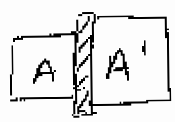
much intro: have seen can make direct connection to av'd gen. F's and W
Therm. rels: can I find param which governs Q?

CH 3 HEAT, TEMP, ENTROPY

HEAT FLOW

- WILL DEF. MACRO QTY'S TO DESCRIBE Q EXCHANGE
- ALL BASED ON PROB. (i.e. FUND. POST.) { will one system give E to another? yes, on ave, if it's more likely; ie, if there are many more ways for that to happen than not }

ex  w/ E, E' FIXED :

ISOLATED : TOTAL $E^{(0)} = E + E'$ CONST

INSULATED : E, E' CONST

$\Rightarrow$ MACRO CONSTRAINTS : $E^{(0)}, E$ (Then $E' = E^{(0)} - E$)
(choose these rather than E & E' since $E^{(0)}$ will remain a const.)

KEEP ALL X_α FIXED (NO W)

$\Rightarrow$ ALL SYS IN ENS. SATISFY THESE

IN EQUIL : ALL POSS. STATES EQ. LIKELY

NOW :

REMOVE INSUL :

- RELAXES CONSTRAINT ;
  $E^{(0)}$ CONST , BUT E (& E') CHG
- $\Omega_f \geq \Omega_i$: { # MICRO STATES POSSIBLE } { by defn of constraint }
  - HAVE ALL ORIGINAL STATES + (USUALLY MANY) MORE w/ DIFF E, E'
  - MANY NEW STATES APPEAR IN ENS. (assume big enough that all still well-represented)

- NOT IMMEDIATELY IN EQUIL (USUALLY):

  Q FLOWS UNTIL ALL P_r = CONST. ⇒ NEW EQUIL

  IN ENS: $E\,(\,\&\,E')$ WILL DIFFER

- CAN'T UNDO BY PUTTING INSUL. BACK IN

  (would need to do more → work on system)

  (won't get heat to flow back)

10/18/02

example of

IRREVERSIBLE PROCESS:

- ISOLATED SYS.
- RELAX CONSTRAINT:

say { ANYTHING RESTRICTING STATES IN ENS.

(REIF: y_α — MORE GENERAL THAN x_α)

ex: $E, N, V, p \ldots$

anything known or set which can be used to exclude states from ens.

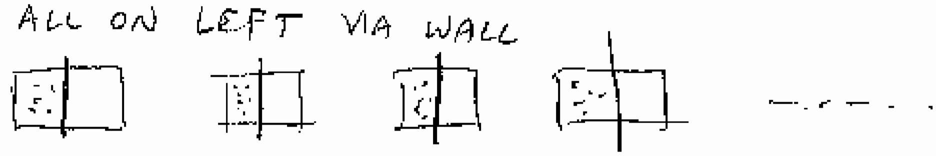
$\Rightarrow \Omega_f > \Omega_i$

⇒ CAN'T UNDO BY REIMPOSING

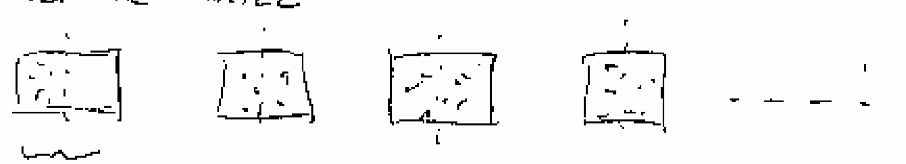
say (already covered in intro)

ex PARTICLES IN BOX:

CONSTR: ALL ON LEFT VIA WALL

ENS: 

RELAX: REMOVE WALL

ENS: 

SMALL SUBSET

$\Omega_f \gg \Omega_i$

HOW MUCH? (twice? not even close)

IN EQUIL: P(ALL LEFT) NOW $= \frac{1}{2^N}$ $N \sim 10^{24}$

FROM: $\underbrace{P(\text{LEFT}) = \frac{\Omega_i}{\Omega_f}}_{\text{FUND. POST}} = \frac{1}{2^N} \sim \frac{1}{2^{10^{24}}}$ (we only needed to pay attn to location, since $\vec{p}$ played no role)

⇒ INITIAL CONFIGS SWAMPED BY NEW CONFIGS

⇒ VERY UNLIKELY TO FIND IN ORIG. CONFIG AFTER NEW EQUIL

- CAN'T UNDO BY PUTTING WALL BACK
  (like Pandora's box; doesn't restore orig. configs in ens.)

- CAN IF INTERACT w/ ANOTHER SYS (but then not isolated)
  DOES WORK

skip

REVERSIBLE PROCESS

- RARE

- RELAX ⇒ $\Omega_f = \Omega_i$ (i.e. doesn't gain access to new state)

  ⇒ REALLY WASN'T A CONSTRAINT

- CAN RESTORE BY REIMPOSING (dumb example: $0 < E < E^{(0)}$; or putting wall out & in again)
  (no additional work)

skip

BACK TO: $\boxed{A \,|\, A'}$ $E^{(0)} = E + E'$

ALL PROB. DEPENDS ON Ω (IN EQUIL)

skip

⇒ WHAT'S NEW $\bar{E}$?

⇒ IS Q LIKELY?

How can I find most likely new $\bar{E}$? Look at Ω_{TOT} from prev post, which tells me how many states as fn of E & look for peak

NEW $\bar{E}$, Q DETERMINED BY

$$\Omega(E^{(0)}, E)_{TOTAL} = \Omega(E)\,\Omega'(E') \qquad \text{(with insul)}$$

$$E' = E^{(0)} - E$$

(# STATES AVAIL AT VARIOUS E's)

FROM POST: $P(E) = \dfrac{\Omega_{TOT}(E^{(0)}, E)}{\sum_E \Omega_{TOT}(n)}$

BEHAVIOR:

EXPECT:

- $\Omega(E)$, $\Omega'(E')$ GROW RAPIDLY w/ E, E'
- Ω_{TOTAL} SHARPLY PEAKED AT SOME $E = \tilde{E}$ (MAX):

$$\Omega_{TOTAL} = \underbrace{\Omega(E)}\;\underbrace{\Omega'(E^{(0)} - E)}$$

AS E INCR: GROWS FAST / SHRINKS FAST (and v.v.)

(falls to zero when $E \geq E^{(0)}$)

EST:

$$\Omega(E) \sim E^f \qquad \Omega'(E') \sim (E')^{f'}$$

$$\Omega_{TOTAL} \sim E^f (E^{(0)} - E)^{f'}$$

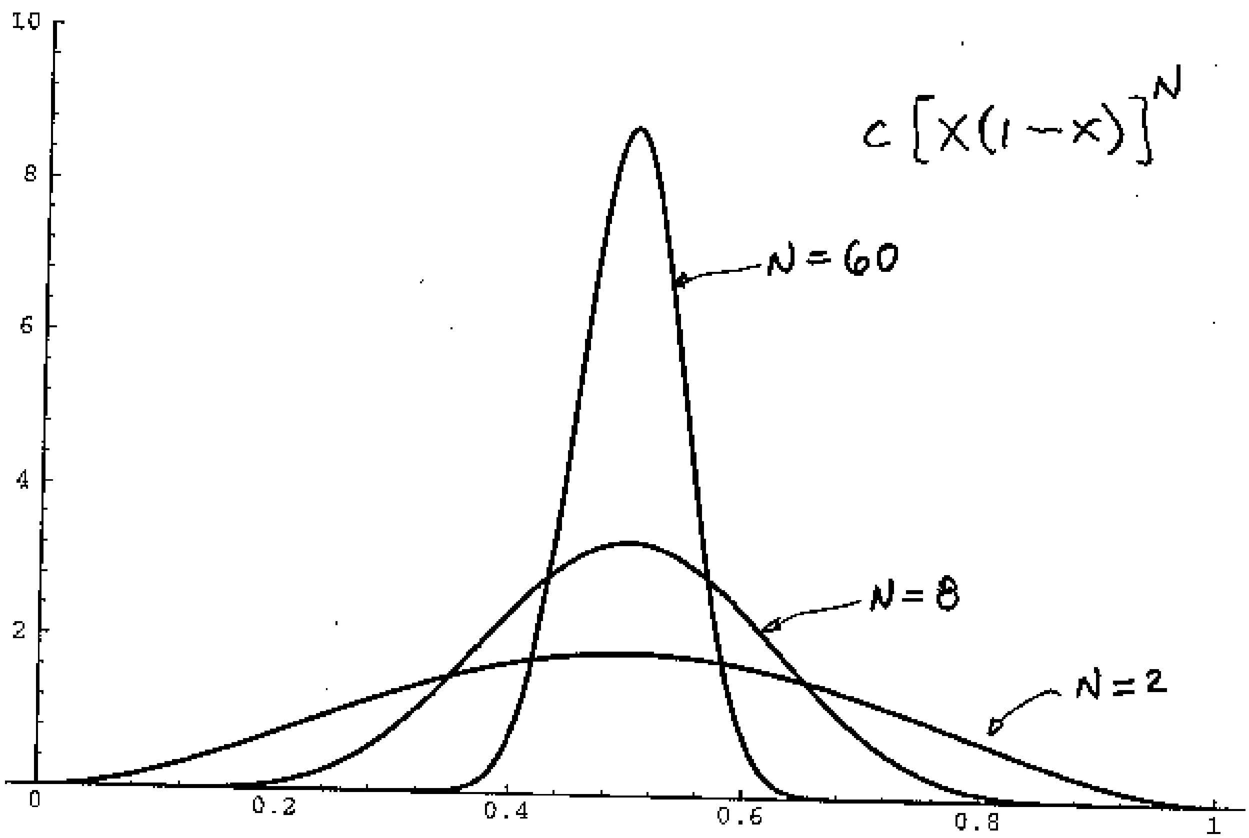
$\Rightarrow$ SHARP PEAK FOR $f \sim 10^{24}$

eg **~~EXAM~~ HW PROB:**

LET $f = f'$ $\qquad x \equiv \dfrac{E}{E^{(0)}}$ (fraction of E_{TOTAL} in sys A)

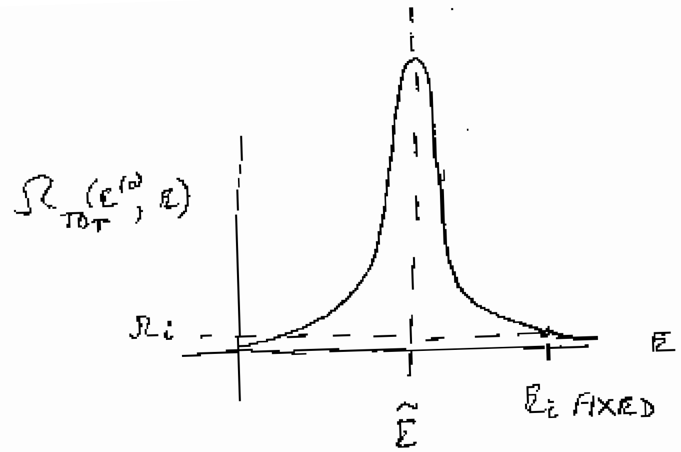
$$\Omega_{TOT} \sim E^{(0)2f} x^f (1-x)^f$$

SHARP PEAK NEAR $x \sim \frac{1}{2}$ (cf PLOT)



$c\left[x(1-x)\right]^N$
N = 60
N = 8
N = 2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

THEN



REMOVE INSUL:

(1) MANY MORE STATES AVAIL, MOST NEAR $\tilde{E}$ (i.e. many more ways can have $\tilde{E}$ than E_i)

(2) MOST SYS. IN ENS. EXCHANGE ENERGY SO $E \approx \tilde{E}$

(3) NEW EQUIL

$$\bar{E} \cong \tilde{E} \quad \Rightarrow \quad \text{DETERMINES } \underset{= \bar{E} - E_i}{Q} \quad \text{(ave effect of } E\text{)}$$

E NOT FIXED, BUT LITTLE PROB TO FIND $E \neq \bar{E} \approx \tilde{E}$

(so E near $\tilde{E}$ is most important,
since that's most probable, tho any E is possible up to $E^{(0)}$)

(→)

{IF KNOW $\Omega(E)$, $\Omega'(E')$} CAN PREDICT $\overline{E}$: ("is what happens when bring together (depends on 1 par")

$$P(E) = C\,\Omega_{TOTAL}(E^{(0)}, E) = C\,\Omega(E)\,\Omega'(E')$$

AT $E = \tilde{E}$ P (OR $\ln P$) IS MAX:

use $\ln P$ ⇒ smoother, more convenient can do a better job approximating; also, sum rather than product

$$\ln P = \ln C + \ln \Omega(E) + \ln \Omega'(E')$$

MAX

$$\frac{\partial \ln P}{\partial E} = \underbrace{\frac{\partial \ln \Omega(E)}{\partial E}}_{\beta(E)} + \underbrace{\frac{\partial \ln \Omega'(E')}{\partial E'}}_{\beta'(E')} \underbrace{\frac{\partial E'}{\partial E}}_{-1} = 0$$

$\beta(E)$ (rate at which ln(#) states grows w/ E)

$(E' = E^{(0)} - E)$

MAX ⇒ $\boxed{\beta(\tilde{E}) = \beta'(\tilde{E}') = \beta'(E^{(0)} - \tilde{E})}$

AT MAX: rate at which number states gained by $\Omega(E)$ = rate lost by $\Omega'(E')$

DEFNS:

ABSOLUTE TEMP $kT \equiv \frac{1}{\beta}$ } units = energy

T ← DIMENSIONLESS

k: CONST W/ UNITS OF ENERGY (depends on units)

SMALL T ⇒ LARGE β ⇒ Ω GROWS A LOT W/ E

LARGE T → SMALL β " " LITTLE

⇒ $\ln \Omega_{TOT}$ GROWS BY ENERGY E MOVE, FROM LARGE T TO SMALL T

(AS MOVE TO $\tilde{E}$, Q FROM LARGE T TO SMALL UNTIL EQUAL)

{note T is abstract → can't look at a single microstate & pick out its T; macro qty; have in mind an ens.}

ENTROPY $\boxed{S \equiv k \ln \Omega}$ (UNITS: ENERGY)

- $\ln$ OF AVAILABLE STATES
- MEASURE OF RANDOMNESS OR DISORDER: (have probably heard)

$\Rightarrow$ LESS KNOWN ABOUT SYSTEM, MORE STATES AVAIL, LARGER S (less organized, more choices system can & will take)

THEN $\boxed{\dfrac{1}{T} = \dfrac{\partial S}{\partial E}}$ (lower T, more S incr. with E)

$\Rightarrow$ $P(E)$ IS MAX WHEN:

$$\boxed{\begin{aligned} E &= \tilde{E} \\ \beta(\tilde{E}) &= \beta'(\tilde{E}') \\ T &= T' \\ \underbrace{S + S'}_{\text{additive}} &= \text{MAX} \end{aligned}}$$

(A)

ALL PROPERTIES OF $\Omega(E)$; NOTE THESE ARE STATISTICAL QTY'S BY DEFN. CAN'T LOOK INSIDE ONE SYS, & SEE T OR S

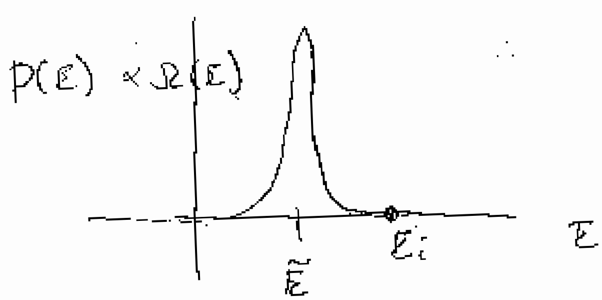
(note def. S as $\ln \Omega$ means add S's)
MULT Ω's $\rightarrow$ ADD S's

skip it

SUMMARY (just say)

APPROACHING EQUIL:

skip drawing again?



AFTER REMOVE INSUL:
MOST AVAIL. STATES HAVE
$E = \tilde{E}$ $\therefore$ IN EQUIL
$\bar{E} = \tilde{E}$

~~AS OCCUPY NEWLY ALLOWED STATES:~~

~~$\bar{E}(t) \rightarrow \tilde{E} = \bar{E}_f$~~

OTHER PROPERTIES OF ABS TEMP T:

(1) SIGN

$$\frac{1}{kT} \equiv \beta \equiv \frac{\partial \ln \Omega}{\partial E} > 0$$

more E, more Ω

$$\boxed{T > 0}$$

(somewhat artificial)

(EXCEPTION: SPIN SYSTEM – ~~MOLECULES~~ TIED DOWN (NO KE)
HAS MAX E → ALL ALLIGNED OPP.;

AS APPROACH E_{MAX}, Ω DECREASES,
$T < 0$)

(2) RELN TO AVE E OF PARTICLES:

$$\Omega(E) \propto E^{f}$$

$$\ln \Omega \sim f \ln E + \text{const}$$

$$\beta = \frac{\partial \ln \Omega}{\partial E} \sim \frac{f}{E}$$

AT EQUIL: $\beta(\bar{E}) = \frac{f}{\bar{E}}$ $\boxed{kT \sim \frac{\bar{E}}{f}}$

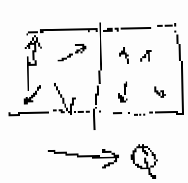
$\Rightarrow$ $\boxed{kT \sim \text{AVE ENERGY PER DOF}}$ (very useful est.)

(familiar – high T, lots of E per molecule, etc)
incl KE, INTERNAL E, ETC

~~B 3/6/97~~

skip

$\Rightarrow$ AS GO TO EQUIL



UNTIL ALL DOF HAVE ~ SAME E
(WE'VE SEEN IN COUNTING EXERCISES:
RARE FOR FEW PARTICLES TO HAVE LOTS OF E)

GAUSS APPROX FOR Ω_{TOT}:

SHARPLY PEAKED — EXPAND $\ln \Omega_{TOT}$ AT $E = \tilde{E}$:

$$\ln \Omega_{TOT}(E) = \ln[\Omega(E)\, \Omega'(E')] = \ln\Omega + \ln\Omega'$$

NEED

$$\frac{\partial \ln \Omega_{TOT}}{\partial E} = \frac{\partial \ln \Omega}{\partial E} + \frac{\partial \ln \Omega'}{\partial E'} \underbrace{\left(\frac{\partial E'}{\partial E}\right)}_{-1}$$

$$(E' = E^{(0)} - E)$$

$$= \beta(E) - \beta'(E')$$

$$\frac{\partial^2 \ln \Omega_{TOT}}{\partial E^2} = \frac{\partial^2 \ln \Omega}{\partial^2 E} + \frac{\partial^2 \ln \Omega'}{\partial^2 E'} \underbrace{\left(\frac{\partial E'}{\partial E}\right)^2}_{+1}$$

$$\equiv -\left(\lambda(E) + \lambda'(E')\right)$$

since expect < 0 at max

MAX: $$\left.\frac{\partial \ln \Omega_{TOT}}{\partial E}\right|_{E=\tilde{E}} = 0 \Rightarrow \beta(\tilde{E}) = \beta'(\tilde{E}')$$

$$\Rightarrow T = T' \quad \text{WHEN} \quad E = \tilde{E} = \bar{E}$$

(KNOW THIS: same T at max)

$$\Rightarrow \ln \Omega_{TOT}(E) \sim \ln \Omega_{TOT}(\tilde{E}) - \frac{1}{2}\underbrace{\left(\lambda(\tilde{E}) + \lambda'(\tilde{E}')\right)}_{\equiv \lambda_0}(E - \tilde{E})^2 + \cdots$$

3.11

equality rare; forget equal case (skip)

$\Rightarrow \quad \lambda_0 \geq 0 \quad$ IF AT MAX $\quad$ (if no max, wouldn't reach equil after some ΔQ)

$\Rightarrow \quad \lambda(\tilde{E}) \;\&\; \lambda'(\tilde{E}') \quad$ BOTH ≥ 0

(IF for ex $\lambda(\tilde{E}) < 0$, could put two of these ident. sys. together $\Rightarrow \lambda_0 = 2\lambda(\tilde{E}) < 0$)

$$\Rightarrow \quad \boxed{\Omega_{TOT}(E) \approx \Omega_{TOT}(\tilde{E})\, e^{-\frac{1}{2}\lambda_0 (E-\tilde{E})^2}}$$

$$\Rightarrow \quad P(E) \propto \Omega_{TOT}(E) \qquad \text{(FUND POST)}$$

$$\Rightarrow \quad \boxed{P(E) = P(\tilde{E})\, e^{-\frac{1}{2}\lambda_0 (E-\tilde{E})^2}}$$

GAUSSIAN:

$$\bar{E} = \tilde{E} \qquad \text{(exact in this approx)}$$

$$\sigma = \Delta^* E = \frac{1}{\sqrt{\lambda_0}} \qquad \text{all determined by } \Omega$$

HOW SHARP?

$$\Omega(E) \sim E^f \qquad \ln \Omega \sim f \ln E$$

$$\frac{\partial^2 \ln \Omega}{\partial E^2} \sim -\frac{f}{E^2}$$

$$\text{AT } \tilde{E} \simeq \bar{E}: \quad \lambda_0 \sim +\frac{f}{\bar{E}^2}$$

$$\Rightarrow \quad \frac{\Delta^* E}{\bar{E}} \sim \frac{1}{\sqrt{f}} \sim 10^{-12}$$

VERY SHARP:

$P(E) \sim 0 \quad$ IF $\quad \Delta E \gtrsim 10^{-12}\, \bar{E}$

$\Rightarrow$ RARE THAT MEAS. FLUCTUATION OF PART/TRILLION

$\Rightarrow$ GAUSS IS ESSENTIALLY EXACT

ALMOST IDENTICAL TO FIXING E AT $\tilde{E}$

CONSEQUENCES OF SMALL $\Delta^* E$: (ie VERY SHARP PEAK)

PUT IN CONTACT :

ENERGY FLOWS (ON AVE) FROM LOW β (HIGH T) ("HOTTER") TO HIGH β (LOW T) ("COLDER") : RATE AT WHICH HIGH β GAINS AVAIL STATES > RATE AT WHICH LOW β LOSES (cp  then put hole in wall)

UNTIL

NEW EQUIL :

sharp!

$$\bar{E} = \tilde{E} \quad \Rightarrow \quad Q = \bar{E} - E_i = -Q' \qquad \Big\{ \text{DEFN OF } Q$$

determined by

$$\beta(\bar{E}) = \beta'(\bar{E}') \quad \text{OR} \quad T(\bar{E}) = T'(\bar{E}') \qquad \Big\{ \text{FROM BEFORE, } w/\ \bar{E} = \tilde{E}$$

$$S(\bar{E}) + S'(\bar{E}') \quad \text{AT MAX} \qquad \Big\{ S = k \ln \Omega$$

AT $E = \bar{E}$, RATE AT WHICH A GAINS AVAIL. STATES

= " " A' LOSES " "

NO FURTHER FLOW ON AVE, JUST FLUCTS. AROUND $\bar{E}$

(0) GAUSS APPROX ~ EXACT ; $\bar{E} = \tilde{E}$

(a) FLUCTS ARE TINY $\Rightarrow$ VERY FEW SYS. IN ENS. HAVE E FAR FROM $\bar{E}$

(b) VERY SIMILAR TO CASE WHERE E IS EXACTLY $\bar{E}$

E' " " $E^{(0)} - \bar{E}$

AND TWO ARE AGAIN ISOLATED

(c) IF $\Delta^* E < \delta E$ (ABILITY TO MEASURE), THEN

IDENTICAL TO THIS CASE

(d) CAN'T THINK OF $T(\bar{E})$ STRICTLY AS TEMP FOR A

– T DEF'D FOR SYS. AT DEFINITE E

– SYS IN ENS. FOR A HAVE $0 < E < E^{(0)}$

BUT
CAN IN SO FAR AS (b)-(c) TRUE

WHAT ABOUT MY (SINGLE) SYS?

(e) Q, T DEF'D FOR ENS., NOT " ; TELLS ABOUT AVES

~~ABOUT AVES?~~

NARROW: KNOW (W/ LITTLE ERROR)

$$E_f = \bar{E}$$

$$\Delta E = \bar{E} - E_i = Q$$

⇒ INSERT ⇐

(f) IN THIS SPIRIT: 0^{TH} LAW

~~⇒ ALMOST IDENTICAL TO FIXING A AT $\tilde{E}$, A' AT $\tilde{E}' = E^{(0)} - \tilde{E}$ (though E is allowed to vary)~~

INSERT

NOTE:

This is why we think that computing prob. & qtys related to them (Q, β, T, $\bar{E}$, S ...) can say something useful about a single system. Although we only know $\bar{E}$, $\Delta^* E \ll \bar{E}$ and it's almost indistinguishable from knowing E exactly. Q is an ave flow of energy, but the odds are so small that for an system that $\Delta E \neq Q = \bar{E}_f - \bar{E}_i$ That it tells us what will happen to 1 system.

Have to look very hard (or at small system) to see fluctuations.

AT EQUIL:

MOST SYSTEMS IN ENS. FOUND NEAR MAX $P(E) \propto \Omega^{(0)}(E^{(0)}, E)$

SO $\bar{E} \cong \tilde{E}$

$\beta(\bar{E}) = \beta'(\bar{E}')$ { at peak, rate at which A gains states balance rate at which A' loses ξ v.v.; systems hover around this equil pt

OR $T(\bar{E}) = T'(\bar{E}')$

$S(\bar{E}) + S'(\bar{E}')$ AT MAX { another way of saying that $\bar{E}$ is at most probable value

skip

0^{TH} LAW: (originally from observation)

THERMAL EQUIL. DEPENDS ON SINGLE PARAM:

IF $\beta(E) = \beta'(E')$ (OR $T = T'$):

- AT MAX FOR $\Omega_{TOT} = \Omega \cdot \Omega'$
- DON'T GAIN BY Q
- IN EQUIL

IF	SYS	A	IN	THERM.	EQ.	W/	SYS C
AND	"	B	"	"	"	"	" C
THEN		A		"		"	B

ie $\boxed{\text{IF } \beta_A = \beta_C \ \xi \ \beta_B = \beta_C \text{ THEN } \beta_A = \beta_B}$

(g) APPLY TO S:

~~3.11.2~~
3.11.5

LARGE NUMBERS / SHARP PEAKS:

skip (1) δE (ACC. OF MEAS.) DOESN'T MATTER:

(a) $\Omega(E) \equiv \omega(E)\,\delta E$ (i.e. $\Omega \propto \delta E$)

$$\ln \Omega = \ln \omega + \ln \delta E$$

$$\beta = \frac{\partial \ln \Omega}{\partial E} = \frac{\partial \ln \omega}{\partial E}$$

$\Rightarrow$ β OR T INDEP OF δE

(b) $S = k \ln \Omega$

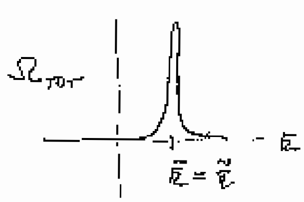
DIFF CHOICE FOR δE: $\delta E \to \delta E^*$

$$\Omega^* = \omega(E)\,\delta E^* = \Omega \cdot \left(\frac{\delta E^*}{\delta E}\right)$$

$$S^* = k \ln \Omega^* = k\Big[\underbrace{\ln \Omega}_{\sim f} + \underbrace{\ln\left(\frac{\delta E^*}{\delta E}\right)}_{\sim 1}\Big]$$

$\therefore\ S^* = S$

cover this one (2) TWO SYSTEMS: can show $S_{ALL\ E} \approx S(\bar{E}) + S'(\bar{E}')$ $\boxed{A}\boxed{A'}$



- ANY E POSSIBLE

$$S_{A.E.} = k \ln \Omega_{AE}^{TOT}$$

$$\Omega_{AE.} = \int_0^{E^{(0)}} dE\ \omega_{TOT}(E^{(0)}, E)$$

$$\frac{\Omega_{TOT}(E^{(0)}, E)}{\delta E}$$

NARROW:

$$\int dE\, \Omega_{TOT}(E^{(0)}, E) \approx \Omega_{TOT}(E^{(0)}, \bar{E})\, \Delta^* E$$

$$\Rightarrow \Omega_{AE} \sim \Omega_{TOT}(E^{(0)}, \bar{E}) \left(\frac{\Delta^* E}{\delta E}\right)$$

so

$$S_{AE} \sim \underbrace{k \ln \Omega_{TOT}(\bar{E})}_{\sim f} + \underbrace{k \ln \left(\frac{\Delta^* E}{\delta E}\right)}_{\sim 1} \rightarrow \text{NEGLIGIBLE}$$

$$\sim k \ln \Omega(\bar{E}) + k \ln \Omega'(\bar{E}')$$

$$= S(\bar{E}) + S'(\bar{E}') \qquad (\bar{E} = \tilde{E})$$

(this is example of statement that almost same as if system were locked exactly at $E = \bar{E} = \tilde{E}$)

f. 2
3/12

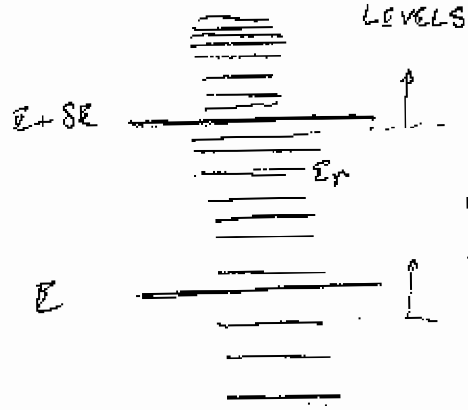
ALLOW BOTH E AND X_α TO CHANGE;

HOW DOES $\Omega(E, X_\alpha)$ CHANGE WITH EXT. PARAMS x_α ?

(TAKE ONE X FOR NOW)

(don't worry about writing X_α as x_α)

LEVELS FOR FIXED X



$\Omega(E, X) = $ # OF MICROSTATES (AT THIS X)

(each E_r is level for entire system)

CHG X : $E_r(x)$ CHG DIFFERENTLY

— SOME INSIDE LEAVE

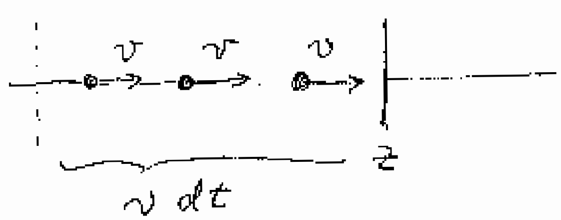
— " OUTSIDE ENTER

{recall: in thermal equil when rates of chg of Ω w/ E via α were equal; will see same for chg of Ω via x

COMPUTE NET CHG IN Ω INSIDE:

1/13/97 →

COMMON ANALOGOUS PROBLEM : FLUID OR PARTICLE FLOW (1 d)



IF ALL HAVE VEL v :

RATE CROSS PT Z :

IN TIME dt, ALL PARTICLES IN LENGTH $v\,dt$ CROSS Z IN dt

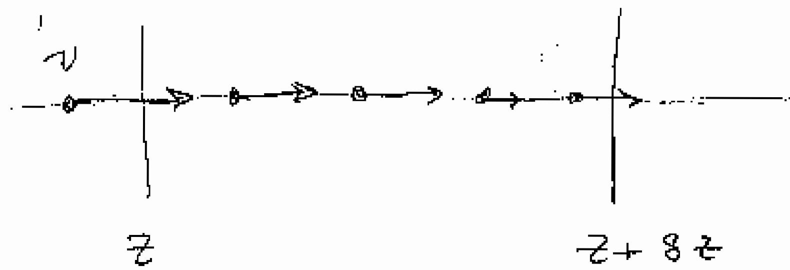
$$\Rightarrow \quad \underbrace{dN}_{\text{\# CROSSING}} = \left(\frac{\text{\# PARTS.}}{\text{LENGTH}}\right) \text{LENGTH} = \underset{\text{DENSITY}}{\rho} \cdot v\,dt$$

(ie chg in # on LHS of wall)

$$\text{RATE} \quad \frac{dN}{dt} = \rho v = \frac{\text{\# CROSSING}}{\text{TIME}} \equiv \underline{\text{FLUX}}$$

MORE COMPLICATED: IF ρ DEPENDS ON v & z:

ie $\rho_v(z) \equiv \dfrac{\#\text{ PARTICLES W/ VEL } v \text{ AT } z}{\text{LENGTH}}$



⇒ CAN HAVE PARTICLES BUILD-UP IN δz:

ENTERING FROM LEFT IN dt:

(a) WITH VEL v:

$$\rho_v(z)\, v\, dt$$

(IF $v < 0$, THESE ARE REALLY LEAVING AT LEFT)

(b) ALL v:

$$\sum_v \rho_v(z)\, v\, dt$$

LEAVING FROM RT:

$$\sum_v \rho_v(z+\delta z)\, v\, dt$$

CHG IN # INSIDE IN dt : # ENTERING − # LEAVING

$$dN_{\text{INSIDE}} = \left[\sum_v \rho_v(z)\, v - \sum_v \rho_v(z+\delta z)\, v\right] dt$$

FINALLY, RECOGNIZE ρ WILL DEP. ON t (AS BUILD UP, SPREAD OUT):

$$\Rightarrow \boxed{\frac{dN_{\text{INSIDE}}}{dt} = \left[\sum_v \rho_v(z,t)\, v - \sum_v \rho_v(z+\delta z, t)\, v\right]}$$

{ for me: $\dfrac{\sum_v \rho_v(z)\, v}{\rho(z)} = \bar{v}(z)$ ⇒ $dN_{\text{INS.}} = \rho(z)\bar{v}(z) - \rho(z+\delta z)\bar{v}(z+\delta z)$

($\rho(z)$: density for all v)

$= \left[-(\partial_z \rho)\bar{v} - \rho\, \partial_z \bar{v}\right]\delta z$

$= \partial_z(\rho(z)\bar{v}(z))\, \delta z = \partial_z(\text{AVE FLUX}) \cdot \delta z$ }

OUR PROBLEM:

PARTICLE # (1D) $\rightarrow$ r

$$z \rightarrow E_r$$

$$t \rightarrow x$$

$$v = \frac{dz}{dt} \rightarrow \frac{\partial E_r}{\partial x} \equiv -X_r \qquad \text{(GEN FORCE)}$$

(ex. $V \leftrightarrow p$)

{capital $x \equiv X$
{small x

LET $\Omega_X(E, x) \equiv$ # STATES w/ E TO $E + \delta E$
X TO $X + \delta X$

(ie break up Ω acc to X values)

$$\rho v(z,t) \rightarrow \frac{\Omega_X(E, x)}{\delta E} \quad (\equiv \text{DENSITY OF STATES} \equiv \omega_X(E,x))$$

THEN

$$\frac{dN_{\text{INSIDE}}}{dt} \rightarrow \frac{\partial \Omega(E,x)}{\partial x} = \sum_X \left\{ \frac{\Omega_X(E,x)}{\delta E}(-X) - \frac{\Omega_X(E+\delta E, x)}{\delta E}(-X) \right.$$

enter from bottom — leave thru top

ASSUME QUASI-STATIC (SO CAN USE FUND. POST.)

$$\text{THEN} \Rightarrow \bar{X}(E,x) = \frac{\sum_X \Omega_X(E,x)\cdot X}{\Omega(E,x)}$$

{IN EQUIL, all states equally likely}

$$\frac{\partial \Omega(E,x)}{\partial x} = \left[-\Omega(E,x)\bar{X}(E,x) + \Omega(E+\delta E, x)\bar{X}(E+\delta E, x) \right] \frac{1}{\delta E}$$

EXPAND IN δE:

$$\sim \frac{\partial \Omega}{\partial E}\bar{X} + \Omega \frac{\partial \bar{X}}{\partial E}$$

IN TERMS OF $\ln \Omega$:

$$\frac{\partial \ln \Omega}{\partial E} = \frac{1}{\Omega}\frac{\partial \Omega}{\partial E}$$

$$\frac{\partial \ln \Omega}{\partial x} = \frac{1}{\Omega}\frac{\partial \Omega}{\partial x}$$

apply to Ω on both sides

$$\Omega \frac{\partial \ln \Omega}{\partial x} = \Omega \underbrace{\frac{\partial \ln \Omega}{\partial E}}_{\beta} \bar{X} + \Omega \frac{\partial \bar{X}}{\partial E}$$

$$\boxed{\frac{\partial \ln \Omega}{\partial x} = \beta \bar{X} + \frac{\partial \bar{X}}{\partial E}}$$

TYPICALLY:

$$\Omega \sim E^f \qquad \beta \bar{X} \sim \frac{f}{E} \bar{X} \gg \frac{\partial \bar{X}}{\partial E} \sim \frac{\bar{X}}{E}$$

MAIN RESULT OF CH 3

$$\Rightarrow \boxed{\frac{\partial \ln \Omega(E, x_\alpha)}{\partial x_\alpha} = \beta \bar{X}_\alpha(E, x_\alpha)}^{*}$$

general — depends on temp

CP. TO

$$\boxed{\frac{\partial \ln \Omega}{\partial E} \equiv \beta}$$

Q-S $\Rightarrow$ now know how Ω varies in gen; will give more gen. equil conds when both E & x_α can vary

HW

skip

ex SPS V ONLY EXT PARAM

WITH $S \equiv k \ln \Omega$ $\qquad \bar{p} = \overline{\frac{\partial E_r}{\partial V}}$ $\qquad \beta = \frac{1}{kT}$

$$\Rightarrow \frac{\partial S}{\partial V} = \frac{\bar{p}}{T}$$

check this

* plausible: sign?

~ like chain rule:

$$\frac{\partial \ln \Omega}{\partial x_\alpha} \sim -\frac{\partial \ln \Omega}{\partial E} \overline{\frac{\partial E_r}{\partial x_\alpha}}$$

chg. in $\ln \Omega$ via E

$\otimes$ avg. chg in E by x_α

EX MONATOMIC IDEAL GAS (CLASSICAL)

- IF KNOW $\Omega(E, x_\alpha)$ CAN GIVE AVE FOR ALL MACRO VARS (IN EQUIL)
- " " SOMETHING ABOUT Ω, CAN OFTEN AT LEAST RELATE "

IDEAL GAS:

MOST USEFUL VARS: $\bar{E}$, V, T, $\bar{p}$, S. (all related to $\Omega(E, V$

HAVE
$$\beta = \frac{\partial \ln \Omega}{\partial E} \qquad \bar{X}_\alpha = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial x_\alpha}$$

$$\bar{p} = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial V}$$

KNOW

(1) MOLS. EQUALLY LIKELY ANYWHERE: (OBVIOUS)
$$\Omega \propto V^N \chi(E)$$
(we actually know more)

THEN
$$\ln \Omega = \text{CONST} + N \ln V + \ln \chi(E)$$
{ should also const into χ, $\Omega = V^N \chi(E)$)

$$\Rightarrow \quad \bar{p} = \frac{1}{\beta}\frac{N}{V} = \frac{NkT}{V}$$

$$\boxed{\bar{p} V = NkT}$$ (MON) EG. IDEAL GAS LAW

{ OTHER VERSIONS:

IN DENSITY $n = N/V \Rightarrow \bar{p} = nkT$

IN MOLES $\nu \equiv N/N_a \Rightarrow \bar{p} V = \nu \underbrace{N_a k}_{\equiv R \text{ (GAS CONST)}} T = \nu RT$ }

$\Rightarrow$ EQN OF STATE: RELATES MACRO VARS

(sharply peaked $\rightarrow \bar{p}$ not very diff from p meas; 10^{-12} or so
makes sense to talk simply about pressure in single system)

ALSO

$$\beta = \frac{\partial \ln \Omega}{\partial E} = \frac{\partial \ln \chi(E)}{\partial E} \Rightarrow T = T(\bar{E}) \text{ or}$$

$$\therefore \boxed{\bar{E} = \bar{E}(T)}_{I.G.}$$

IDEAL GAS : $\bar{E}$ ONLY DEP. ON T
{IF INCL. INTS. OF MOLS., $\bar{E}$ COULD ALSO DEP. ON AVE DIST. BETW. ∴ $\bar{E}(N, V, T)$ }

CONTENT: $\Omega \propto V^N$ (FOR IDEAL GAS)

{ startling how much for so little input - only used most basic stat reasoning

(2) IF KNOW MORE, CAN SAY MORE : ALSO WORKED OUT ENERGY DEP OF Ω FOR MONATOMIC IDEAL GAS:

$$\Omega \doteq \overset{\text{const}}{B} V^N E^{3N/2}$$

$$\beta(E) = \frac{\partial \ln \Omega}{\partial E} = \frac{\partial}{\partial E}\left(\frac{3N}{2} \ln E\right) = \frac{3}{2}\frac{N}{E}$$

AT EQUIL: $E \approx \bar{E}$

$$\beta(\bar{E}) \equiv \frac{1}{kT} = \frac{3}{2}\frac{N}{\bar{E}}$$

$$\boxed{\frac{\bar{E}}{N} = \frac{3}{2} kT}_{\substack{(MON.)\\ I.G.}}$$

(recall est.: $kT \sim \frac{\bar{E}}{f}$; here $f = 3N$)

⇒ CAN NOW GIVE ALL QTYS FOR IG IF KNOW E (OR $\bar{E}$) AND V:

$$T = \frac{2}{3}\frac{1}{Nk}\bar{E}$$

$$\bar{p} = \frac{2}{3}\frac{\bar{E}}{V}$$