Functional I

$$I = \int_{x_1}^{x_2} f\left[y(x), y'(x); x\right] dx$$

variable: x , function: $y(x)$

$$y' = \frac{dy}{dx}$$

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e.g. $y(x) = x^2$ $\quad y' = 2x$

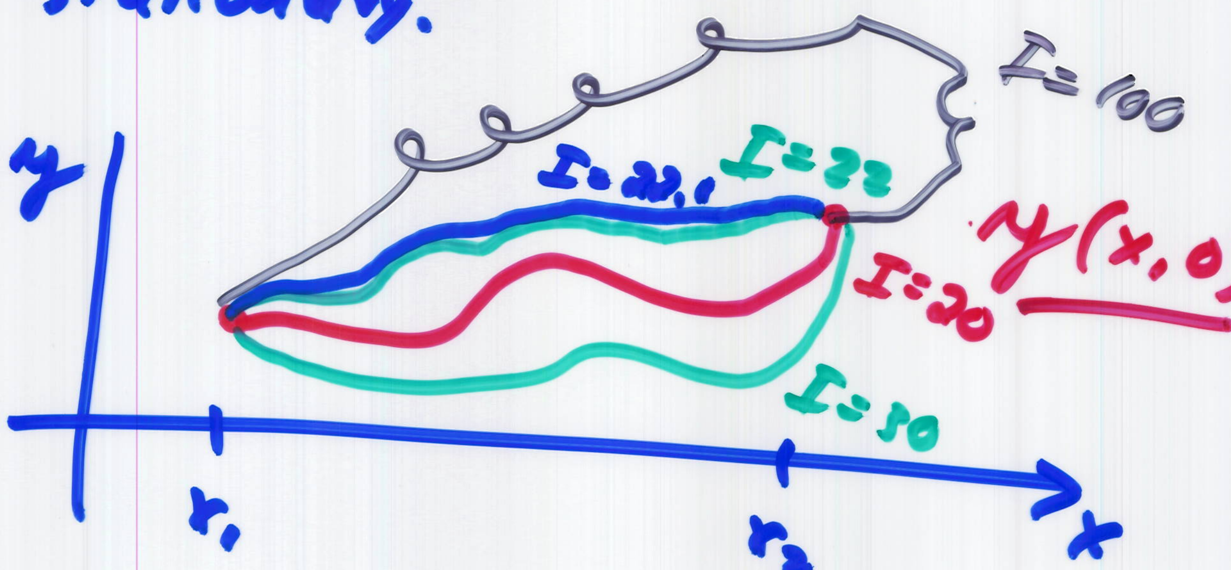
$$f = y^3/y' + \sin(y)\, x$$

$$f = \frac{x^6}{2x} + \sin(x^2)\, x$$

$$I = \int_{x_1}^{x_2} \left[\frac{x^5}{2} + \sin(x^2)\, x\right] dx = \#$$

$$I = \int_{x_1}^{x_2} f[y(x), y'(x); x]\, dx$$

Find paths $y(x)$ that makes I stationary.



$$y(x, \alpha) = \underline{y(x, 0)} + \alpha\, \eta(x)$$

$$y'(x, \alpha) = \underline{y'(x, 0)} + \alpha\, \eta'(x)$$

$$\eta(x_1) = 0 = \eta(x_2)$$

$$\delta I = 0 = \frac{\partial I}{\partial \alpha} \delta\alpha$$

$$\delta I = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y} \underbrace{\frac{dy}{d\alpha}}_{\eta(x)} \delta\alpha + \frac{\partial f}{\partial y'} \underbrace{\frac{dy'}{d\alpha}}_{\eta'(x)} \delta\alpha \right) dx$$

$$\delta I = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\eta + \frac{\partial f}{\partial y'}\eta' \right) \delta\alpha \, dx$$

Use integration by parts to remove the derivative from η'. Then both terms will be proportional to η.

2nd term: $\displaystyle \int_{x_1}^{x_2} \frac{\partial f}{\partial y'} \frac{d\eta}{dx} \delta\alpha \, dx$

$$= \frac{\partial f}{\partial y'} \eta \Big|_{x_1}^{x_2} \delta\alpha \to 0 \; - \int_{x_1}^{x_2} \left(\frac{d}{dx} \frac{\partial f}{\partial y'} \right) \eta \, \delta\alpha \, dx$$

"surface" term

$$\delta I = \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'} \right) \right] \eta(x) \, \delta\alpha \, dx = 0 \quad \forall \, \eta(x)$$

$$\Longrightarrow \frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'} \right) = 0$$