How to integrate? (Like a Boss!)

Consider first the sum

$$\sum_{j=1}^{n} j^3 = \underbrace{1^3 + 2^3 + 3^3 + \dots + n^3}_{\text{function of } n \text{ (not } j\text{)}} = f(n)$$

upper limit

j is a "dummy index", which means j can be replaced without changing the sum

$$f(n) = \sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \dots + n^3$$
$$= \frac{n^2(n+1)^2}{4} \longleftarrow \text{no } j\text{, no } k$$

Now: $v(t) = \frac{dx(t)}{dt} \longrightarrow dx = v(t)dt$

$\int dx = \int v(t)dt$

Now with endpoints

$$\int_{x=x_0}^{x(t)} dx = \int_{t'=t_0}^{t} v(t')dt' = \int_{y=t_0}^{t} v(y)dy$$

smoosh

$x(t_0)$ t' is a dummy variable of integration

$\int_{t_0}^{t} v(t)dt$ is like $\sum_{k=1}^{k} k^3 = ?$

$$x\Big|_{x_0}^{x(t)} = x(t) - x_0 = \int_{t'=t_0}^{t} v(t')dt'$$

$$x(t) = \int_{t'=t_0}^{t} v(t')dt' + x_0$$

One more time!

$a(t) = \frac{dv(t)}{dt} \longrightarrow \int dv = \int a(t)dt$

$$\int_{v=v_0}^{v(t)} dv = \int_{t'=t_0}^{t} a(t')dt'$$

$v(t_0)$

$$v(t) = \int_{t'=t_0}^{t} a(t')dt' + v(t_0)$$

$$x(t) = \int_{t'=t_0}^{t} v(t')dt' + x_0$$

$$x(t) = \int_{t'=t_0}^{t} \left[\int_{t'=t_0}^{t} a(t')dt' + v_0 \right] dt' + x_0$$

non-sensical, like $\sum_{k=1}^{n}\sum_{k=1}^{n} k^3 = ?$

$$x(t) = \int_{t''=t_0}^{t} \underbrace{\left[\int_{t'=t_0}^{t''} a(t')dt' + v_0 \right]}_{[v(t'')]} dt'' + x_0$$

Like $\sum_{j=1}^{n} \boxed{\sum_{k=1}^{j} k^3} \longrightarrow f(j)$

$$\sum_{j=1}^{n} f(j) = g(n)$$

$$f(j) = \frac{j^2(j+1)^2}{4}$$

$$g(n) = \frac{1}{60} n(n+1)(n+3)(3n^2+6n+1)$$

$$g(n) = \sum_{j=1}^{n} \frac{j^2(j+1)^2}{4} = 1 + 9 + 36 + \dots$$

$$x(t) = \int^{t}\left[\int^{t''} a(t')\,dt' + v_0\right]dt'' + X_1$$

Ex: $a(t) = g$ constant $9.8\,\frac{m}{s^2}$

$$v(t'') = \int_{t'=t_0}^{t''} g\,dt' + v_0 = g[t'' - t_0] + v_0$$

$$x(t) = \int_{t''=t_1}^{t} v(t'')\,dt'' + X_1$$

$$x(t) = \int_{t''=t_1}^{t} g[t'' - t_0]\,dt'' + v_0 \int_{t''=t_1}^{t} dt'' + X_1$$

$$= \frac{g[t''^2]}{2}\Big|_{t_1}^{t} - g t'' t_0 \Big|_{t_1}^{t} + v_0 t'' \Big|_{t_1}^{t} + X_1$$

$$= \frac{g}{2}[t^2 - t_1^2] - g t_0 [t - t_1] + v_0 [t - t_1] + X_1$$

Don't recognize this?

$t_0 = 0 = t_1$

$$X(t) = \frac{1}{2} g t^2 + v_0 t + X_0$$

Taylor 2.64-2.65

$m\dot{v}_x = qBv_y$ $\qquad$ $\eta = v_x + i v_y$

$m\dot{v}_y = -qBv_x$ $\quad$ take derivative

$m\ddot{v}_y = -qB\dot{v}_x \longrightarrow \dot{v}_x = \frac{-\ddot{v}_y m}{qB}$

Substitute into fist equation

$$m\left[\frac{-\ddot{v}_y m}{qB}\right] = qBv_y$$

$$\ddot{v}_y + \left(\frac{qB}{m}\right)^2 v_y = 0$$

$$\ddot{v}_y(t) + \left(\frac{qB}{m}\right)^2 v_y(t) = 0$$

$$\frac{d^2}{dt^2} v_y(t) + w_c^2 v_y(t) = 0$$