How to integrate (like a boss).

Consider first the sum

$$\sum_{j=1}^{n} j^3 = \underbrace{1^3 + 2^3 + 3^3 + \cdots + n^3}_{\text{function of } n \text{ (not } j)} = f(n)$$

← upper limit

j is a "dummy index" which means j can be replaced without changing the sum

$$f(n) = \sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \cdots + n^3$$

$$= \frac{n^2(n+1)^2}{4}$$

← no j, no k

Now: $v(t) = \dfrac{dx(t)}{dt} \Rightarrow dx = v(t)\,dt$

$$\int dx = \int v(t)\,dt$$

now with endpoints

$$\int_{x=x_0}^{x(t)} dx = \int_{t'=t_0}^{t} v(t')\,dt' = \int_{y=t_0}^{t} v(y)\,dy$$

match

$x_0 = x(t_0)$

t' is a dummy variable of integration.

$$\int_{t=t_0}^{t} v(t)\,dt \text{ is like } \sum_{k=1}^{k} k^3 = ?$$

$$x\Big|_{x_0}^{x(t)} = x(t) - x_0 = \int_{t'=t_0}^{t} v(t')\,dt'$$

$$x(t) = \int_{t'=t_0}^{t} v(t')\,dt' + x_0$$

One more time

$$a(t) = \frac{dv(t)}{dt} \Longrightarrow \int dv = \int a(t)\,dt$$

$$\int_{v=v_0}^{v(t)} dv = \int_{t'=t_0}^{t} a(t')\,dt'$$

$$v_0 \uparrow v(t_0)$$

$$v(t) = \int_{t'=t_0}^{t} a(t')\,dt' + v(t_0)$$

$$x(t) = \int_{t'=t_0}^{t} v(t')\,dt' + x_0$$

$$x(t) = \underbrace{\int_{t'=t_0}^{t} \left[\int_{t'=t_0}^{t} a(t')\,dt' + v_0 \right] dt'}_{\text{nonsensical - like } \sum_{k=1}^{n} \sum_{k=1}^{n} k^3 = ?} + x_0$$

S+3

$$x(t) = \int_{t''=t_1}^{t} \underbrace{\left[\int_{t'=t_0}^{t''} a(t')\,dt' + v_0 \right]}_{[v(t'')]} dt'' + x_0$$

Like

$$\sum_{j=1}^{n} \boxed{\sum_{k=1}^{j} k^3} \rightarrow f(j)$$

$$\sum_{j=1}^{n} f(j) = g(n)$$

$$f(j) = \frac{j^2 (j+1)^2}{4}$$

$$g(n) = \frac{1}{60}\, n(n+1)(n+2)(3n^2+6n+1)$$

$$g(n) = \sum_{j=1}^{n} \frac{j^2 (j+1)^2}{4} = 1 + 9 + 36 + \ldots$$

$$x(t) = \int_{t''=t_1}^{t} \left[\int_{t'=t_0}^{t''} a(t')\,dt' + v_0 \right] dt'' + x_1$$

E.g. $a(t) = g$ constant $9.8\ m/s^2$

$$v(t'') = \int_{t'=t_0}^{t''} g\,dt' + v_0 = g[t'' - t_0] + v_0$$

$$x(t) = \int_{t''=t_1}^{t} v(t'')\,dt'' + x_1$$

$$x(t) = \int_{t''=t_1}^{t} g[t'' - t_0]\,dt'' + v_0 \int_{t''=t_1}^{t} dt'' + x_1$$

$$= \frac{g[t''^2]}{2}\Big|_{t_1}^{t} - g t'' t_0 \Big|_{t_1}^{t} + v_0 t'' \Big|_{t_1}^{t} + x_1$$

$$= \frac{g}{2}[t^2 - t_1^2] - g t_0 [t - t_1] + v_0 [t - t_1] + x_1$$

Don't recognize this?

$$t_0 = 0 = t_1$$

$$x(t) = \frac{1}{2} g t^2 + v_0 t + x_0$$

Taylor: 264-65 $\qquad \eta = v_x + i v_y$

$$m \dot{v}_x = qB v_y$$

$$m \dot{v}_y = -qB v_x$$ take derivative

$$m \ddot{v}_y = -qB \dot{v}_x \rightarrow \dot{v}_x = -\frac{\ddot{v}_y m}{qB}$$

Substitute into first equation

$$m\left[-\frac{\ddot{v}_y m}{qB}\right] = qB v_y$$

$$\ddot{v}_y + \left(\frac{qB}{m}\right)^2 v_y = 0$$

$$\ddot{v}_y(t) + \left(\frac{qB}{m}\right)^2 v_y(t) = 0$$

$$\frac{d^2}{dt^2} v_y(t) + \omega_c^2 v_y(t) = 0$$