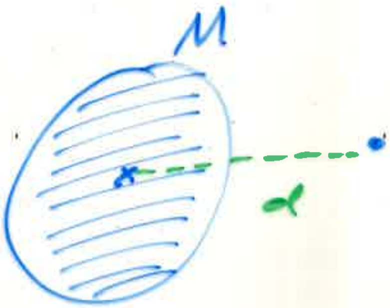
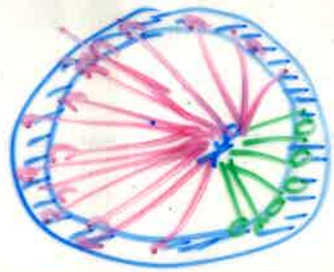
Newton's Theorem: (Gauss' Law) aka Shell Theorem



Replace

No force in the cavity.

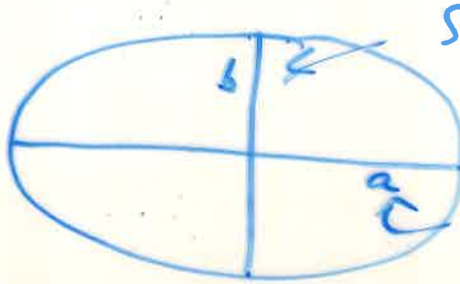


Weightless

Central Force: $\vec{F} \propto \hat{r} \Rightarrow$ Law of Equal Areas

$\Rightarrow$ Conservation of Angular Momentum.

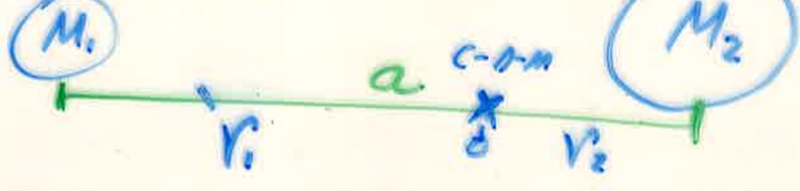
semi minor axis



semi major axis

Center of Mass

$r_1 + r_2 = a$

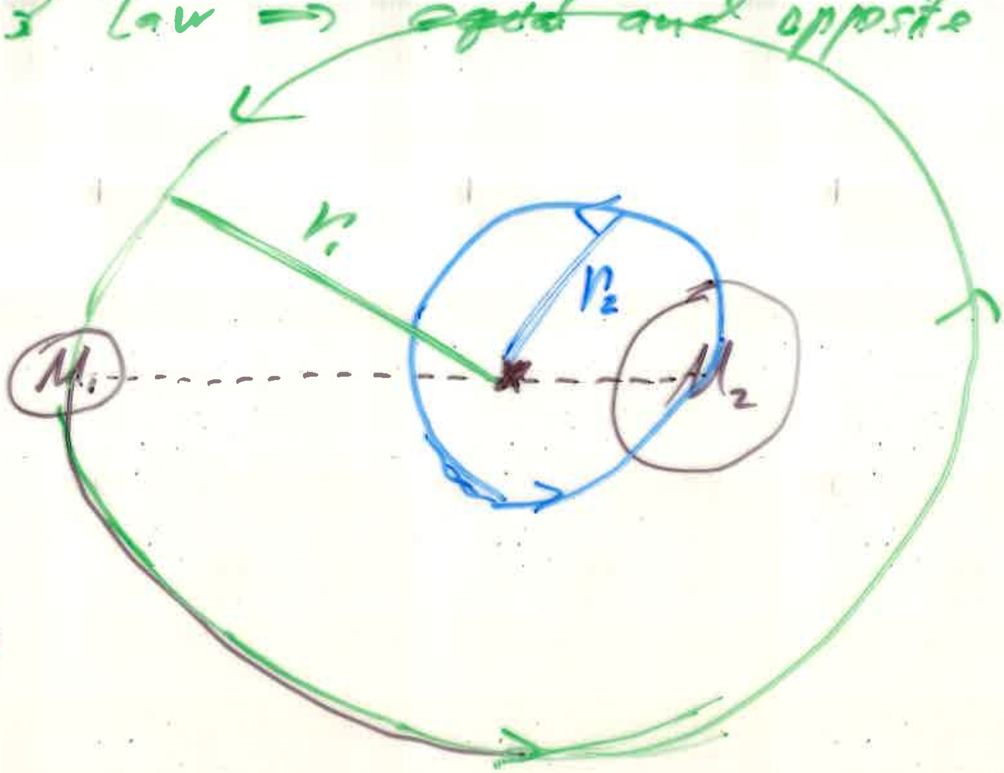


$$X_{cm} = \frac{(-r_1)M_1 + r_2 M_2}{M_1 + M_2} = 0$$

$$\Rightarrow \boxed{r_1 M_1 = r_2 M_2}$$

$$\left.\begin{array}{l} \text{Force on } M_1 \text{ caused by } M_2 = \dfrac{G M_1 M_2}{a^2} \\ \text{Force on } M_2 \text{ caused by } M_1 = \dfrac{G M_1 M_2}{a^2} \end{array}\right\} \text{action reaction pair}$$

Newton's 3rd law $\Rightarrow$ equal and opposite forces.



$M_2 > M_1$

$v_1 > v_2$

$\uparrow$ in m/s

Both M_1 and M_2 have the <u>same</u> angular speed ω

$\uparrow$ in $\frac{\text{radians}}{\text{sec}}$

Newton's 2nd law : Force = Mass · Acceleration

Force on M_1 : $\dfrac{G M_1 M_2}{a^2} = M_1 \underbrace{\omega^2 r_1}_{\text{acceleration}}$

$r_1 + r_2 = a$, $r_1 M_1 = r_2 M_2 \Longrightarrow$ 2 equations
2 unknowns r_1, r_2

$\downarrow$

$r_2 = a - r_1$, $r_1 M_1 = (a - r_1) M_2$

$r_1 M_1 = a M_2 - r_1 M_2$

$r_1 M_1 + r_1 M_2 = a M_2$

$r_1 (M_1 + M_2) = a M_2 \longrightarrow r_1 = \dfrac{a M_2}{M_1 + M_2}$

By symmetry : $r_2 = \dfrac{a M_1}{M_1 + M_2}$

$$\frac{G \cancel{M_1} \cancel{M_2}}{a^2} = \cancel{M_1}\, \omega^2 \left(\frac{a \cancel{M_2}}{M_1 + M_2} \right)$$

$$\frac{G(M_1 + M_2)}{a^3} = \omega^2 = \left(\frac{2\pi}{T} \right)^2$$

↑ period

Kepler's 3rd Law

Average Angular Quantities

$f(\theta, \varphi)$ e.g. $\cos^2\theta \sin\theta + \theta$

$$\langle f \rangle = \frac{1}{4\pi} \underbrace{\iint}_{\text{whole sphere}} f(\theta,\varphi)\, d\Omega = \frac{1}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} f(\theta,\varphi) \sin\theta \, d\theta \, d\varphi$$

azimuthal   polar

$$= \frac{1}{4\pi}\left(0 + 2\pi^2\right) = \frac{\pi}{2}$$

$\theta = 0^\circ$ is north pole , $\theta = 180^\circ = \pi$ rad is south pole.

angle of inclination