Algebraic Equations

unknown : x

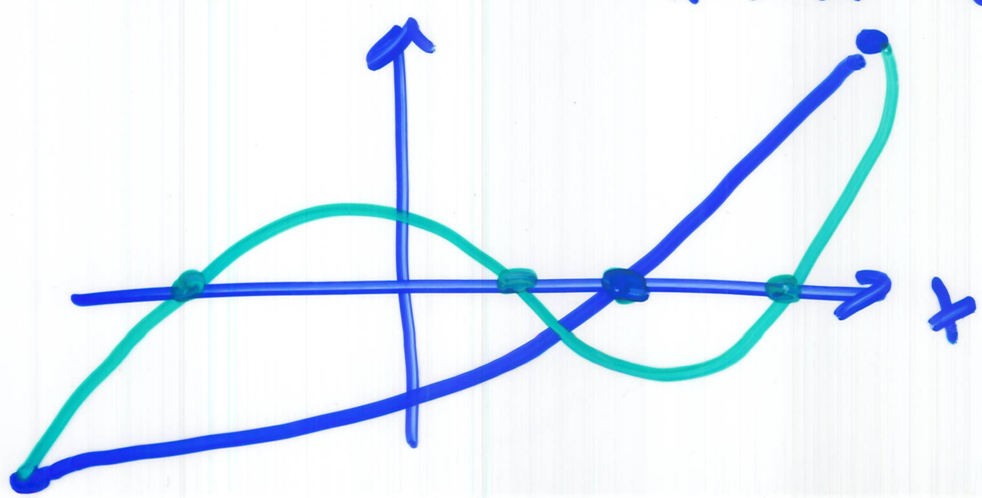
e.g.: $x^2 + 7x + 3 = 0$

goal: find the solutions (roots)
These are <u>numbers</u> that make the equation true.

The number of roots is the same as the highest power.

e.g. $x^2 - 4 = 0 \longrightarrow x_1 = +2$
$x_2 = -2$

$x^3 + bx^2 + cx + d = 0$



Differential Equations

unknown: $x(t)$

e.g.: $\ddot{x}(t) + \omega_0^2 \, x(t) = 0$ simple harmonic oscillator

goal: find the solutions - functions of time - that make the differential equation (D.E.) true.

The number of independent solutions is equal to the <u>order</u> of the D.E.

<u>order</u> - highest derivative

e.g. $$\frac{d^2 x(t)}{dt^2} + \omega_0^2 \, x(t) = 0$$

order = 2

$\Rightarrow$ two constants of integration. These are fixed by boundary conditions (initial conditions)

Eg. initial conditions

① $x(t_0) = 3\,\text{m}$, $\dot{x}(t_0) = 5\,\text{m/s}$

② $x(t_0) = 3\,\text{m}$, $\dot{x}(t_1) = 7\,\text{m/s}$

③ $x(t_0) = 3\,\text{m}$, $x(t_1) = 4\,\text{m}$

Linearity (in x)

Every term has one or zero derivatives of $x(t)$, including the zeroeth derivative which is just $x(t)$ itself.

$x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, ...

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e.g. $\ddot{x}(t) + \omega_0^2 x(t) = 0$ Linear
mass on a Hooke's law spring.

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e.g. $\ddot{x}(t) + \dfrac{g}{\ell}\sin[x(t)] = 0$

Nonlinear

pendulum w/o small angle approximation

"The study of non-linear physics is like the study of non-elephant biology." – Stanislaw Ulam

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Homogeneity

A D.E. is homogeneous if there are no terms without x or its derivatives.

e.g. $\ddot{x}(t) + \omega_0^2 x(t) = 0$ homogeneous

e.g. Damped Driven SHO

$$\ddot{x}(t) + 2\beta \dot{x}(t) + \omega_0^2 x(t) = \frac{F_0}{m} \cos(\omega t)$$

non-homogeneous

The solution to a homogeneous D.E. is called the complementary solution

$x_c(t)$ ← this has n arbitrary constants where n = order

The solution to a non-homogeneous D.E. is the general solution

$$x(t) = x_{gen}(t) = x_c(t) + x_p(t)$$

↗ particular solution

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e.g. $\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = 0$

homogeneous

solution is $x_c(t)$

$$\ddot{x}_c(t) + 2\beta\dot{x}_c(t) + \omega_0^2 x_c(t) = 0$$

$$\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = \frac{F_0}{m}\cos(\omega t)$$

$$x_{gen}(t) = x_c(t) + x_p(t)$$

$$= x_c(t) + \frac{F_0/m}{\omega_0^2 - \omega^2}\cos(\omega t - \delta)$$

Taylor: Horizontal Motion with linear Drag

Newton 2: $\sum \vec{F} = m\vec{a}$

$$-bv = ma \quad x\text{-component}$$

$$m\ddot{x}(t) + b\dot{x}(t) = 0$$

2nd order, linear (in x), homogeneous ordinary, differential equation

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$$m\dot{v}(t) + bv(t) = 0$$

1st order " " " "

Vertical Motion w/ linear Drag

$$\sum \vec{F} = m\vec{a}$$

$$-bv - mg = ma \quad (x\text{-comp})$$

$$m\ddot{x}(t) + b\dot{x}(t) = -mg$$

2nd order, linear, <u>non</u> homogeneous ordinary, D.E.