Black-body Radiation (Cavity Radiation)

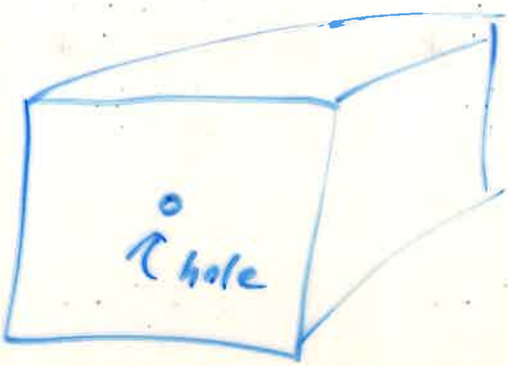
A "black body" absorbs all incident electromagnetic radiation (photons) of all frequencies. (ν = nu)

ν in Hz = hertz, $1\ Hz = 1\ \frac{\text{cycle}}{\text{sec}}$

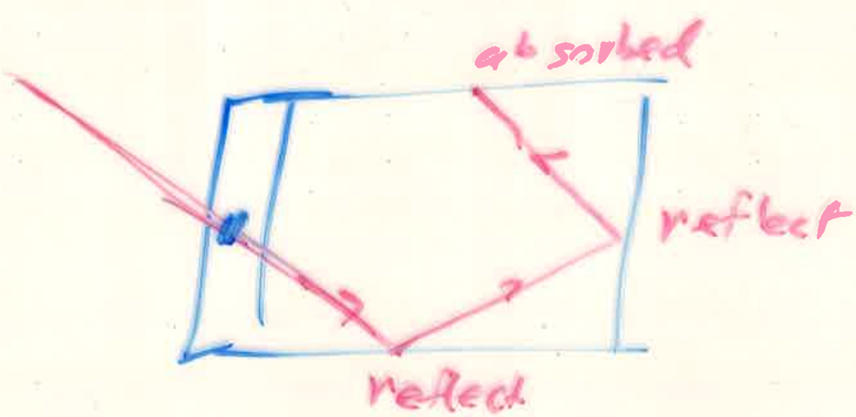
ω = angular frequency in $\frac{\text{radians}}{\text{sec}}$

$\omega = 2\pi\nu$

A black body is a perfect absorber (and perfect emitter)



hole is the black body.

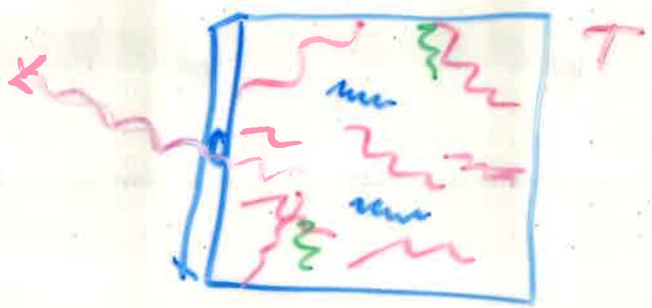


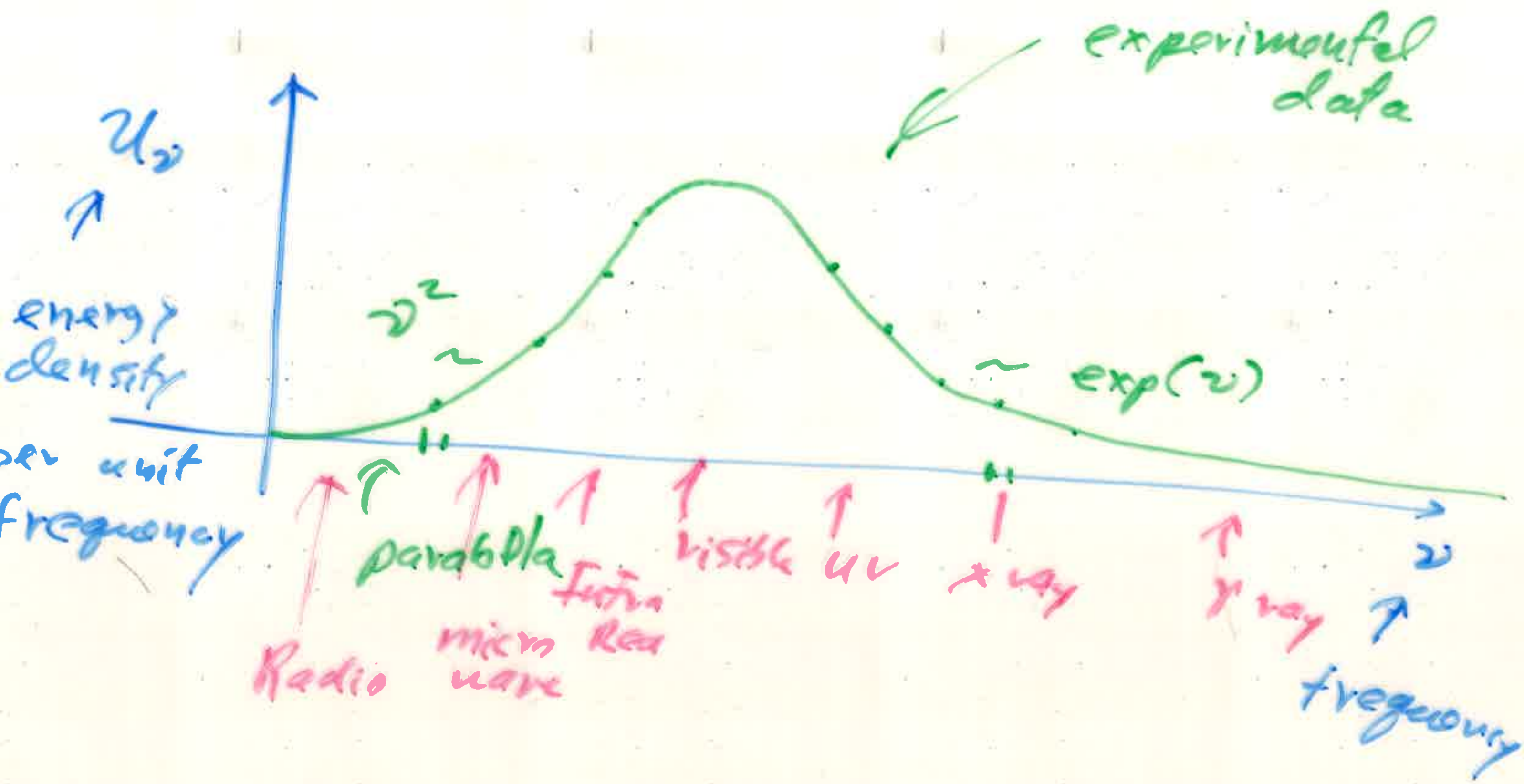
Light rays can't find their way back out through the hole.

Eventually thermal equilibrium is reached

T is constant ; energy in = energy out

Atoms that make up the container begin to vibrate and emit photons — some of those photons come out of the hole.





U is an energy (in joules J in MKS).

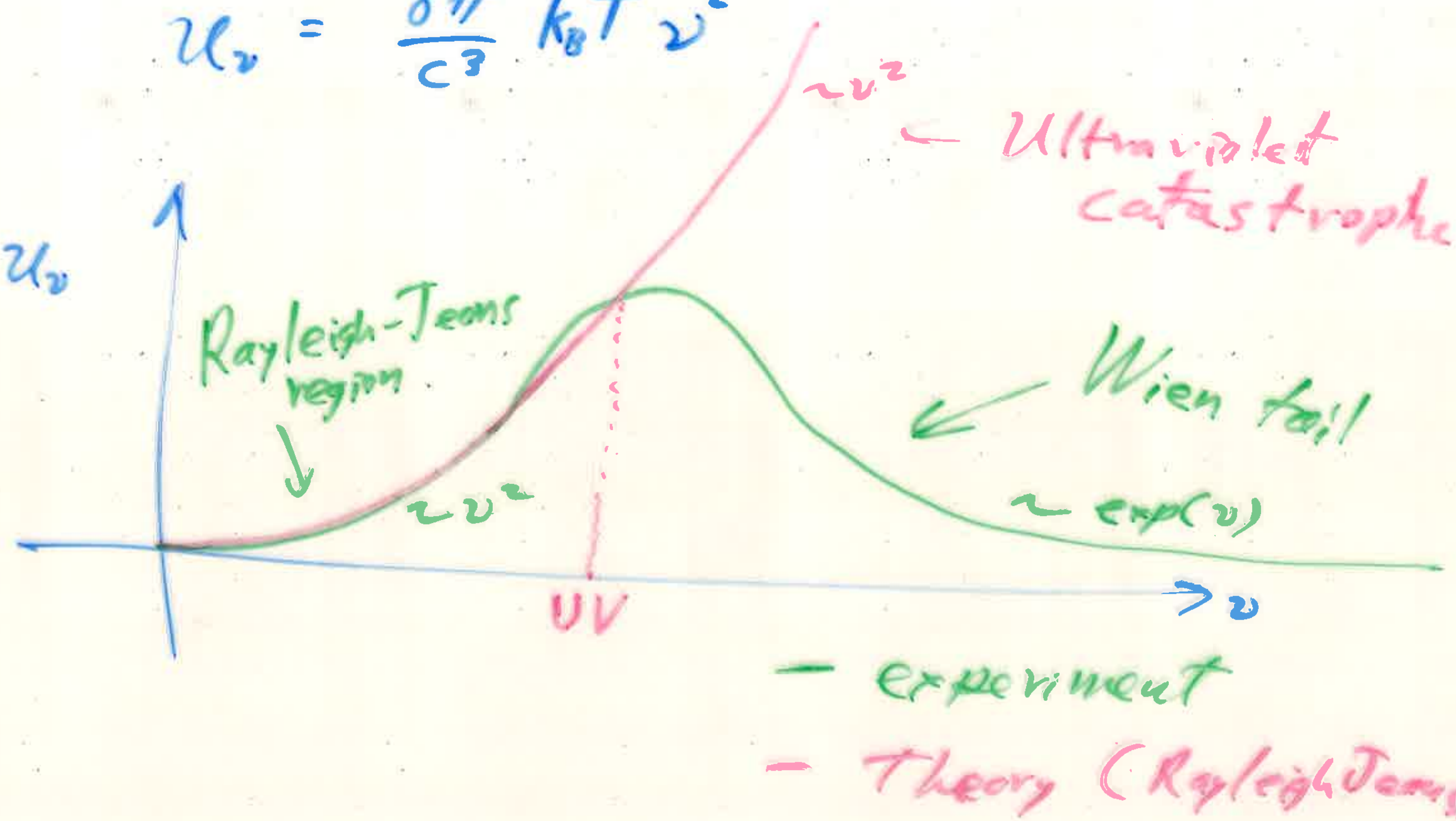
u is an energy density : $u = \frac{U}{V}$ $\left(\frac{J}{m^3}\right)$

$u_\nu = \frac{du}{d\nu}$ is an energy density per frequency $\left(\frac{J}{m^3\,Hz}\right)$

$$u = \int du = \int \left(\frac{du}{d\nu}\right) d\nu = \int_{\nu=0}^{\infty} u_\nu \, d\nu =$$ area under curve

Rayleigh + Jeans used classical (non-QM) physics called equipartition.

$$u_\nu = \frac{8\pi}{c^3} k_B T \nu^2$$



Max Planck : $u_\nu = \frac{8\pi \nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}$

① $h\nu \ll k_B T$ (can ignore QM effect)

$$e^{\frac{h\nu}{k_B T}} \equiv e^x = 1 + x + \cancel{\frac{x^2}{2!} + \frac{x^3}{3!}}$$ small neglect.

$$= 1 + \frac{h\nu}{k_B T} + \dots$$

$$u_\nu = \frac{8\pi \nu^2}{c^3} \frac{h\nu}{\left(\frac{h\nu}{k_B T}\right)} = \frac{8\pi \nu^2 k_B T}{c^3} = RJ$$

② $h\nu \gg k_B T$ (QM effects important

$$u_\nu \propto \exp\left(-\frac{h\nu}{k_B T}\right)$$

Flux : $f_\nu = \frac{c}{4} u_\nu$; $f = \int_{\nu=0}^{\infty} f_\nu \, d\nu$

Luminosity (Power) : $L_\nu = \oiint f_\nu \, dA =$

$$= f_\nu \, 4\pi r^2$$

$$\frac{du}{d\nu} = u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{k_BT}}-1}$$

$$u = \int du = \int \left(\frac{du}{d\nu}\right) d\nu = \int_{\nu=0}^{\infty} u_\nu \, d\nu = \int_{\nu=0}^{\infty} \frac{8\pi\nu^2}{c^3}\frac{h\nu \, d\nu}{e^{\frac{h\nu}{k_BT}}-1}$$

change of variable $\quad x \equiv \frac{h\nu}{k_BT}$, $\quad dx = \frac{h}{k_BT} d\nu$

$\nu = 0 \leftrightarrow x = 0$, $\quad \nu = \infty \Leftrightarrow x = \infty$

$$u = \frac{8\pi h}{c^3}\int_{x=0}^{\infty} \frac{\left(\frac{k_BT}{h}\right)^4 x^3 \, dx}{e^x - 1} = \frac{8\pi k_B^4}{c^3 h^3} T^4 \underbrace{\int_{x=0}^{\infty} \frac{x^3 dx}{e^x - 1}}_{\frac{\pi^4}{15}}$$

(some number)

$$u = \left(\frac{8\pi^5 k_B^4}{15c^3h^3}\right) T^4 = a T^4$$

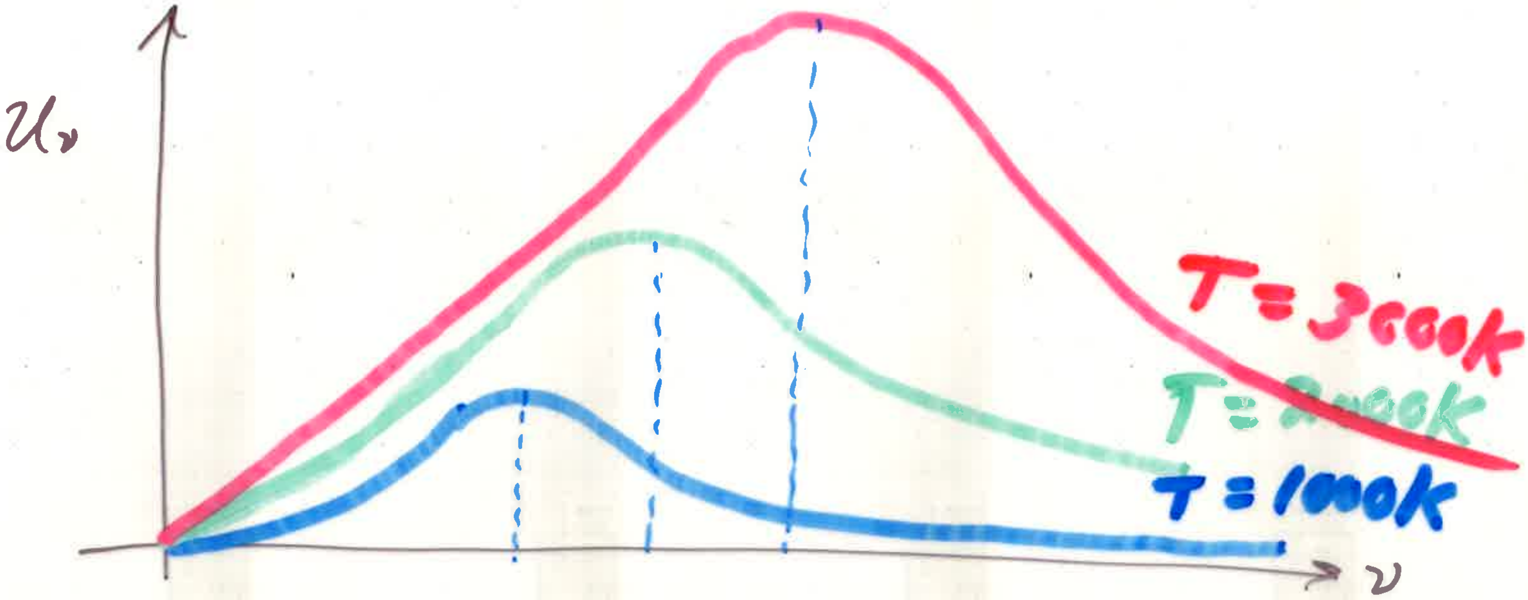
a ← radiation constant

Stefan-Boltzmann Law

$$\sigma = \frac{c}{4} a$$

$$f = \frac{c}{4} u = \sigma T^4$$

σ ← Stefan-Boltzmann constant



$u = \int u_\nu \, d\nu$ = area under curve $\propto T^4$

area under red curve $= 3^4 = 81 \times$ area under Blue curve.

$\nu_{peak} \propto T$ e.g. $\nu_{\substack{peak \\ red}} = 3 \; \nu_{\substack{peak \\ Blue}}$

$$\left. \frac{d}{d\nu} u_\nu \right|_{\nu = \nu_{peak}} = 0 \Longrightarrow \left. \frac{d}{d\nu}\left(\frac{8\pi}{c^3} \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1} \right) \right|_{\nu = \nu_{peak}} = 0$$

transcendental equation
↑ can not be solve algebraically: $e^x = x$

$\nu_{peak} = 2.8 \frac{k_B}{h} T$ Wien's Law