Monatomic gas (e.g. He)

d.f. = 3

$$\left.\begin{array}{l} \frac{1}{2}mv_x^2 \\ \frac{1}{2}mv_y^2 \\ \frac{1}{2}mv_z^2 \end{array}\right\} \text{Translation}$$

Diatomic gas (N_2, NO, CO, O_2, H_2 ...)

d.f. = 7

$$\left.\begin{array}{l} \frac{1}{2}I\omega_z^2 \\ \frac{1}{2}I\omega_y^2 \end{array}\right\} \text{Rotation}$$

↑ moment of inertia ("angular mass")

$$\left.\begin{array}{ll} \text{K.E.} & \frac{1}{2}\mu v_{vib}^2 \\ \text{P.E.} & \frac{1}{2}k x^2 \end{array}\right\} \text{Vibration}$$

↑ spring constant $\vec{F} = -k\vec{x}$

Equipartion: every d.f. contributes $\frac{1}{2}k_B T$

Ideal Gas Equation of State

$$PV = N k_B T$$

$$P = \left(\frac{N}{V}\right) k_B T = n k_B T$$

density

$$\frac{N}{V} = \frac{M_{TOT}}{\bar{m}} \frac{1}{V} = \left(\frac{M_{TOT}}{V}\right)\frac{1}{\bar{m}} = \frac{\rho}{\bar{m}}$$

average mass of molecule

Approximations: $m_H \approx m_p \approx m_n$

hydrogen proton neutron

$m_e \ll m_p$

$m_e = \frac{1}{1836} m_p$, $m_e \approx 0$

e.g. molecular hydrogen H_2

$\bar{m} = 2 m_H$

---

monatomic hydrogen H

$\bar{m} = m_H$

---

ionized hydrogen $\dot{e}$ $\dot{p}$

$\bar{m} = \frac{1}{2}(\cancel{m_e}^{\,0} + m_p) \approx \frac{m_H}{2}$

---

doubly ionized C_{12}

$\bar{m} = \frac{4}{} m_H$

$\frac{1}{3}(\cancel{m_e}^{\,0} + \cancel{m_e}^{\,0} + C_{12}^{++})$

---

ionized deuterium ${}^2_1H$

$\bar{m} = \frac{1}{} m_H = \frac{1}{2}(\cancel{m_e}^{\,0} + 2m_p)$

Volume Averaged Quantities

$$\overline{P} = P_{AVG.} = \frac{1}{V}\int P(r)\,dV$$

spherical symmetry

$$= \frac{1}{V}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{R_*} P(r)\, r^2\, dr\, d\theta\, d\varphi$$

$$= \frac{1}{V}\int_{r=0}^{R_*} P(r)\, 4\pi r^2\, dr$$

$$\overline{T} = \frac{1}{V}\int_{r=0}^{R_*} T(r)\, 4\pi r^2\, dr$$

etc.

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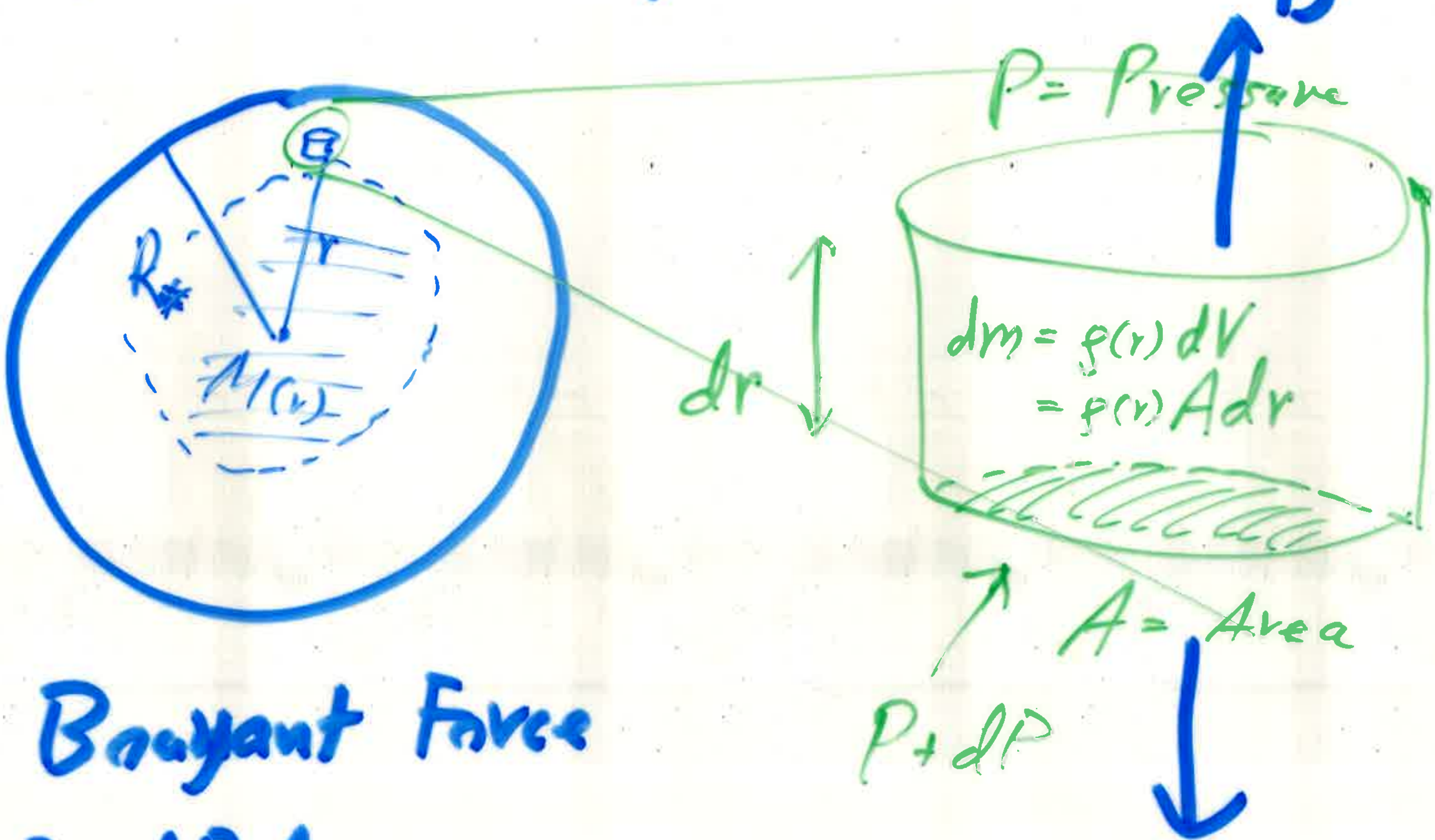
electrons + protons

kinetic pressure: $P_{kin} = \frac{N}{V} k_B T$ (local)

radiation pressure: $P_{rad} = \frac{1}{3} u = \frac{1}{3}\frac{U}{V} = \frac{1}{3} a T^4$

photons only

Hydrostatic Equilibrium



Bouyant Force

$B = dP\,A$

$$F_{Gravity} = \frac{G M(r)\, dm}{r^2}$$

$$dP\,\cancel{A} = -\frac{G M(r)\, \rho(r)\, \cancel{A}\, dr}{r^2}$$

$$\frac{dP}{dr} = -\frac{G\,(M(r))\,\rho(r)}{r^2}$$

Boundary condition $P(R_*) = 0$

$$\frac{dM}{dr} = \rho(r)\, 4\pi r^2$$ Mass Continuity