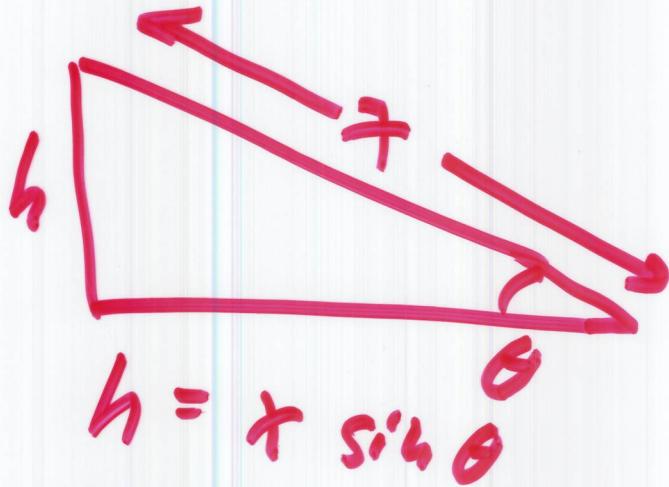


What is time  $t$ ?

$$x = \frac{1}{2} g \sin \theta t^2 = \frac{h}{\sin \theta}$$



$$t = \sqrt{\frac{2h}{g}} \cdot \frac{1}{\sin \theta}$$

$$h = x \sin \theta$$

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$$V = \frac{2}{3} g \sin \theta \quad t = \frac{2}{3} g \sin \theta \sqrt{\frac{3h}{g \sin \theta}}$$

$$V = \sqrt{\frac{hg}{3}} = \sqrt{\frac{4hg}{3}}$$

1) Newton:  $\vec{F} = m\vec{a} = m \frac{d^2\vec{r}}{dt^2}$

2) Energy Conservation:  $dE = 0$

3) Lagrange

Calculate the kinetic energy

$$K = \sum_i \frac{1}{2} m_i v_i^2 = T_{(t)}$$

Calculate the potential energy <sup>in time</sup>

$$U_{(t)} = \sum_i m_i g y_i + \text{constant}$$

The quantity  $T_{(t)} - U_{(t)} = L_{(t)}$

this is the Lagrangian

The integral  $\int_{-\infty}^{+\infty} L_{(t)} dt = S$

The actual path, speed, acceleration etc. is the one that extremizes  $S$  (usually, but not always, minimizes  $S$ ). <sup>Action</sup>

## 4) Hamilton

Define generalized coordinates  
 $\{q_1, q_2, \dots, q_n\}$

lengths, angles, charges, Fourier coefficients ...

$$L = T - U = L(q_1, q_2, \dots, q_n)$$

Form the "conjugate momentum"

$$P_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

Form Hamiltonian

$$H \equiv (\sum_i P_i \dot{q}_i) - L$$

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$$\dot{q}_i = \frac{\partial H}{\partial P_i} \quad ; \quad \dot{P}_i = -\frac{\partial H}{\partial q_i}$$

two 1st order Differential Equations  
for each coordinate  $q_i$ .

$$\vec{D} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{D} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$D_x = \rho / \begin{vmatrix} A_x & A_z \\ B_y & B_z \end{vmatrix}$$

$$D_y = \rho (A_y B_z - A_z B_y)$$

$$D_z = A_z B_y - A_y B_z$$


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Kronecker Delta Symbol

$$\delta_{ij} = \begin{cases} +1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\delta_{11} = 1, \quad \delta_{22} = 1, \quad \delta_{12} = 0$$

$$\begin{aligned}
 \vec{r} &= x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3 \\
 &= x\hat{e}_x + y\hat{e}_y + z\hat{e}_z \\
 &= x\hat{x} + y\hat{y} + z\hat{z}
 \end{aligned}$$

$$v_1 = v_x = \frac{dx}{dt} \quad ; \quad v_2 = v_y = \frac{dy}{dt}$$

$\frac{df}{dt}$  Leibniz notation

$\dot{f}$  Newton notation

$$a_1 = a_x = \frac{d^2 x}{dt^2} = \ddot{x}$$

$$a_3 = a_z = \frac{d^2 z}{dt^2} = \ddot{z}$$