Continued...

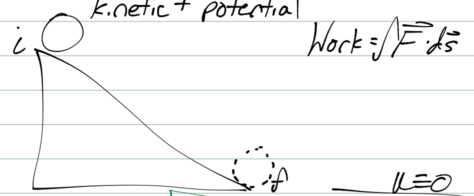
Again with Energy Conservation

Total Mechanical Energy

$E = K + U$

kinetic + potential

Work $= \int \vec{F} \cdot d\vec{s}$



$E_i = E_f \Rightarrow \boxed{K_i + U_i = K_f + U_f}$

$(K_f - K_i) + (U_f - U_i) = 0$

$\Delta K + \Delta U = 0$

$\Delta E = 0$

$\omega = \frac{v}{R}$

$0 + Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + 0$

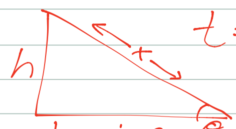
$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$

$= \frac{3}{4}Mv^2$

$v = \sqrt{\frac{4gh}{3}}$

What is time t?

$x = \frac{1}{3}g\sin\theta\, t^2 = \frac{h}{\sin\theta}$



$t = \sqrt{\frac{3h}{g}\frac{1}{\sin\theta}}$

$h = x\sin\theta$

$V = \frac{2}{3}g\sin\theta\, t = \frac{2}{3}g\sin\theta\sqrt{\frac{3h}{g}\frac{1}{\sin\theta}}$

$V = \sqrt{\frac{4hg}{3}}$

1) Newton: $\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$

2) Energy Conservation: $\Delta E = 0$

3) Lagrange

Calculate the kinetic energy

$K = \sum_i \frac{1}{2}m_i v_i^2 = T_{(t)}$ — time

$U_{(t)} = \sum_i m_i g y_i +$ constant

The quantity $T_{(t)} - U_{(t)} = \mathcal{L}_{(t)}$

This is the Lagrangian

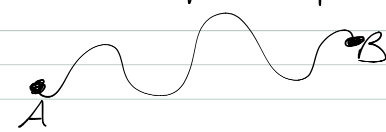
The integral $\int_{-\infty}^{\infty} \mathcal{L}(t)\,dt = \underset{\text{action}}{\underset{\uparrow}{S}}$

$T(t) + V(t) = \underset{\text{constant}}{\underset{\uparrow}{E}}$

The actual path, speed, acceleration, etc., is the one that extremizing (usually, but not always, minimizes).

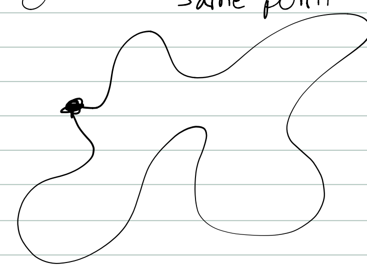
Remember

- $\vec{F} = -\vec{\nabla} U$
- $\vec{F}$ is conservative
- Work does not depend on path



- Zero work if start and end at the same point

$W = 0$ same point



- $\vec{\nabla} \times \vec{F} = 0$

4) Hamilton

Define generalized coördinates

$\{q_1, q_2, q_3, \ldots q_n\}$

lengths, angles, energies, Fourier coefficients...

$L = T - U = L(q_1, q_2, q_3, \ldots q_n)$

Form the "conjugate momentum"

$P_i = \frac{\partial L}{\partial \dot{q}_i}$

Form Hamiltonian

$H \equiv \left(\sum_i P_i \dot{q}_i\right) - L$

$\dot{q}_i = \frac{\partial H}{\partial P_i}$; $\dot{P}_i = -\frac{\partial H}{\partial q_i}$

Two 1st order differential equations for each coördinate q.

$\vec{D} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{D} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ $D_x = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}$ $D_x = (A_y B_z - A_z B_y$

$D_1 = A_2 B_3 - A_3 B_2$

$\vec{A} \times \vec{B} \cdot \vec{C}$

$\vec{B} \times \vec{C} \cdot \vec{A}$

$\vec{C} \times \vec{A} \cdot \vec{B}$

$\vec{A} \cdot \vec{B} \times \vec{C}$

$\vec{B} \cdot \vec{C} \times \vec{A}$

$\vec{C} \cdot \vec{A} \times \vec{B}$

A useful vector identity

$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

Kronecker Delta Symbol

$\delta_{ij} = \begin{cases} +1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

$\delta_{11} = 1, \quad \delta_{22} = 1, \quad \delta_{13} = 0$

$\vec{r} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$

$= x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$

$= x\vec{i} + y\vec{j} + z\vec{k}$

$= x\vec{x} + y\vec{y} + z\vec{z}$

$v_1 = v_x = \frac{dx}{dt}$; $v_2 = v_y = \frac{dy}{dt}$

$\frac{df}{dt}$ Leibnitz notation

$\dot{f}$ Newton notation

$a_1 = a_x = \frac{d^2x}{dt^2}$

$a_3 = a_z = \frac{d^2z}{dt^2}$

$v = |\vec{v}| = |\dot{\vec{r}}| \neq \dot{r}$