Last Time:

- Reviewed solid angle.
- Reviewed atomic structure and the hydrogen atom.

The Lyman and Balmer series have special names for some transitions.

$$\text{Ly}\alpha: 2 \leftrightarrow 1, 1216 \text{ Å};$$
$$\text{Ly}\beta: 3 \leftrightarrow 1, 1025 \text{ Å};$$
$$\text{Ly}\gamma: 4 \leftrightarrow 1, 972 \text{ Å};$$

Photon wavelengths are in UV region.

$$\text{Lyman continuum} = \infty \leftrightarrow 1, <911.5 \text{ Å}$$
$$\vdots$$

$$\text{H}\alpha: 3 \leftrightarrow 2, 6563 \text{ Å}$$
$$\text{H}\beta: 4 \leftrightarrow 2, 4861 \text{ Å}$$
$$\text{H}\gamma: 5 \leftrightarrow 2, 4340 \text{ Å}$$

Photon wavelengths are in optical region.

$$\text{Balmer continuum} = \infty \leftrightarrow 2, <3646 \text{ Å}$$
$$\vdots$$

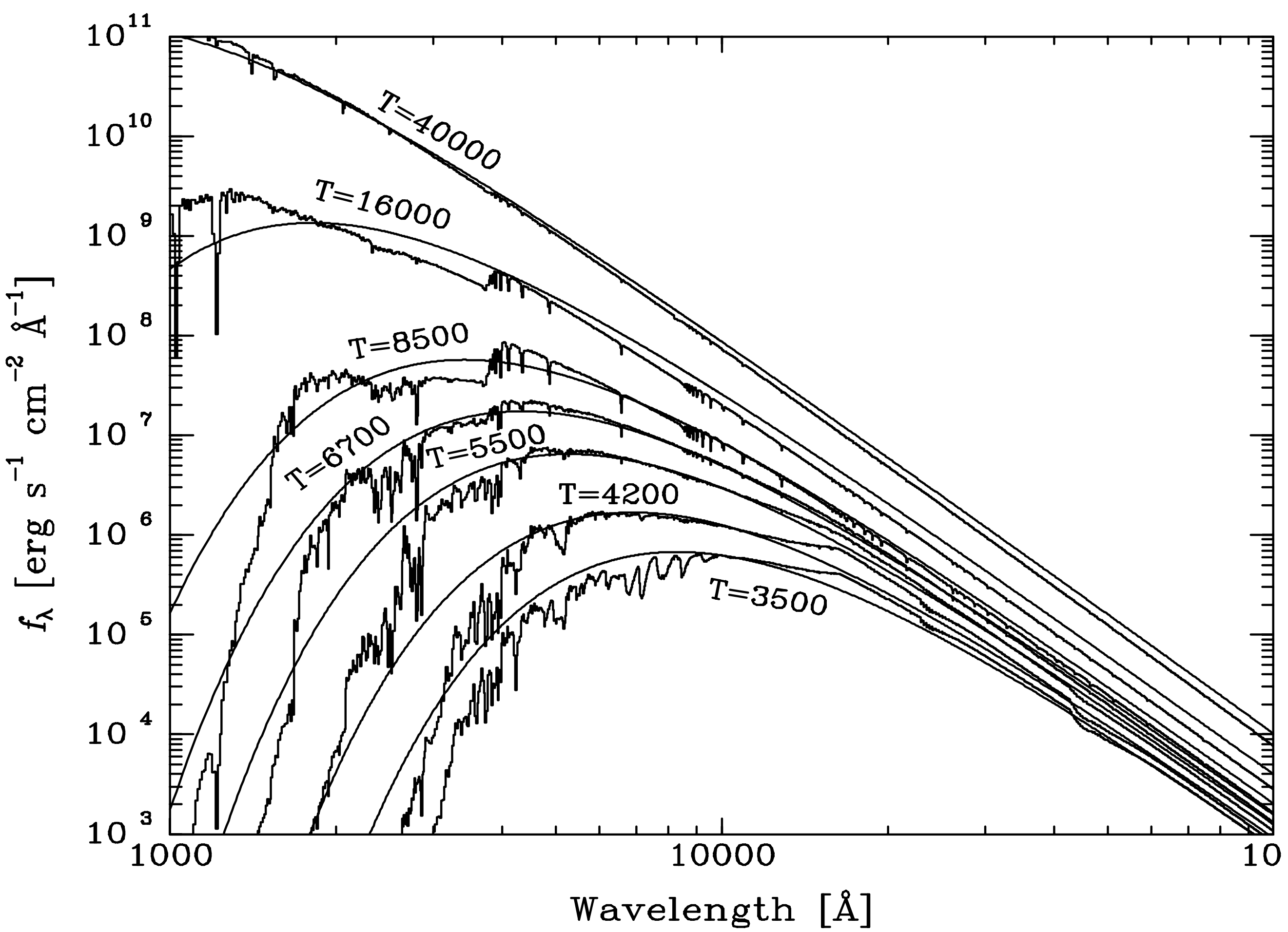
Last Time:

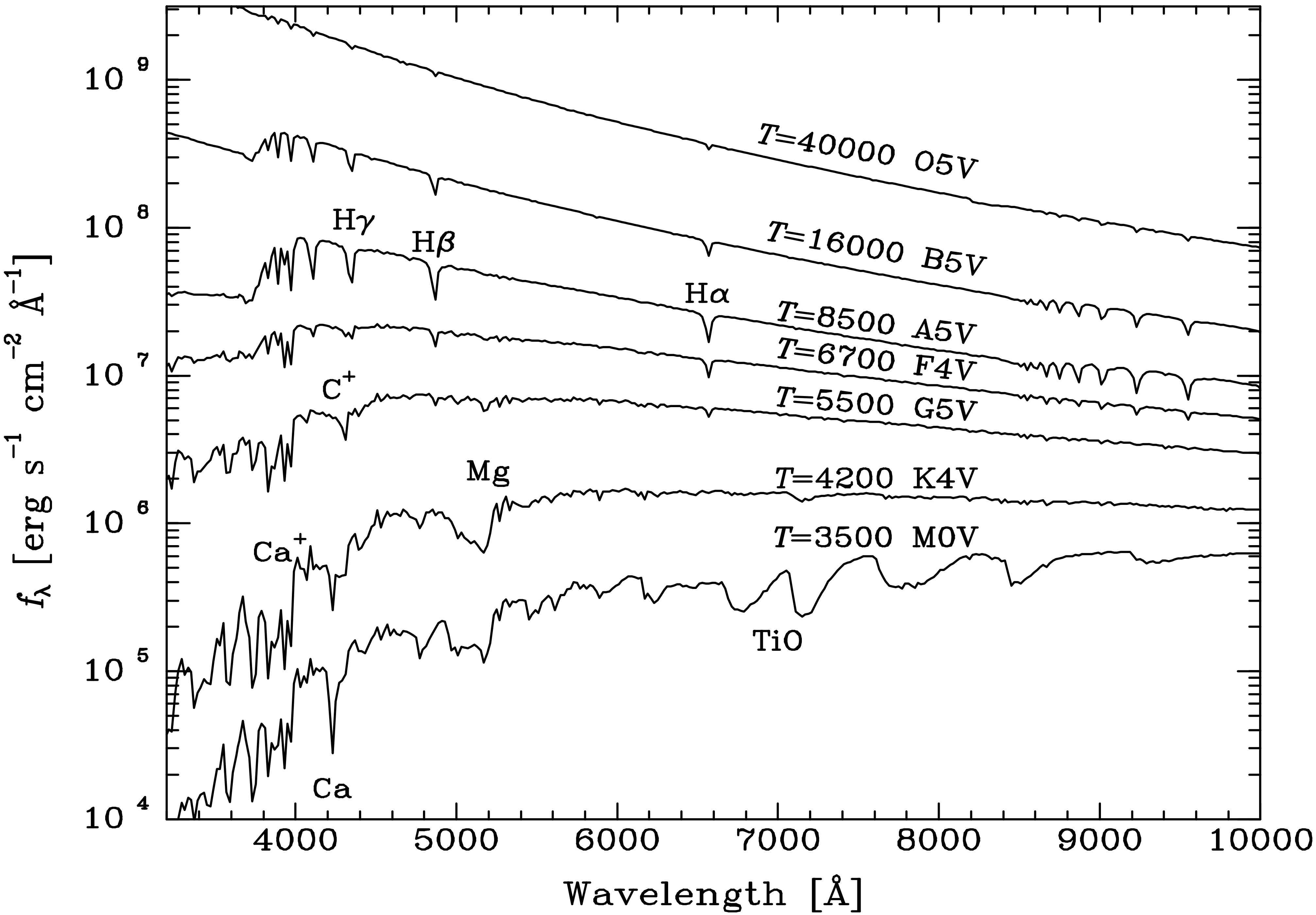
Classification of Stars

- Stars are classified according to their surface (color) temperature.

- Spectral types are OBAFGKM with a digit 0 - 9 in order from hottest (O1) to coldest (M9).

- A Roman numeral is added to the classification to indicate size: I = giant and V = dwarf.

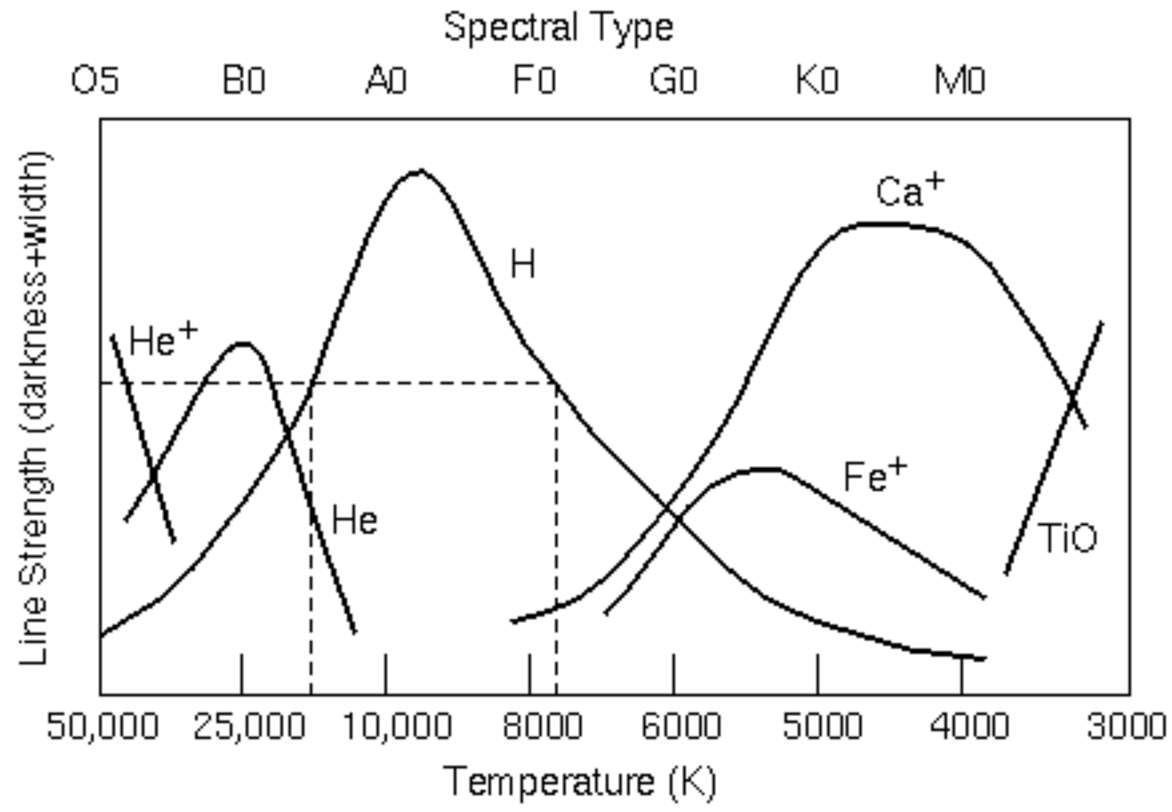




Atomic spectral lines produced in the photosphere also depend on temperature and provide another means of classification.

Why do A-*type* stars have strong hydrogen lines (Balmer series) while cooler and hotter stars do not?

To produce a strong H-absorption line in the visible spectrum, electrons need to start in the second energy level. If the temperature is too low, electrons are in the ground state. If the temperature is too high, most electrons are in higher excited states.



Luminosity and Radius

Luminosity is defined as:

$$L = f 4\pi d^2$$

Recall: **Bolometric Luminosity** is the luminosity integrated overall wavelengths.

From this you can derive a relationship between the star radius, temperature of the star and luminosity.

$$L = 4\pi r_*^2 \sigma T^4$$

The temperature derived from this equation is the **effective temperature**, T_E. It is the temperature of a blackbody that has the same luminosity per unit surface area as the star.

Example: Effective Temperature of the Sun

Calculate the effective temperature of the sun.

$$L = 4\pi r_*^2 \sigma T^4$$

$$T = (\frac{L}{4\pi\sigma r_*^2})^{\frac{1}{4}} = (\frac{3.8 \times 10^{33}\ erg\ s^{-1}}{4\pi\ (5.7 \times 10^{-5}\ erg\ cm^{-2}\ s^{-1}\ K^{-4})(7.0 \times 10^{10}\ cm)^2})^{\frac{1}{4}}$$

$$T = 5700\ K$$

5800 K is often quoted as the temperature of the surface of the sun. However, this is not entirely true. The surface of the sun has hotter and colder regions. However, this is the temperature of the material that emits the bulk of the suns power.

Binary Star Systems

A **binary star system** is composed of two stars whose gravitational attraction causes them to orbit each other.

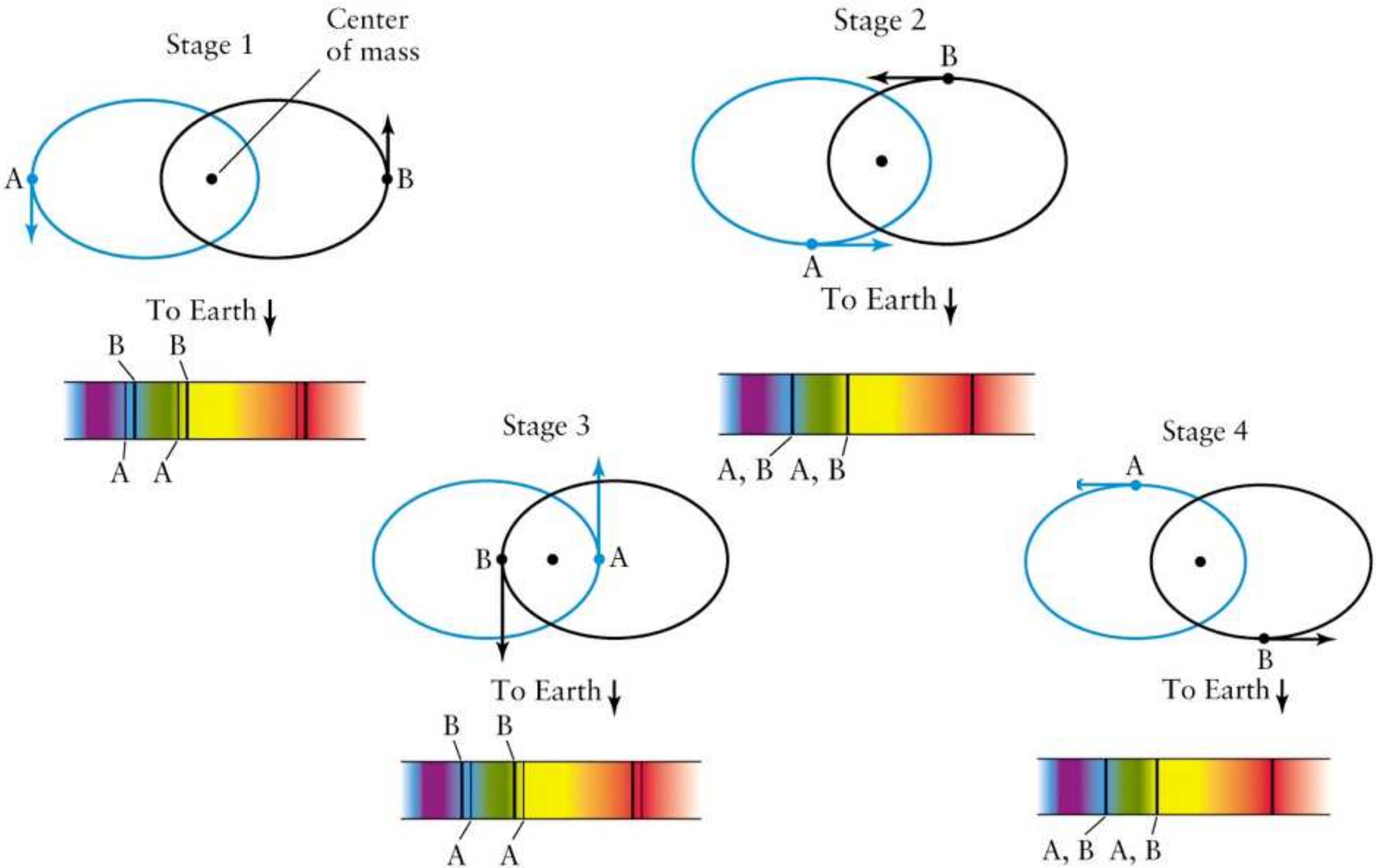
Visual Binaries:

Stars are sufficiently close to the Earth that they can be seen and are enough apart from each other that they can be resolved.

Long - term observations of the system allow observers to track the stars motion over time.

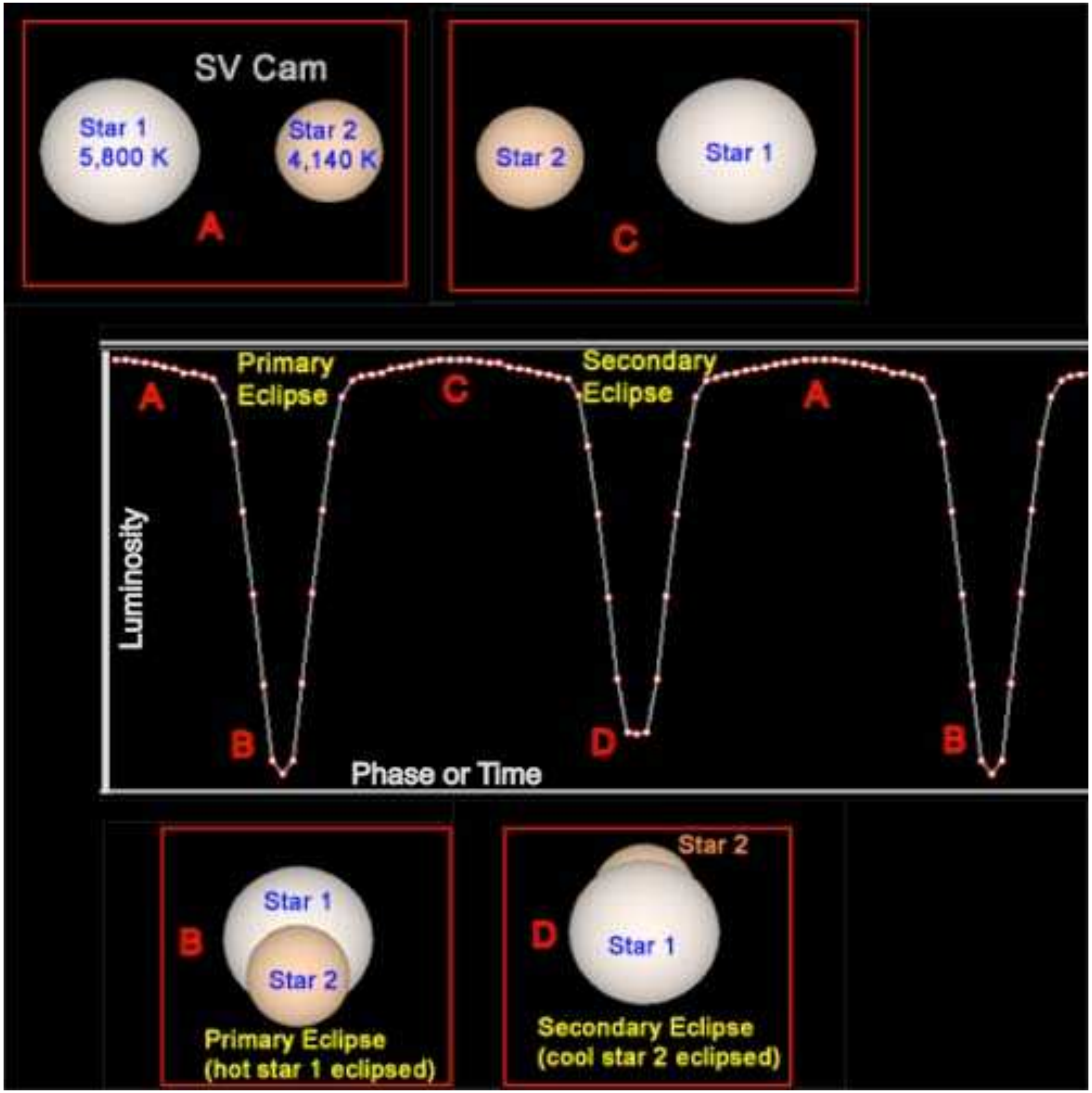
Distance from Earth: ~1.3 parsec
Separation Distance: ~23 AU

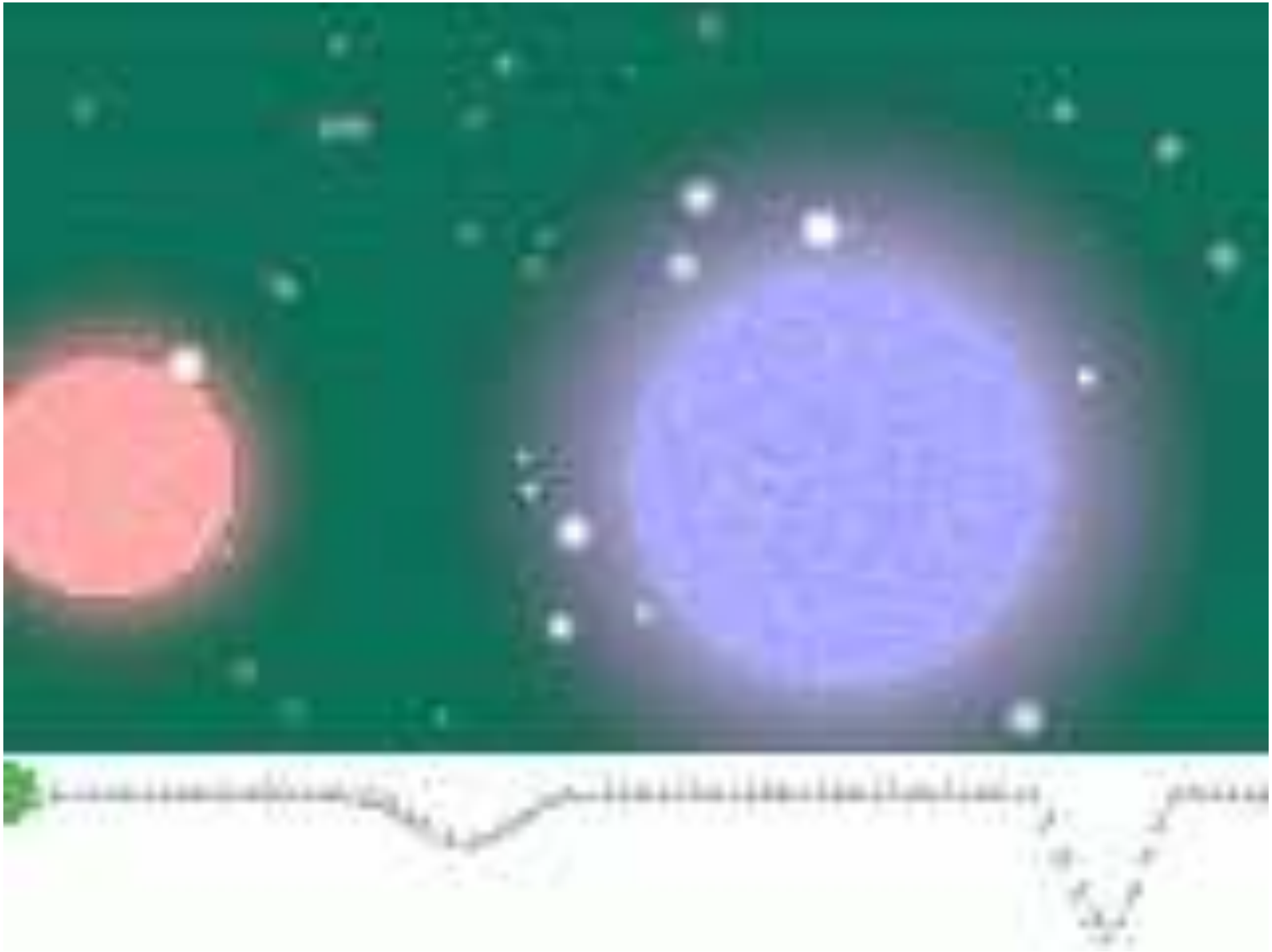
Spectroscopic Binaries: Stars are too close together to be resolved. The pair are revealed by their spectrum.



Eclipsing Binaries:

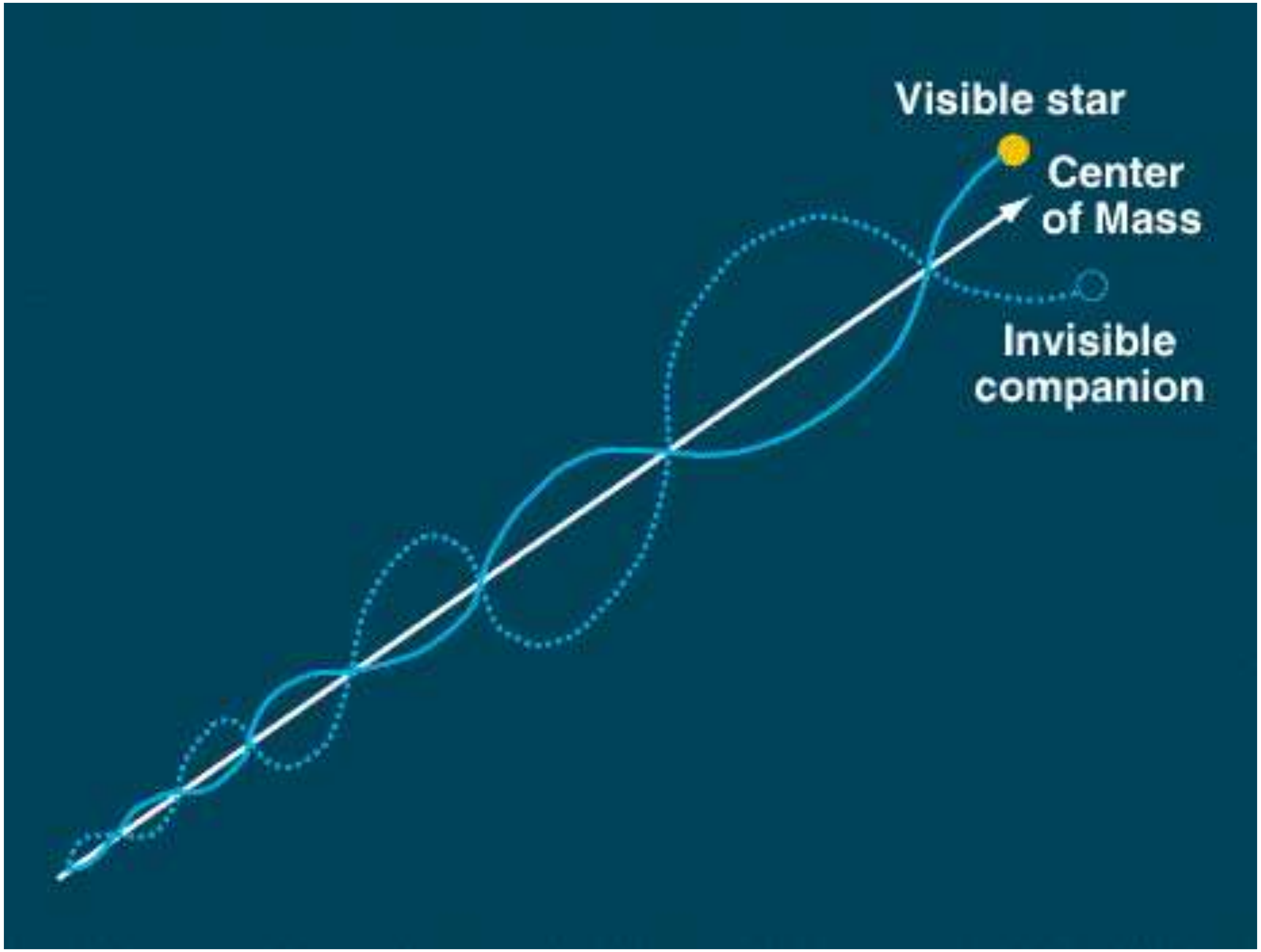
The orbital plane of the stars is inclined such that in our line of sight one member of the pair eclipses the other.





Astrometric Binaries:

Repeated observations over time reveal a perturbation or "wobble" in the stars proper motion.



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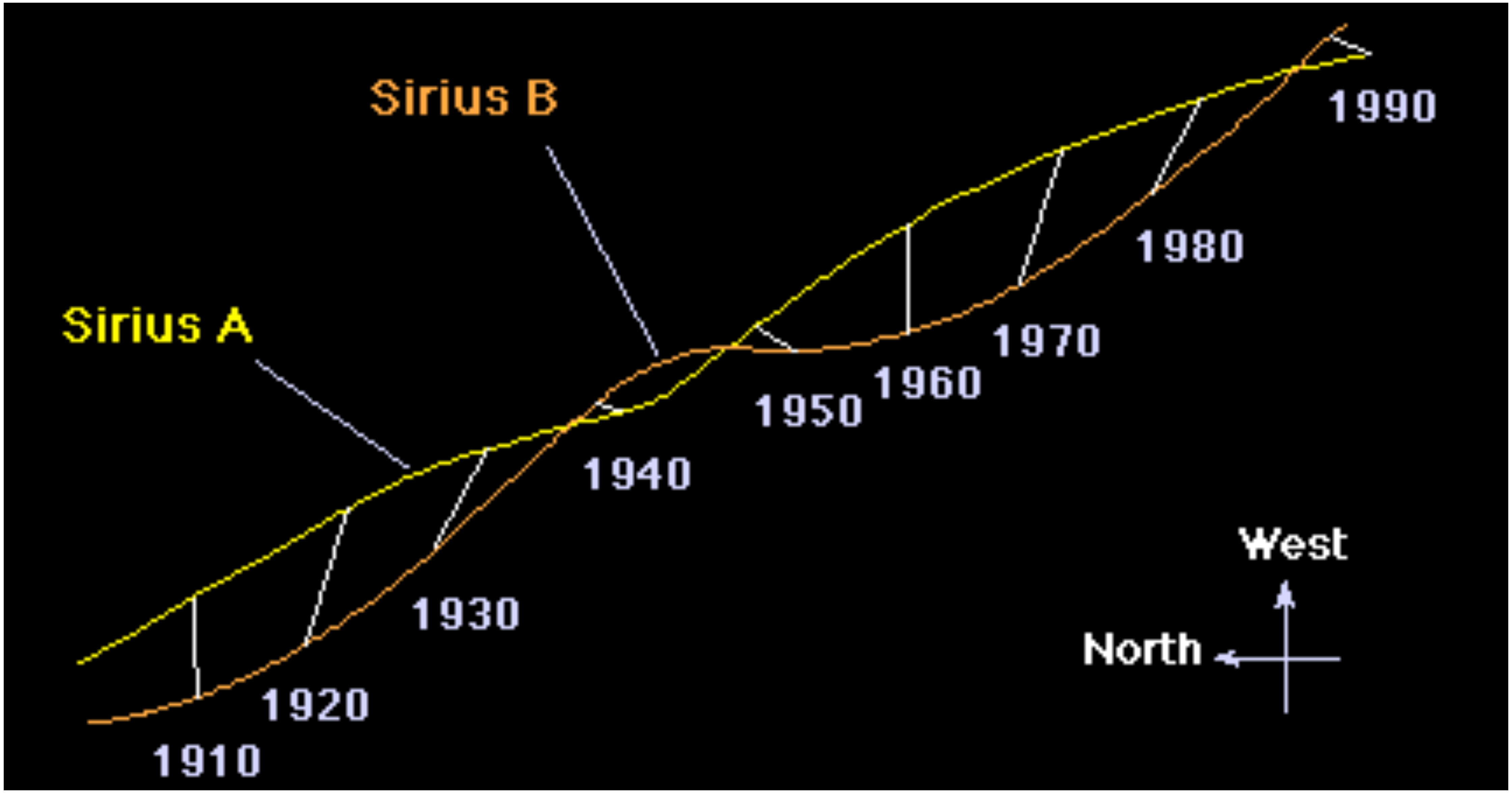
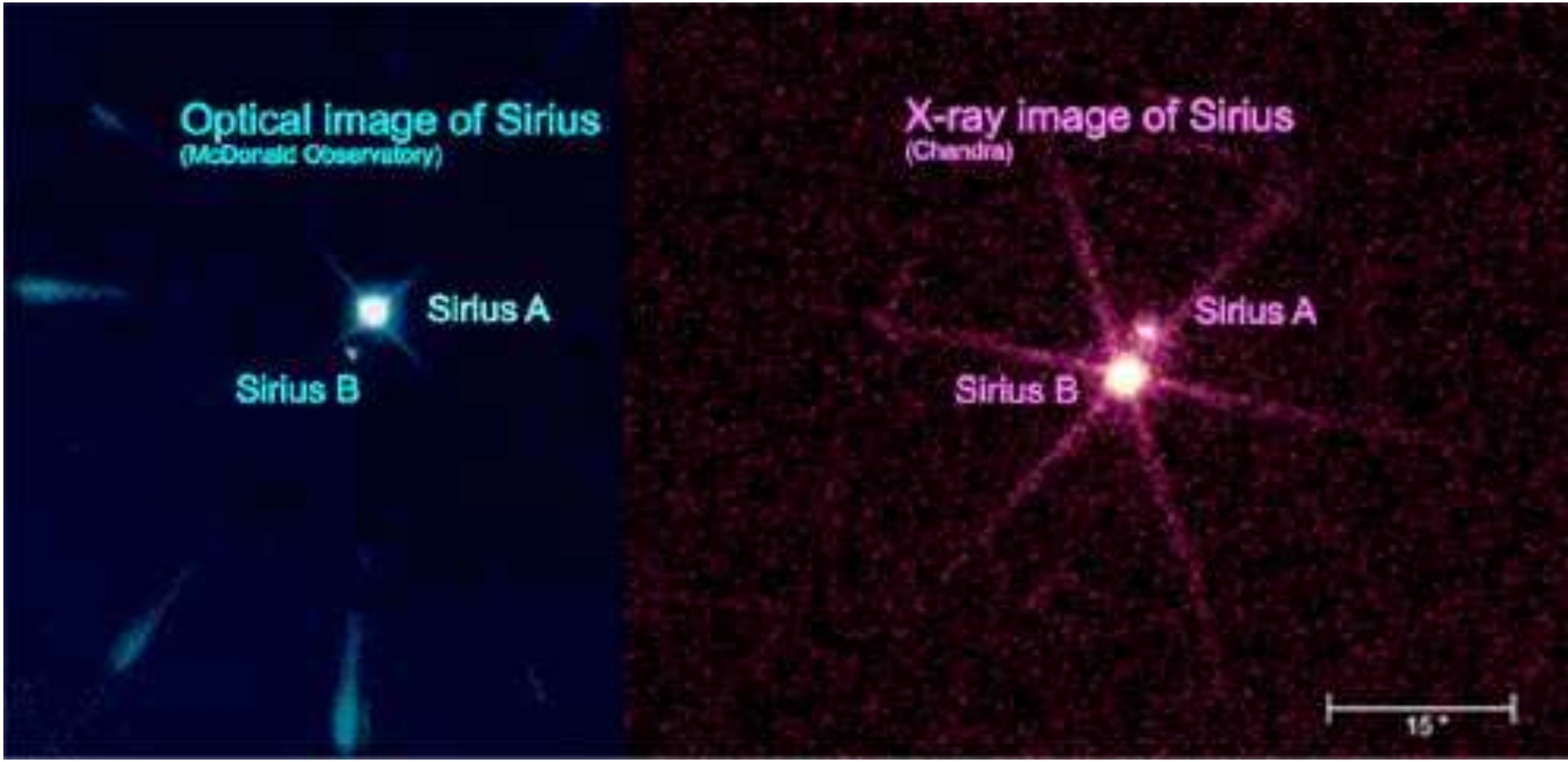
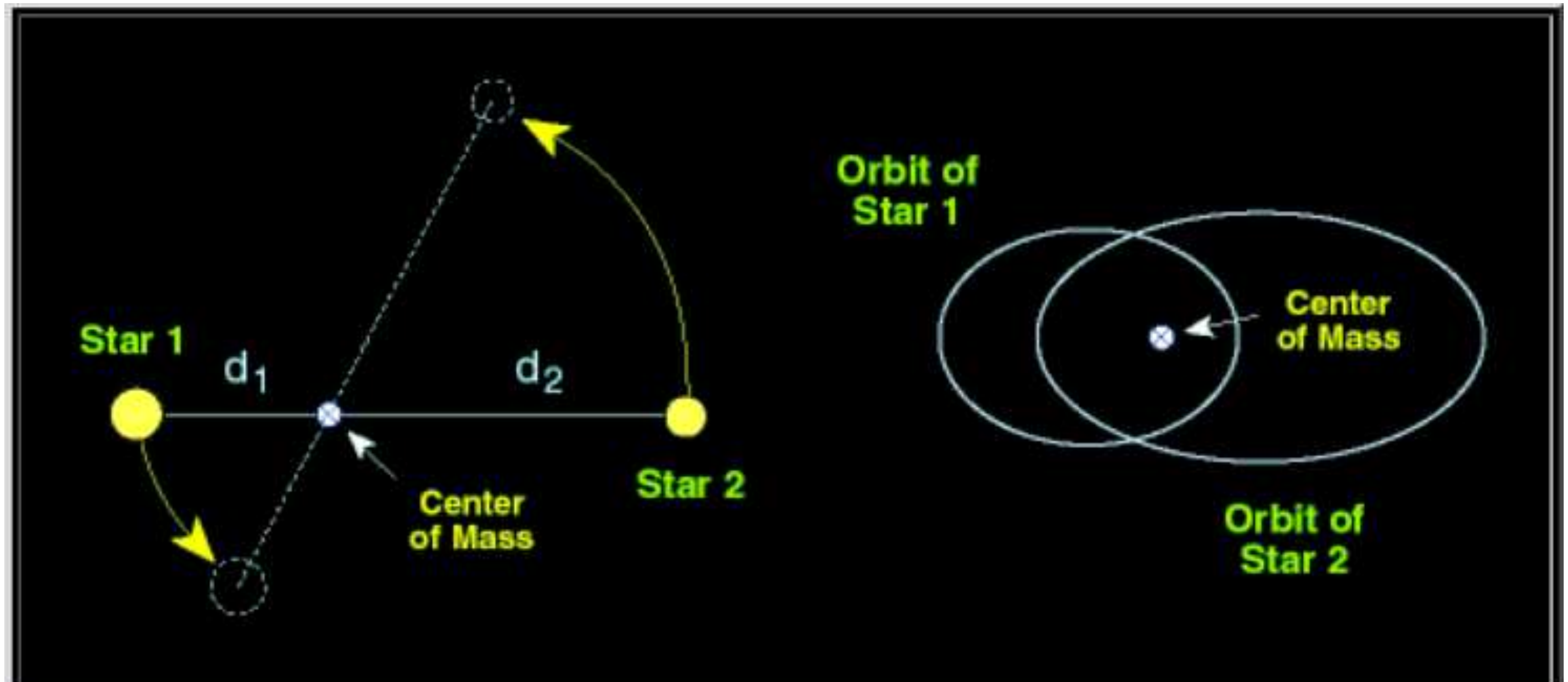




Sirius A and Sirius B are now considered visual binaries.

Stellar Mass Determination

Direct measurements of stellar mass is possible in certain binary systems.

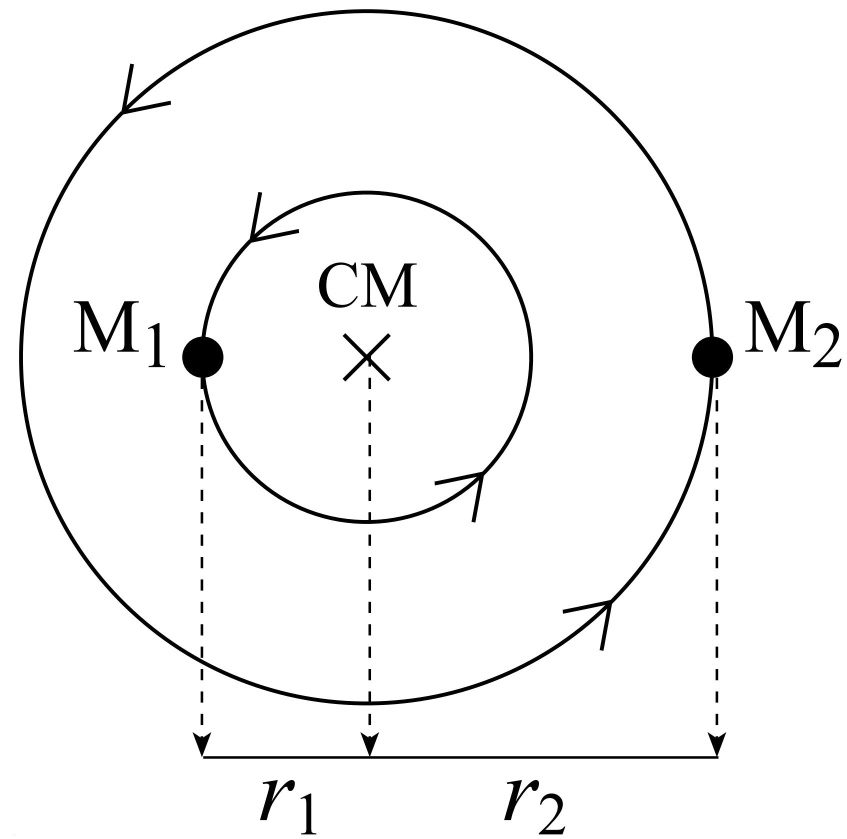


Orbits for binary star systems

Review: Keplerian Two-Body Problem

Assume two masses orbiting each other about their common center of mass. Assume their orbits are circular.

From the definition of center of mass:



$$r_1 M_1 = r_2 M_2$$

Let $a = r_1 + r_2$.

$$r_1 = \frac{M_2}{M_1}(a - r_1)$$

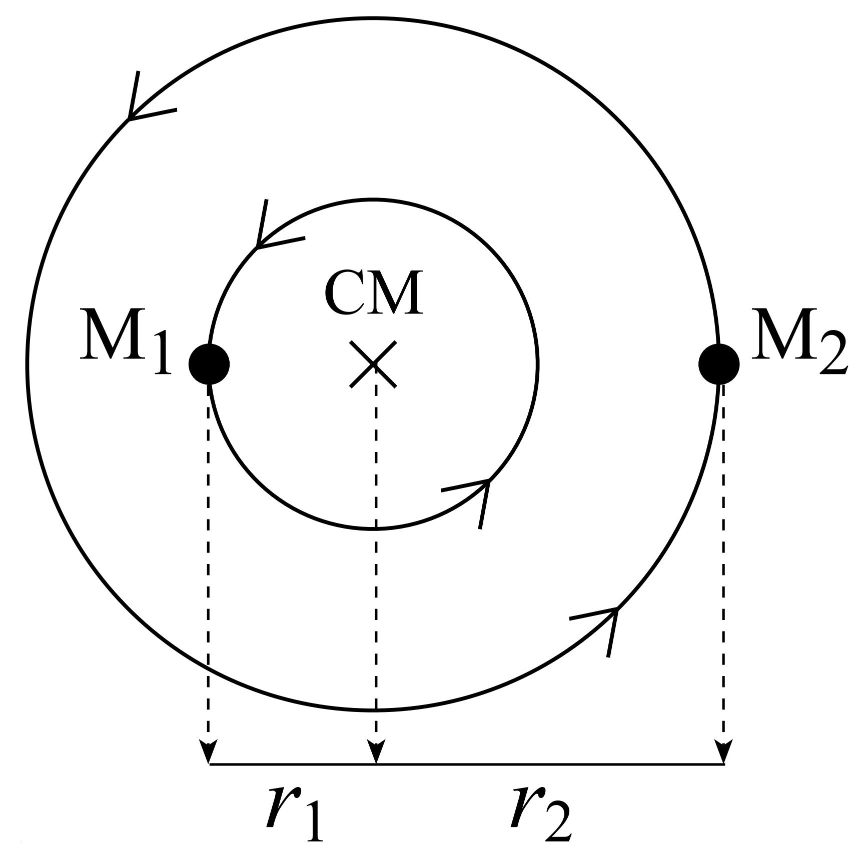
Which can be rewritten as

$$\boxed{r_1 = \frac{M_2}{M_1 + M_2} a \qquad \text{or} \qquad r_2 = \frac{M_1}{M_1 + M_2} a}$$

$$r_1 = \frac{M_2}{M_1 + M_2} a$$

Recall the first equation of motion (for angular motion):

$$M_1 \omega^2 r_1 = \frac{G M_1 M_2}{a^2}$$



Substituting in our eqn for r_1 and solving for ω yields

$$\cancel{M_1} \omega^2 \frac{\cancel{M_2}}{M_1 + M_2} a = \frac{G \cancel{M_1} \cancel{M_2}}{a^2}$$

$$\boxed{\omega^2 = \frac{G(M_1 + M_2)}{a^3}}$$

Now let's see how we can use this equation do determine mass.

Consider the Earth - Sun system. $M_{Earth} << M_{Sun}$. Thus,

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3} \longrightarrow M_\odot \approx \frac{\omega^2 a^3}{G}$$

Let $\tau = 2\pi/\omega$ and substitute for ω.

$$M_\odot = \frac{4\pi^2 a^3}{\tau^2 G}$$

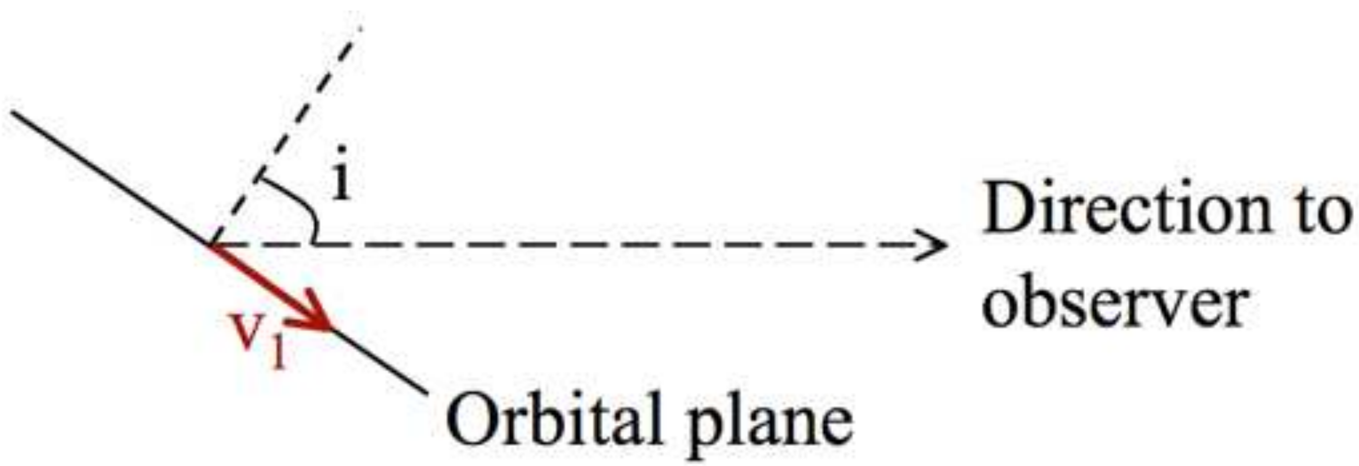
Using this formula, calculate the mass of the sun.

mass of sun

$$M_\odot = \frac{4 \times \pi^2 (1.5 \times 10^{13}\ \mathrm{cm})^3}{(3.15 \times 10^7\ \mathrm{s})^2 \times 6.7 \times 10^{-8}\ \mathrm{erg\ cm\ g^{-2}}} = \boxed{2.0 \times 10^{33}\mathrm{g}}$$

Spectroscopic Binaries:

We can not directly measure the separations a, r_1, and r_2. Amplitudes in the line of site velocities can be deduced by Doppler shift. In most cases the perpendicular to the orbital plane is inclined to the line of sight, the measured velocities are related to the true orbital velocities by



$$|v_{1\mathrm{obs}}| = |v_1| \sin i$$

and

$$|v_{2\mathrm{obs}}| = |v_2| \sin i$$

What is the relationship between linear and angular velocity?

$$\omega = \frac{v}{r}$$

$$|v_{1\mathrm{obs}}| = |v_1| \sin i$$
$$|v_{2\mathrm{obs}}| = |v_2| \sin i.$$

We can use these relationships to

$$\omega = vr \quad \text{and} \quad \omega = \frac{2\pi}{\tau}$$

To write

$$|v_1| = \frac{2\pi r_1}{\tau}, \quad |v_2| = \frac{2\pi r_2}{\tau}$$

Taking the ratio of the observed velocities yields

$$\frac{|v_{1\mathrm{obs}}|}{|v_{2\mathrm{obs}}|} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$

Going through a bit of math (exercise for the student), we find

$$(M_1 + M_2) \sin^3 i = \frac{\tau(|v_{1\mathrm{obs}}| + |v_{2\mathrm{obs}}|)^3}{2\pi G}$$

$$(M_1 + M_2)\sin^3 i = \frac{\tau(|v_{1\text{obs}}| + |v_{2\text{obs}}|)^3}{2\pi G}$$

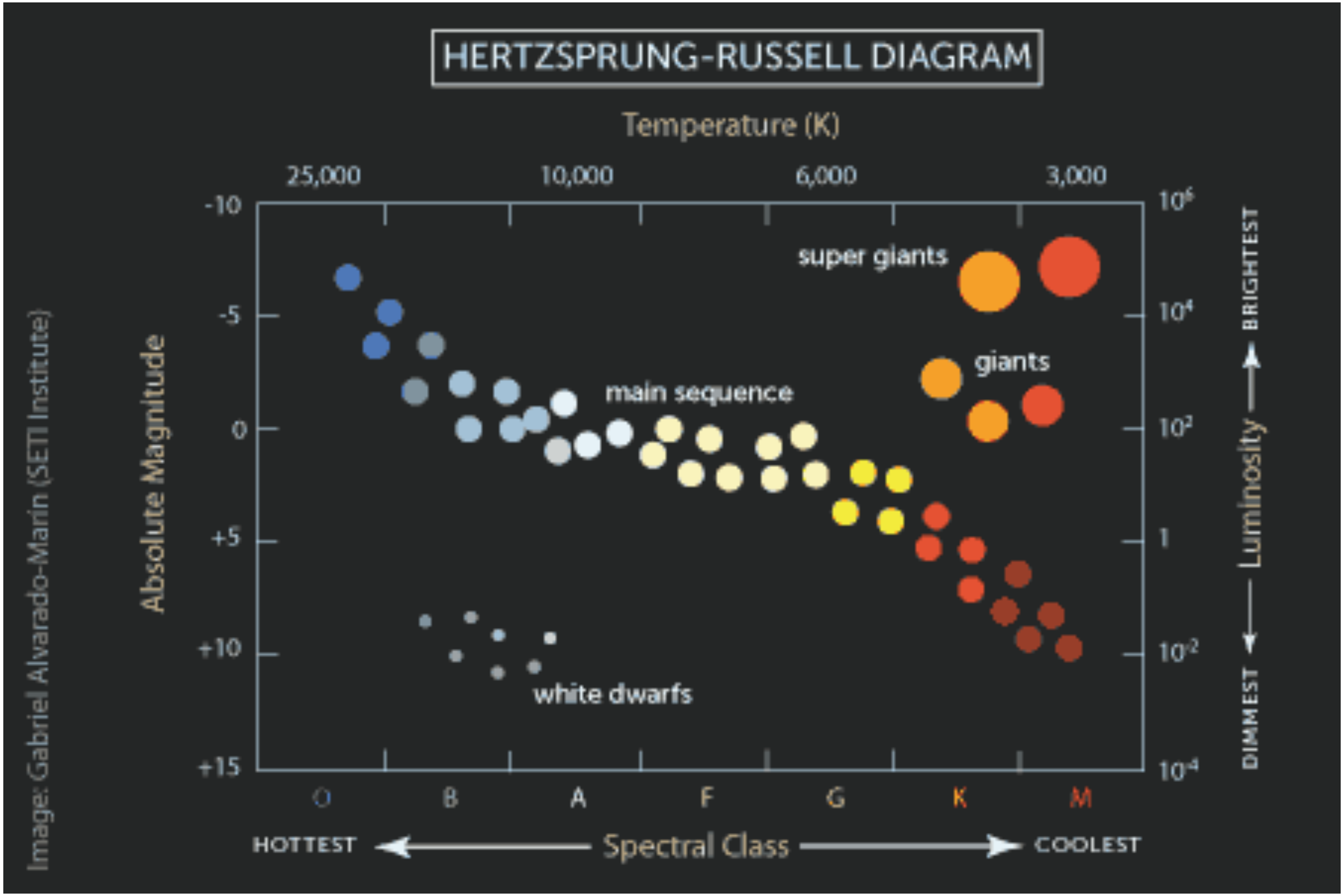
Notice, we can only determine the sum of the masses if we can determine the inclination angle *i*.

This requires that the stars are also eclipsing:

- detailed shape of the light curve of the eclipse gives *i*.
- for an eclipse (obviously?), the members of the pair must be close to 90°.

Your textbook goes through some special cases, faint second object and the case that $M_2 << M_1$. You should review those cases.

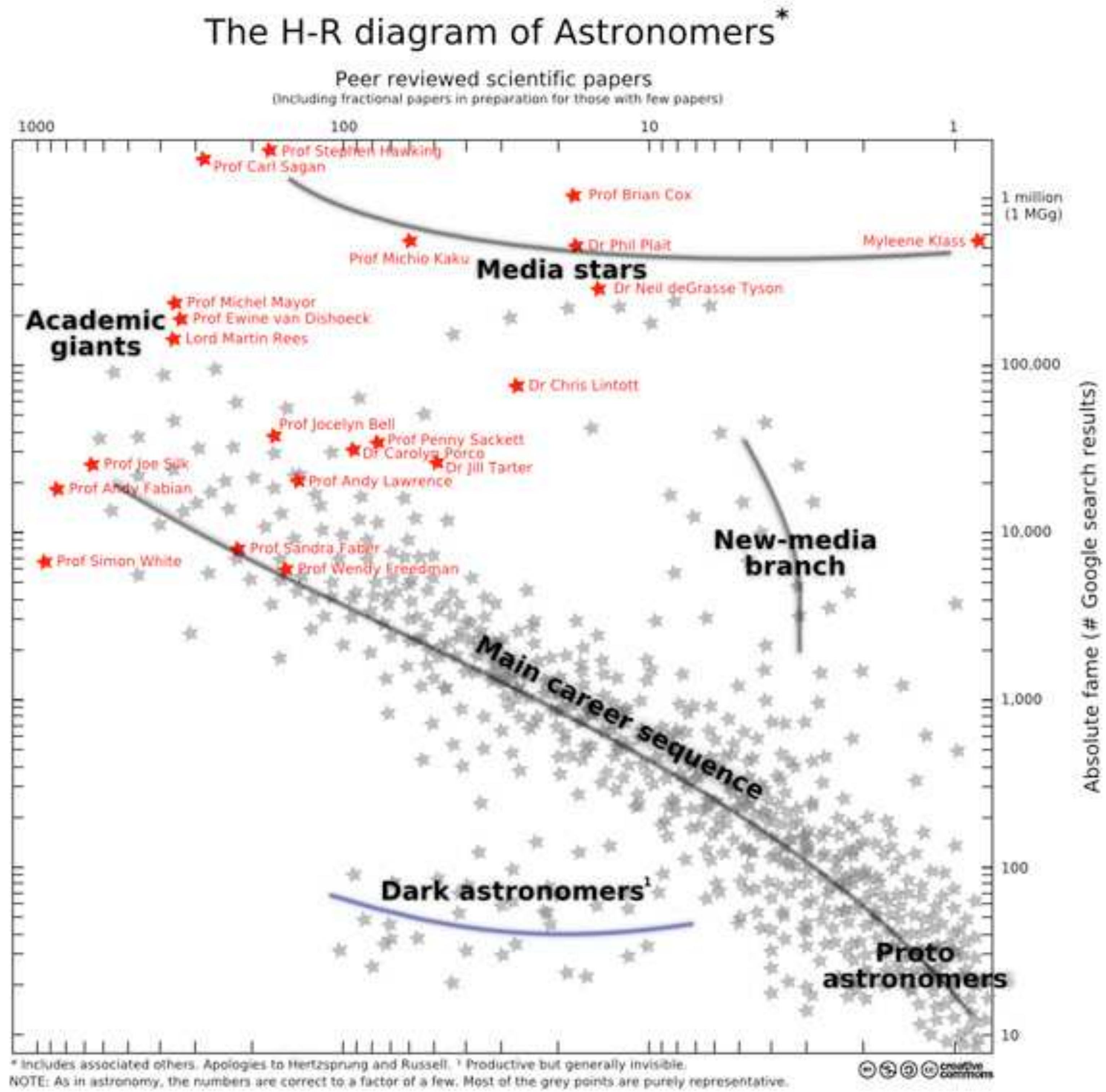
Hertzsprung-Russell Diagram



Physical Meaning

- It was first (incorrectly) thought the main sequence was a cooling sequence, in which stars were born hot and then moved along the sequence as they cooled.
- Measurements of binary stars made it clear that the main sequence is a **mass sequence** with high-mass stars at high luminosities and high T_E and low-mass stars at low luminosities with low T_E.
- Stars spend most of their lifetime at the same location on the main sequence.
  - Stars less massive than $8M_{sun}$ eventually shed outer layers and become white dwarfs.
  - Stars more massive than $8M_{sun}$ past through the giant stage undergo gravitational core collapes that sometimes ends in a supernova explosion.
  - Neutron stars and black holes are stellar remnants of SN explosions. They are more company and even hotter. They are not generally plotted on H-R diagrams.

The End (for today)!



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