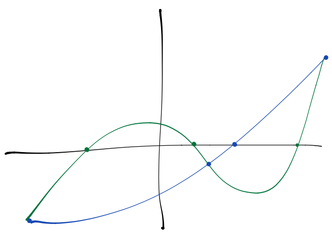
Cartesian $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$
$h_x = 1,\ h_y = 1,\ h_z = 1$

Spherical Polar $\vec{r} = r\hat{e}_r + 0\hat{e}_\theta + 0\hat{e}_\phi$
$\hat{e}_r(\theta, \varphi)$

Cylindrical Polar $\vec{r} = \rho\hat{e}_\rho + 0\hat{e}_\varphi + z\hat{e}_z$
$\hat{e}_\rho(\varphi)$ $\hat{e}_\varphi(\varphi)$

Algebraic Equations

Unknown: x
Ex: $x^2 + 7x + 3 = 0$
Goal: Find the solutions (roots). These are numbers that make the equation true.
The number of roots is the same as the highest power.
Ex: $x^2 - 4 = 0 \Rightarrow x_1 = +2,\ x_2 = -2$
$x^3 + bx^2 + cx + d$



Differential Equations

Unknown: $x(t)$
Ex: $\ddot{x}(t) + \omega_0^2 + x(t) = 0$
Goal: Find the solutions - functions of time - that makes the differential equations true
The number of independent solutions is equal to the order of the differential equation
Order - Highest derivative
Ex: $\frac{d^2x}{dt^2} + \omega_0^2 x(t) = 0$; order $= 2 \Rightarrow$ Two constants of integration. These are fixed by boundary conditions (initial conditions).
Ex: Initial Conditions
1. $x(t_0) = 3m,\ \dot{x}(t_0) = 5\frac{m}{s}$
2. $x(t_0) = 3m,\ \dot{x}(t_1) = 7\frac{m}{s}$
3. $x(t_2) = 3m,\ x(t_1) = 4m$

Linearity (in x)

Every term has one or zero derivatives of $x(t)$, including the zeroeth derivative which is just $x(t)$ itself.
$x(t), \dot{x}(t), \ddot{x}(t), \ldots$
Ex: $\ddot{x}(t) + \omega_0^2 x(t) = 0$ Linear mass on a Hooke's Law spring
$\ddot{x}(t) + \frac{g}{l}\sin[x(t)] = 0$ Non-linear pendulum without small-angle approximation

"The study of non-linear physics is like the study of non-elephant biology."
~ Stanislaw Ulam

Homogeneity

A differential equation is homogeneous if there are no terms without x or its derivatives

Ex: $\ddot{x}(t) + \omega_0^2 x(t) = 0$ Homogeneous

Damped driven simple harmonic oscillator (SHO)

$\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = \frac{F_0}{m}\cos(\omega t)$ Non-homogeneous

The solution to a homogeneous differential equation is called the complementary solution

$X_c(t)$; this has n arbitrary constants where n = order

The solution to a non-homogeneous differential equation is the general solution

$X(t) = X_{gen}(t) = X_c(t) + \underbrace{X_p(t)}_{\text{particular solution}}$

Ex: $\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = 0$ Homogeneous

Solution is $X_c(t)$

$\ddot{X}_c(t) + 2\beta\dot{X}_c(t) + \omega_0^2 X_c(t) = 0$

$\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = \frac{F_0}{m}\cos(\omega t)$

$X_{gen}(t) = X_c(t) + X_p(t)$

$= X_c + \frac{F_0/m}{\omega_0^2 - \omega^2}\cos(\omega t - \delta)$

Horizontal Motion With Linear Drag

Newton's Second Law: $\Sigma\vec{F} = m\vec{a}$

$-bv = m\vec{a}$ (x-components)

$m\ddot{x}(t) + b\dot{x}(t) = 0$

2nd order, linear, homogeneous, ordinary differential equation

$m\dot{v}(t) + bv(t)$

1st order, linear, homogeneous, ordinary differential equation

Vertical Motion with Linear Drag

$\Sigma\vec{F} = m\vec{a}$

$-bv - mg = ma$ (x-component)

$m\ddot{x}(t) + b\dot{x}(t) = -mg$

2nd order, linear, non-homogeneous, ordinary differential equation