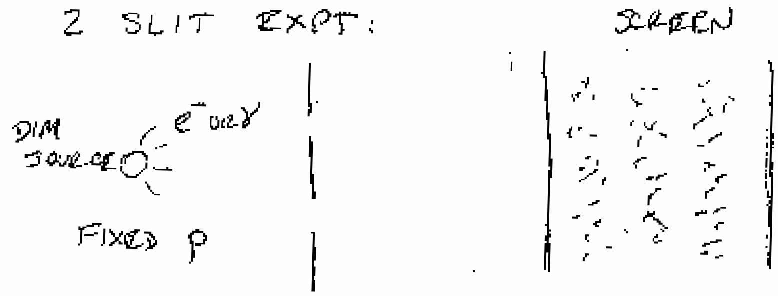
QM REVIEW:

FUND PARTICLES PROPAGATE AS WAVES
INTERACT AS PARTICLES

{talk about light 1st ⇒ how to show it's a wave, give result for dim source; same for e^-}

ex 2 SLIT EXPT:



(FROM ABOVE)

DEPOSIT E IN LUMPS, LOCALLY RANDOM

LARGE # ⇒ SEE INTERFERENCE PATTERN

⇒ EACH PARTICLE HAS WFN WHICH DETERMINES PROB. OF WHERE TO HIT

PARTICLE / WAVE CONNECTION:

{put on side board}

$$\boxed{\begin{aligned} E &= h\nu = \hbar\omega \\ p &= h/\lambda = \hbar k \end{aligned}}$$

$(\omega = 2\pi\nu, \; k = 2\pi/\lambda$
$\hbar = h/2\pi)$

(BLACKBODY SPECTRUM, PHOTOELECTRIC EFFECT,
COMPTON SCATTERING, DAVISSON-GERMER EXPT, ...)

INTERPRETING WAVEFUNCTIONS: (RESTRICT TO 1 DIM FOR SIMPLICITY)

skip

- SOLVED

$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x,t)$$

NOW WHAT? WHAT DOES IT PREDICT?

- NATURE AT MOST BASIC IS RANDOM (ex 2-SLIT EXPT)

∴ CAN'T PREDICT WHERE 1 PART. WILL HIT;
CAN PREDICT PROBABILITY DISTRIBUTIONS: (have many ident. expts in mind)

$$\mathcal{P}(x,t)\,dx = \text{PROB } e^- \text{ (FOR ex) IS BETWEEN } x \text{ \& } x+dx \text{ AT } t$$

$$= |\Psi(x,t)|^2 dx = \Psi^*(x,t)\,\Psi(x,t)\,dx$$

THEN

(1) $\displaystyle\int_{-\infty}^{\infty} \mathcal{P}(x,t)\,dx = \int_{-\infty}^{\infty} \Psi^* \Psi\,dx = 1$ (ANY t)

(2) POSITION

$$\overline{x(t)} \equiv \langle x(t) \rangle = \int \mathcal{P}(x,t)\, x\, dx = \int \Psi^*(x,t)\, x\, \Psi(x,t)\, dx$$

AVE OR EXPECTATION VALUE

IF Ψ LOCAL ~  ~ BEHAVES AS CLASSICAL $x(t)$

HOW WELL KNOW x?

$$[\Delta^* x(t)]^2 = \overline{(\Delta x)^2} = \int \Psi^*(x,t) \cdot (x-\bar{x})^2 \cdot \Psi(x,t)\, dx$$

(3) OTHER $f(x)$:

$$\overline{f(x)} = \int \Psi^* \cdot f(x) \cdot \Psi\, dx \quad \text{AS USUAL}$$

(4) MOMENTUM

CLASSICALLY, NEED x & p TO SPECIFY PARTICLE.

– WFN HAS ALL AVAIL INFO; WHERE'S p?

PRESCRIPTION:

USE "MOM. OPERATOR"

$$\boxed{\hat{p} \equiv -i\hbar \frac{\partial}{\partial x}}$$

FOR $\overline{p}$ IN SAME WAY USED x TO GET $\overline{x}$:

$$\Rightarrow \quad \overline{p(t)} = \int \Psi^*(x,t)\, \underbrace{\hat{p}}_{-i\hbar\frac{\partial}{\partial x}}\, \Psi(x,t)\, dx$$

$$\overline{p^2(t)} = \int \Psi^* \hat{p}^2 \Psi \, dx$$

$$= \int \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial}{\partial x}\right) \Psi \, dx$$

ETC

WHY WORKS:

WFN W/ SINGLE $k = 2\pi/\lambda$:

$$\Psi = \frac{1}{\sqrt{L}} e^{i(kx - \omega t)}$$

} MUST BE COMPLEX WAVE IN GEN'L

CARRIES MOM. $p = \hbar k$ WHEN HITS SOMETHING

SUPERPOSITION:

CAN HAVE WAVE W/ 2 DIFF λ'S MIXED:

$$\Psi = C_1 \frac{1}{\sqrt{L}} e^{i(k_1 x - \omega_1 t)} + C_2 \frac{1}{\sqrt{L}} e^{i(k_2 x - \omega_2 t)}$$

WHEN HITS, CAN MEAS. $p_1 = \hbar k_1$ OR $p_2 = \hbar k_2$

C_1, C_2 TELL HOW MUCH OF EACH

$\Rightarrow$ p IS NOW UNCERTAIN

FOR THIS WFN:

$$\bar{p} = \int \Psi^* \underbrace{\left(-i\hbar \frac{\partial}{\partial x}\right)}_{\hat{p}} \Psi \, dx$$

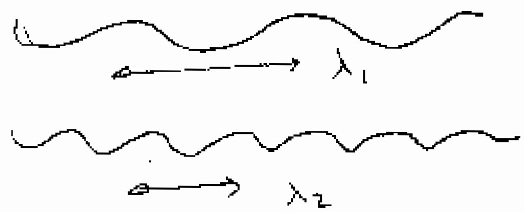
$$= \int \left[c_1^* \tfrac{1}{\sqrt{2}} e^{-i(k_1 x - \omega_1 t)} + c_2^* \tfrac{1}{\sqrt{2}} e^{-i(k_2 x - \omega_2 t)} \right] \times$$

$$\times \left[(\hbar k_1) c_1 \tfrac{1}{\sqrt{2}} e^{i(k_1 x - \omega_1 t)} + (\hbar k_2) c_2 \tfrac{1}{\sqrt{2}} e^{i(k_2 x - \omega_2 t)} \right]$$

DERIV. BRINGS DOWN p_1, p_2 ** KEY **

- EACH SEP. WFN WAS NORM'D TO 1

- CROSS TERMS VANISH:



PRODUCT IS AS OFTEN NEG. AS POS $\Rightarrow$ INTEGRATE TO ZERO
("ORTHOGONAL FNS" OF MECHANICS, FOUR. TRANS)

THEN AVE

$$\boxed{\bar{p} = |c_1|^2 \underbrace{\hbar k_1}_{p_1} + |c_2|^2 \underbrace{\hbar k_2}_{p_2}}$$

ALSO, NORM REQMT: $\boxed{\int \Psi^* \Psi \, dx = |c_1|^2 + |c_2|^2 = 1}$

INTERPRET:

$$|c_1|^2, |c_2|^2 = \text{PROB. OF MEAS. } p_1, p_2$$

$\Rightarrow$ USUAL RULE FOR COMPUTING AVES

- WORKS BECAUSE $-i\hbar \frac{\partial}{\partial x}$ PULLS DOWN $\hbar k = p$ FOR EACH WAVE

- WORKS IN GENERAL BECAUSE ANY WFN $\Psi(x,t)$
CAN BE WRITTEN AS SUM OF WAVES w/ DIFF λ (OR k)
$\Rightarrow$ FOURIER THM

$|\hat{\Psi}|^2$ = PROB DENSITY FOR $\hbar$

- SOMETIMES NEED CONTINUOUS SUM: $\Psi(x,t) = \int dk \, \hat{\Psi}(k,t) \, e^{-i(kx - \omega t)}$

(5) HEISENBERG UNCERTAINTY RELN:

{ both a statement about the process of measurement and a precise statistical statement about wfns

$$(\Delta^* p)^2 = \overline{(p-\bar{p})^2} = \int \Psi^* (\hat{p} - \bar{p})^2 \Psi$$

$$(\Delta^* x)^2 = \overline{(x-\bar{x})^2} = \int \text{"} \; (\hat{x}-\bar{x})^2 \; \text{"}$$

{CAN THINK OF X AS OPERATOR, TOO; JUST MULT BY X}

FOR ANY Ψ:

$$(\Delta^* x)(\Delta^* p) \geq \frac{\hbar}{2}$$

$\Rightarrow$ JUST A STATISTICAL STATEMENT

(RESULT FROM FOURIER ANALYSIS)

(EQUAL? GAUSSIAN WFN)

- smaller spread in $x \Leftrightarrow$ bigger range of p ∴ λ's to add together & v.v.
- the more your wave looks like a particle, the less like a wave

(6) OTHER OPERATORS: (ONE FOR EACH OBSERVABLE QTY)

(c) ENERGY:

$$\hat{E} = i\hbar \frac{\partial}{\partial t} \qquad \text{(PULLS DOWN } \hbar\omega\text{)}$$

THEN SCHRÖDINGER EQN IS JUST (1DIM)

$$\hat{E}\,\Psi = \left(\frac{\hat{p}^2}{2m} + V(\hat{x})\right)\Psi$$

$$\Rightarrow \quad i\hbar\frac{\partial}{\partial t}\Psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(\hat{x})\right)\Psi$$

ANGULAR MOMENTUM: $\hat{L} = \hat{r} \times \hat{p}$

(7) EIGENFUNCTIONS, EIGENVALUES:

(D) EACH OP. HAS ASSOCIATED SET OF SPECIAL FNS OR STATES CALLED EIGENFUNCTIONS; (EIGEN ≡ "ITS OWN")

SATISFY (ex for $\hat{E}$):

$$\hat{E}\,\Psi_n = E_n \Psi_n$$

{ Ψ_n IS STATE W/ DEFINITE ENERGY E_n "STATIONARY STATE"

"EIGENVALUE" ≡ ONLY VALUES EVER MEASURE IN EXPT

CAN SEE:

(a) $\bar{E} = \int \Psi_n^* \hat{E} \Psi_n \, dx = E_n$

(b) $\Delta^* E = 0$

} NO UNCERTAINTY

ANGULAR MOM:

- USED OFTEN FOR EXAMPLES
- NEED FEW SIMPLE RESULTS

$\hat{L} = \hat{r} \times \hat{p}$ (HAVE TO DISCUSS IN 3D) } MEASURES DEGREE TO WHICH PARTICLE IS MOVING ABOUT Z

ALLOWED EIGENVALS (= POSSIBLE MEASMTS)

$\hat{L}^2$: $\ell(\ell+1)\hbar^2$ $\ell = 0, 1, 2, \dots$

(ATOMIC PHYS: s, p, d, f ...)

$\hat{L}_z$: $m\hbar$ $m = -\ell, -\ell+1, \dots, \ell-1, \ell$

(NOTE: JUST AN INTEGER; NOT MASS)

{FROM: (a) WFN SINGLE-VALUED AS GO AROUND 2π ⇒ LIKE MODES ON STRING

(b) WFN FINITE AT $\theta = 0, \pi$ (ie POLES) }

ex $\underline{\ell = 2}$ $L^2 = 6\hbar^2$ $|L| = 2.4\hbar$

$L_z = -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar$

cf PLOT: CLASSICAL PICTURE ~ AS IF VECTOR CAN'T POINT ALONG z PRECISELY, CAN ONLY ^ IN CERTAIN DIRECTIONS

NOTE:

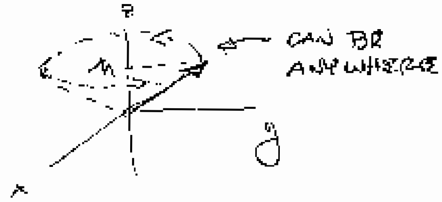
CLASSICAL VECTOR:

CAN SPECIFY L^2, L_x, L_y, L_z

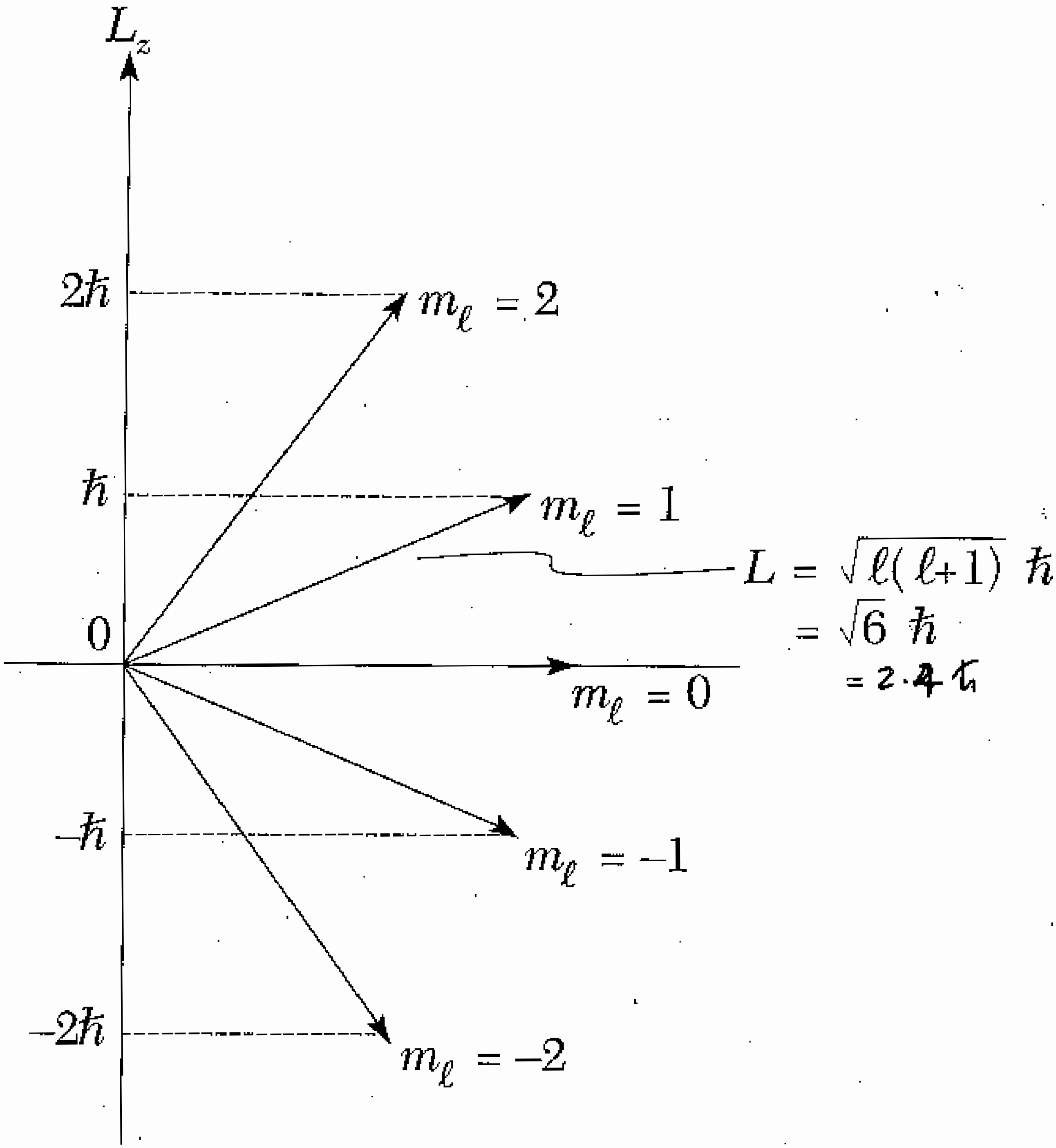
QM:

IN STATE OF DEF. L^2, L_z ⇒ CAN'T KNOW L_x, L_y EXACTLY (ONLY ON AVE)

ex $\underline{\ell=2, m=1}$: AS IF PRECESSING:

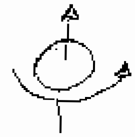


(BUT ALLOWED VALUES FOR $L_{x,y}$ SAME AS FOR L_z)



35. Figure 8.3
Thornton/Rex: Modern Physics for Scientists and Engineers

SPIN

- PARTICLES CAN CARRY INTRINSIC ANG. MOM. EVEN IF NOT MOVING
- AS IF SPINNING  (BUT CAN'T SLOW DN, SPEED UP)

(but better to think of as an intrinsic property, like mass)

- USE $\hat{\vec{S}}$ (TO DIST. FROM $\hat{\vec{L}}$)

E-VALUES:

little s

NEW

$\hat{S}^2$: $s(s+1)\hbar^2$ $\qquad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

$\hat{S}_z$: $s_z\hbar$ $\qquad s_z = -s, -s+1, \ldots s-1, s$

(AS BEFORE)

e^- : $s = \frac{1}{2}$ (SPIN $\frac{1}{2}$)

$S^2 = \frac{3}{4}\hbar^2 \quad S_z = \pm\frac{1}{2}\hbar$ (SPIN UP OR DOWN)

γ : $s = 1$

$S^2 = 2\hbar^2 \quad S_z = \pm 1\hbar$

(ALL KNOWN FUND. PARTICLES HAVE EITHER $s = \frac{1}{2}$ OR 1
(BUT LOTS OF COMPOSITES W/ MORE, OR $s=0$))

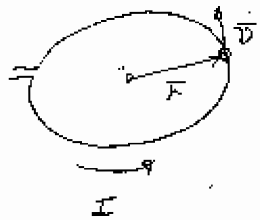
MEASURE?

- FROM ANG. MOM CONS. IN COLLISIONS
- IF CHARGED, HAS MAGN MOMENT ⇒ FEEL $\vec{B}$ FIELD

9.8 ~~2.7.1~~ ~~5.15~~

MAGNETIC MOMENTS

WIRE LOOP



(vectors, not areas)

$$\bar{\mu} = \frac{1}{2c} \oint \bar{r} \times (q\bar{v}) \qquad I\,d\bar{l}$$

(DIR: RH RULE)

{ Tells me.. - force loop feels in $\bar{B}$ - $\bar{B}$ loop creates }

$$\bar{L} : \quad \oint \bar{r} \times \bar{p} = \oint \bar{r} \times (m\bar{v})$$

SO EXPECT $\bar{\mu} \propto \bar{L}$

(HERE: FACTOR $\frac{1}{2c}\frac{q}{m}$)

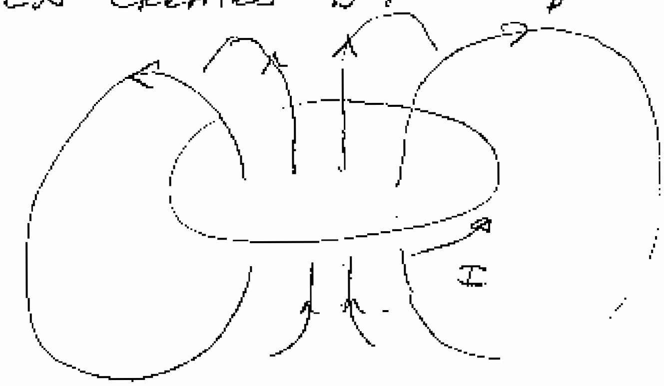
IN CONST. $\bar{B}$: (from Lorentz force law $\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})$)

$$\bar{T} = \bar{\mu} \times \bar{B}$$

$$E = -\bar{\mu} \cdot \bar{B}$$

} TRIES TO LINE UP W/ $\bar{B}$ ⇒ MAKES E LOWER

ALSO CREATES $\bar{B}$:



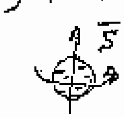
(BIOT-SAVART LAW)

SO LOOPS CAN FEEL OTHER LOOPS

QM: $\bar{\mu} \propto \bar{L}$ ALSO ⇒ BOTH QUANTIZED

ex e^- IN $\bar{B}$ FIELD : (AT REST) : $S_z = \pm\frac{1}{2}\hbar$

TAKE $\bar{B} = B\hat{z}$



$\downarrow \bar{\mu}$ (chg<0)

$E = -\bar{\mu} \cdot \bar{B} = -\mu_z B$

⇒ TWO STATES: $E = \pm |\mu_z| B$

{ NOTATION: REIF USES $H \equiv B$, $\mu \equiv |\mu_z|$ }

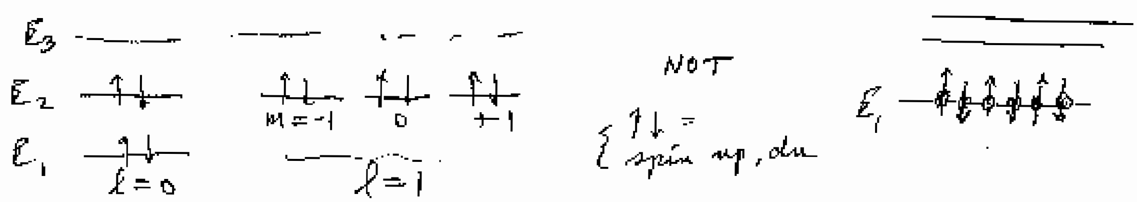
PAULI EXCLUSION PRINCIPLE & IDENT. PARTICLES

- E LEVELS OF ATOMS → BIG SUCCESS OF QM

- e^-'S SPEND MOST TIME (FOR COMMON MATTER) IN LOWEST STATE

{ notation: $L^2 = \ell(\ell+1)\hbar^2$ orbital ang mom
$L_z = m\hbar$ ↑↓ $S_z = \pm\frac{1}{2}\hbar$
$\ell = 0, 1, 2$ $m = -\ell, -\ell+1, \dots, +\ell$

ex NEON: 10 e^-s
FIND



- RESPONSIBLE FOR ALL INTERESTING CHEM. (INCL LIFE).

PAULI EXCLUSION PRINCIPLE:

- ONLY ONE e^- CAN BE IN THE SAME STATE (HAVE SAME WFN) AT SAME TIME (STATE: E, ℓ, m & S_z IN PARTIC. ATOM)
(DIFF ATOMS ⇒ X DIFF → OK)

DEEP CONNECTION TO SPIN:

RULE: $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$ OBEY RULE ⇒ "FERMIONS"
(INCLUDES e^-, n, p, ...)

$S = 0, 1, 2, \dots$ DON'T ⇒ "BOSONS"
(INCLUDES γ, ...)

(IF e^- HAD SPIN 0 ⇒ ALL ATOMS COLLAPSE
⇒ BAD NEWS)

NEITHER ARE CLASSICAL:

CLASSICAL PARTICLES

- CAN ATTACH LABEL W/OUT ALTERING PROPERTIES (ex # ON BILLIARD BALL)

  $\Rightarrow$ INFINITESIMAL DIFF. MAKES SENSE

- CAN WATCH TRAJECTORY W/OUT ALTERING EXPT

QM:

- IDENTICAL PARTICLES REALLY ARE IDENTICAL;
- CAN'T FOLLOW TRAJ. W/OUT ALTERING (ex BOUNCING γ OFF e^- TO SEE IT; BETTER RESOLUTION, SMALLER λ, BIGGER Δp)
- NO EXTRA LABELS AVAILABLE: ($Q = -1$, $S = \frac{1}{2}$, $S_z = +\frac{1}{2}$, $\vec{p}$) COMPLETELY SPECIFIES FREE e^-

  ex CHG Q TO +1 $\Rightarrow$ DIFF PARTICLE (There are no infinitesimal diffs)

$\Rightarrow$ START 2 e^-'s OUT & LATER MEAS. POSITION OF 1 e^-: FINITE PROB. COULD BE EITHER (Which one? not meaningful question)

COUNTING STATES:

MOST CAN SAY IS HOW MANY IN EACH STATE

ex 2 PARTICLES A & B IF ONLY POSSIBLE 1-PART. STATES HAVE E_1 & E_2:

DISTINGUISHABLE (ex: classical or different QM particles)

E_1	E_2
AB	—
A	B
B	A
—	AB

IDENT. BOSONS (ex 2 γ's)

E_1	E_2
2	0
1	1
0	2

more likely in same state

IDENT. FERMIONS

E_1	E_2
1	1

never in same state