Physics 3334
Classical mechanics

Thursday, January 31, 2013

Different Coordinate Systems

- Cartesian

  $$\mathbf{r} = x\,\mathbf{e_x} + y\,\mathbf{e_y} + z\,\mathbf{e_z}$$

- Spherical Polar

  $$\mathbf{r} = r\,\mathbf{e_r} + 0\,\mathbf{e_\theta} + 0\,\mathbf{e_\phi}$$

- Cylindrical Polar

  $$\mathbf{r} = \rho\,\mathbf{e_\rho} + 0\,\mathbf{e_\theta} + z\,\mathbf{e_z}$$

Cartesian is the easiest coordinate system because the scale functions all equal 1
$h_x = 1$; $h_y = 1$; $h_z = 1$

$\mathbf{e_r}(\theta, \phi)$

$\mathbf{e_\rho}(\phi)$

Types of Drag Forces

Taylor mentions two types of drag forces

- Linear

  Like a feather falling in the air

- Quadratic

  Like a cannonball flying through the air

However, there is another kind of drag force…

- Constant

  $f_k = \mu_k N$

  The frictional drag force is independent of the speed of the object.

Introduction to Differential equations

- To start off our exploration of the unknown, differential equations, first we'll explore a more familiar terrain… Algebraic Equations

  unknown: x

  e.g.: $x^2 + 7x + 3 = 0$

  goal: to find the solutions (or roots)

- The solutions are the <u>numbers</u> that make the equation true

Algebraic equations

- The number of roots is given by the highest power of x in the equation

e.g.: $x^2 - 4 = 0$

=> $x_1 = +2$

$x_2 = -2$

Differential Equations

unknown: x(t)

e.g.: $x''(t) + \omega_0^2 x(t) = 0$

Simple Harmonic Oscillator (S.H.O.)

- goal: to find the solutions – functions of time – that make the differential equation (D.E.) true
- The number of independent solutions is equal to the order of the D.E.

Order = highest derivative

e.g. in the S.H.O. above,

order = 2

and solving this D.E. would give two constants of integration

Boundary Conditions

- The constants of integration are fixed by the boundary conditions (or initial conditions)

e.g. initial conditions

(1) $x(t_0) = 3$ m; $x'(t_0) = 5$ m/s

(2) $x(t_0) = 3$ m; $x'(t_1) = 7$ m/s

(3) $x(t_0) = 3$ m; $x(t_1) = 4$ m

But initial conditions of two different velocities, you would not be able to solve for one of the two arbitrary constants of integration, while the other one would be over-constrained.

LINEARITY (IN X)

- An equation is considered linear if and only if every term has one or zero derivatives of x(t), including the 0th derivative, which is just x(t) itself.

e.g. $x''(t) + \omega_0^2\, x(t) = 0$

Linear mass on a Hooke's law spring

e.g. $x''(t) + (g/\, l)\sin[x(t)] = 0$

Nonlinear pendulum without the small angle approximation

"The study of nonlinear physics is like the study of non-elephant biology."
~Stanislaw Ulam

Homogeneity

- A differential equation is homogeneous if there are no terms without x or its derivatives

e.g. $x''(t) + \omega_0^2\, x(t) = 0$ … *homogeneous*!

e.g. Damped Driven S.H.O.

$x''(t) + 2\beta\, x'(t) + \omega_0^2\, x(t) = F_0 \cos(\omega t) / m$

Natural Frequency

Driving Frequency

is *nonhomogeneous* b/c of the last term

General Solutions

- The solution to a homogeneous differential equation is called the complementary solution

  $x_c(t)$ – has n arbitrary constants, where

  n = order

- The solution to a non-homogeneous differential equation is the general solution.

  $$x(t) = x_{gen}(t) = x_c(t) + x_p(t)$$

Particular Solution

Example…

$x''(t) + 2\beta\, x'(t) + \omega_0^2\, x(t) = 0$ … homogeneous,

so the solution is $x_c(t)$

$x_c''(t) + 2\beta\, x_c'(t) + \omega_0^2\, x_c(t) = 0$

$x''(t) + 2\beta\, x'(t) + \omega_0^2\, x(t) = F_0 \cos(\omega t) / m$

$$x_{gen}(t) = x_c(t) + x_p(t)$$

=> $x_{gen}(t) = x_c(t) + F_0 \cos(\omega t - \delta) / [m(\omega_0^2 - \omega^2)]$

Taylor: Horizontal motion with linear drag

- Newton's 2^{nd} Law: $\Sigma \mathbf{F} = m\mathbf{a}$

- Picture a train on a track…

The x-component of Newton's second law yields

$$-bv = ma$$

Which is actually a 2^{nd} order, linear (in x), homogeneous, ordinary differential equation

$$m\,x''(t) + b\,x'(t) = 0$$

Which Taylor manipulates into a 1^{st} order, linear, homogeneous, ordinary differential equation

$$m\,v'(t) + b\,v(t) = 0$$

Taylor: Vertical motion with linear drag

- Newton's 2nd Law: $\Sigma \mathbf{F} = m\mathbf{a}$

The y-component of Newton's second law yields

$$-bv - mg = ma$$

Which is a 2nd order, linear (in x), *non*homogeneous, ordinary differential equation

$$m\,x''(t) + b\,x'(t) = mg$$

Taylor defines a new variable,

$u = x'(t) - mg / b$ (where u is the terminal velocity)

to make the equation homogeneous