy^3z^2$ , what is the gradient of  $\Phi$  in Cartesian coordinates.
2. Suppose that  $\vec{A}(r, \theta, \phi)$  is a vector field with a radial component which can depend on all three spherical coordinates  $A_r(r, \theta, \phi)$ , a polar component which can depend on all three spherical coordinates  $A_\theta(r, \theta, \phi)$ , and an azimuthal component which can depend on all three spherical coordinates  $A_\phi(r, \theta, \phi)$ . What is the polar component of the curl of  $\vec{A}$ ?  $(\vec{\nabla} \times \vec{A})_\theta = ?$
3. A particle moves in a plane elliptical orbit described by the displacement vector

$$\vec{r}(t) = 2b \sin(\omega t) \hat{e}_x + b \cos(\omega t) \hat{e}_y$$

where  $b$  and  $\omega$  are constants. Find

- (a) the velocity vector  $\vec{v}$
  - (b) the scalar speed  $v$
  - (c) the acceleration vector  $\vec{a}$
  - (d) the angle between  $\vec{v}$  and  $\vec{a}$  at time  $t = \frac{\pi}{2\omega}$
4. If the derivative

$$\frac{d}{dt}[\vec{r} \times (\vec{v} \times \vec{r})] = f\vec{a} + g\vec{v} + h\vec{r}$$

find the scalars  $f$ ,  $g$ , and  $h$ .

5. For the vector function of time  $\vec{A}(t)$ , evaluate

$$\int (\vec{A} \times \ddot{\vec{A}}) dt$$