Physics 3344 **Professor Scalise**
Fall 2006 **Assignment #8**

Due: 26 October

1. For each of the following ordinary differential equations, state: the function; the variable; the order of the D.E.; whether it is linear or non-linear; and if linear, whether homogeneous or non-homogeneous.

   (a) $3\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 7 = 0$

   (b) $2\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 6t = 0$

   (c) $4\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x\sin(t) = 0$

   (d) $\frac{d^3x}{dt^3} = 9x$

   (e) $\frac{d^2y}{dt^2} + 2\beta\frac{dy}{dt} + \omega_0^2 y = \sin(y)$ (β and ω_0 are constants)

   (f) $7\frac{d^4y}{dx^4} + 5\frac{d^3y}{dx^3} + y^2 = 0$

   (g) $x^2 + 3\frac{dx}{dt} = 13$

2. For the damped sinusoidally driven oscillator derive the resonance frequencies for

   (a) amplitude of displacement

   (b) potential energy

   (c) speed

   (d) kinetic energy

   that is, find the frequency ω at which the driver must operate in order to maximize the quantities listed above.

3. Two masses m_1 and m_2 slide freely in a horizontal frictionless track and are connected by a spring of force constant k. Find the natural frequency of oscillation for this system. This is a model the the vibrational modes of a diatomic molecule.

4. A critically damped oscillator with initial displacement $x(0) = 0$ and initial velocity $v(0) = 0$ is subjected to a constant force F_0 beginning at time $t = 0$.

   (a) What is the particular solution for times $t > 0$?

   (b) What is the response (i.e. the displacement, i.e. the general solution) of the system for times $t > 0$?

   (c) What is the response of the system after a very long time?

   (d) Sketch the displacement vs. time.

5. Consider a damped oscillator driven with a linearly increasing force, $F(t) = J_0 m t$. (J_0 is a constant jerk.) Find a particular solution for $t > 0$ to

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = J_0 t$$