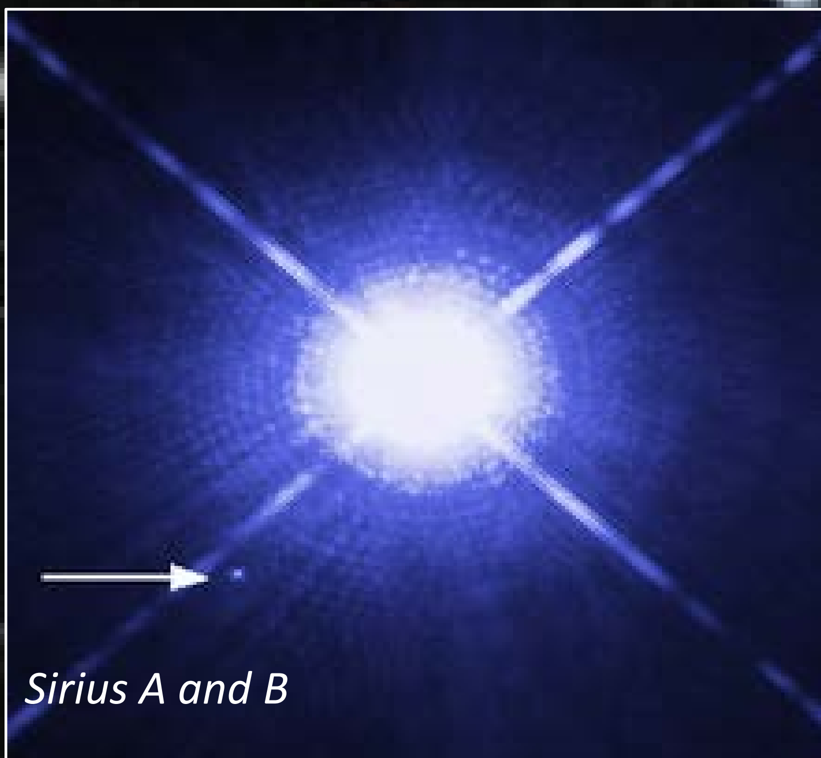
White Dwarfs and Electron Degeneracy

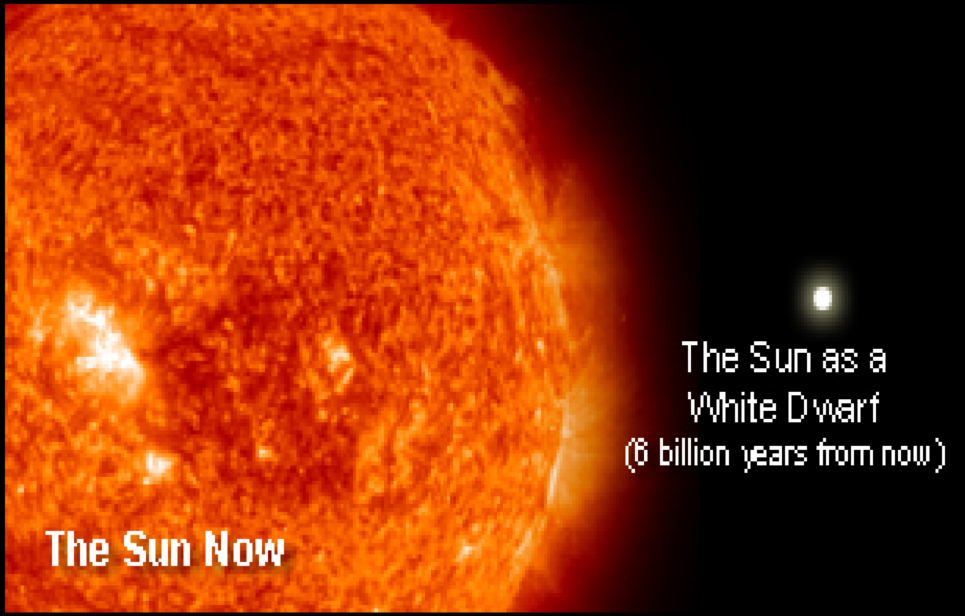


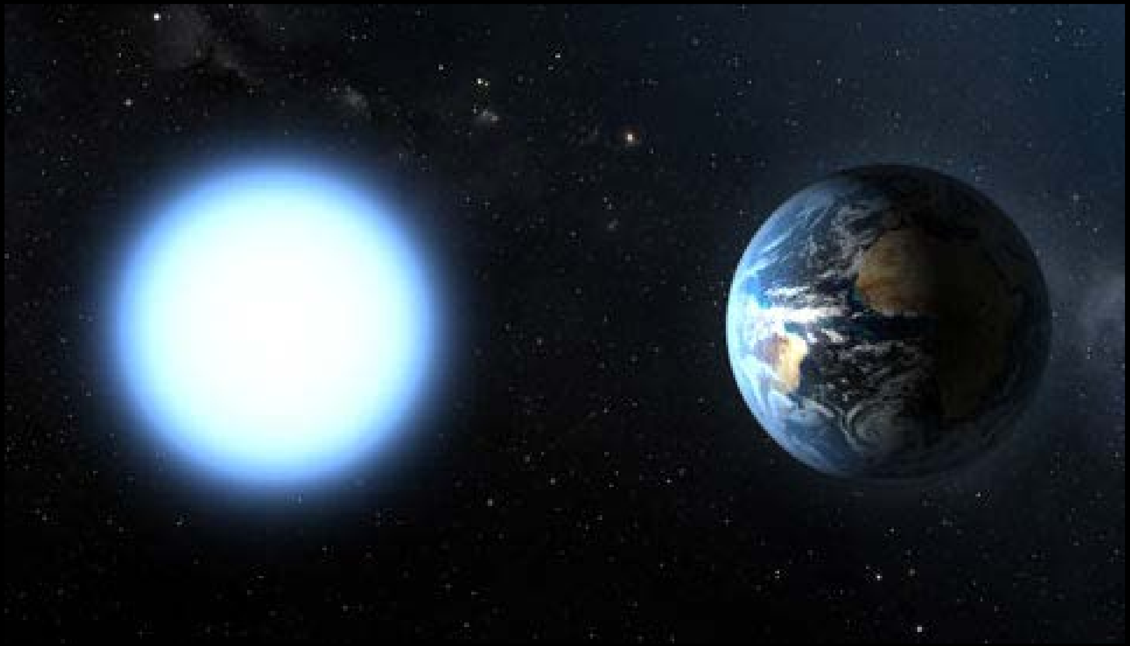
Sirius A and B

Farley V. Ferrante
Southern Methodist University

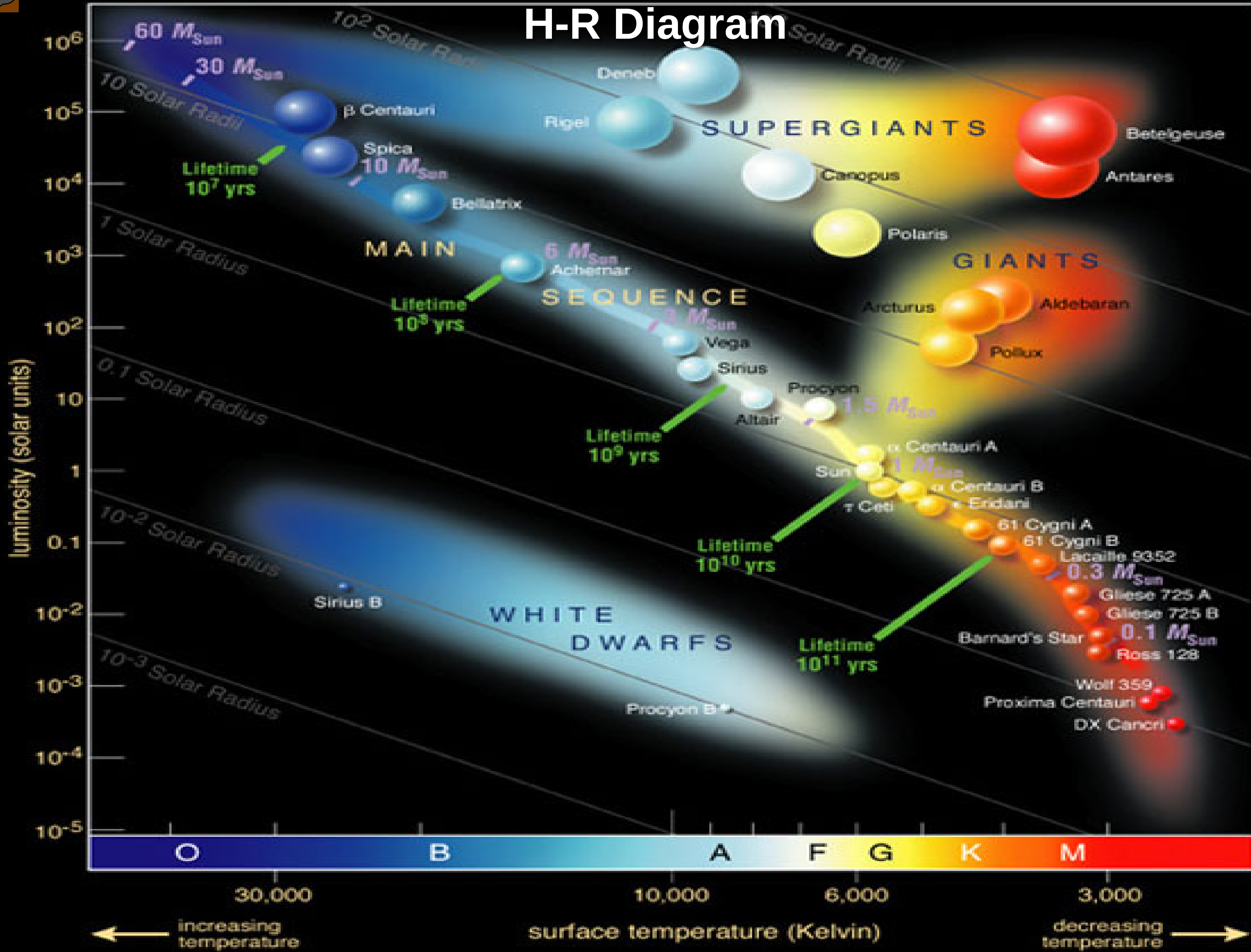
Outline

- Stellar astrophysics
- White dwarfs
  - Dwarf novae
  - Classical novae
  - Supernovae
- Neutron stars

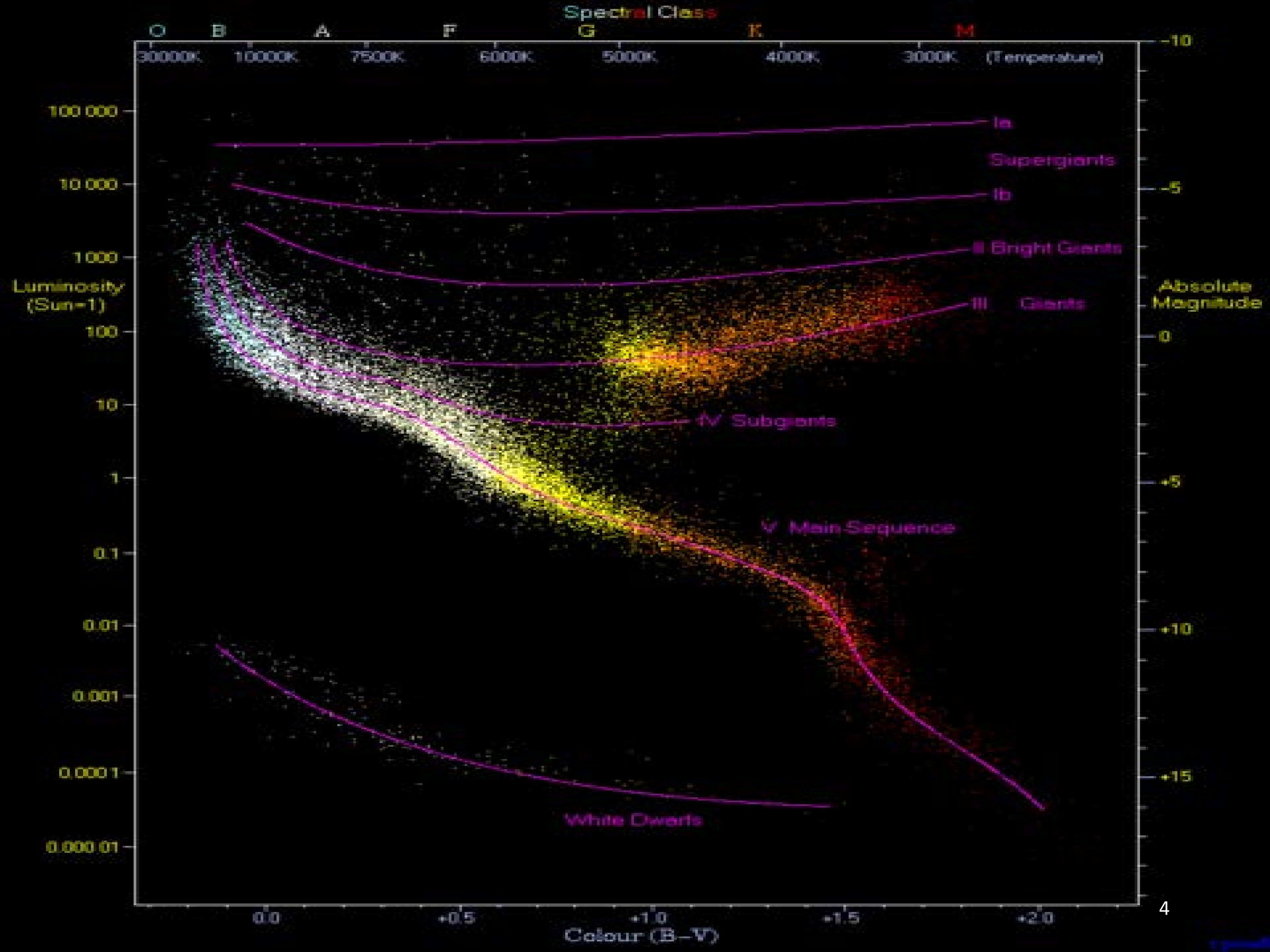




H-R Diagram



60 M_{Sun}
30 M_{Sun}
10 M_{Sun}
6 M_{Sun}
3 M_{Sun}
1.5 M_{Sun}
1 M_{Sun}
0.3 M_{Sun}
0.1 M_{Sun}
10^2 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^{-2} Solar Radius
10^{-3} Solar Radius
Solar Radii
Deneb
Rigel
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Procyon
Altair
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Betelgeuse
Antares
Canopus
Polaris
Arcturus
Aldebaran
Pollux
Sirius B
Procyon B
SUPERGIANTS
GIANTS
MAIN
SEQUENCE
WHITE
DWARFS
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^{10} yrs
Lifetime 10^{11} yrs
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature



Spectral Class
O
B
A
F
G
K
M
30000K
10000K
7500K
6000K
5000K
4000K
3000K
(Temperature)
100 000
10 000
1000
Luminosity
(Sun=1)
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
-10
-5
Absolute
Magnitude
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B−V)

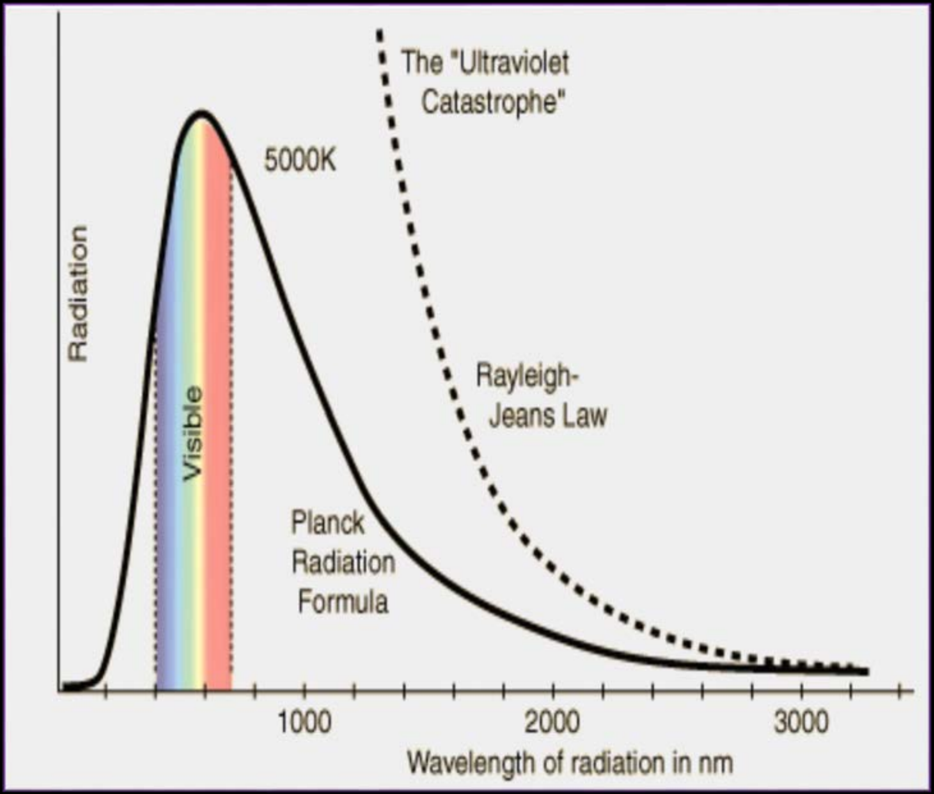
Stellar Astrophysics

- Stefan-Boltzmann Law:

$$F_{bol} = \sigma T^4; \sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67x10^{-5} ergs^{-1}cm^{-2}K^{-4}$$

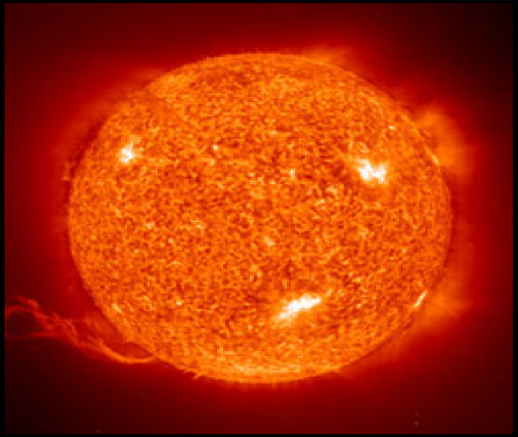
  - Effective temperature of a star: Temp. of a black body with the same luminosity per surface area
  - Stars can be treated as black body radiators to a good approximation
  - Effective surface temperature can be obtained from the B-V color index with the Ballesteros equation:

$$T = 4600\left(\frac{1}{0.92(B-V)+1.70} + \frac{1}{0.92(B-V)+0.62}\right)$$



- Luminosity:

$$L = 4\pi r_*^2 \sigma T_E^4$$



Life History of Stars

Mass	Core Details	Comments
> $0.08M_{sun}$	Low mass ball of gas, not hot enough for hydrogen fusion	Stars in this mass range are not stars, but brown dwarfs of spectral type L and T.
$0.08M_{sun} < M < 0.5M_{sun}$	Fusion of H -> ^{4}He. Star is never hot enough to fuse ^{4}He to ^{12}C or ^{16}O.	Stars in this mass range are M on the main sequence. End up white dwarfs made of helium.
$0.5M_{sun} < M < 5M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O. Center is not hot enough to fuse ^{12}C and ^{16}O.	Stars in this mass range are A, F, G and K on the main sequence. End up white dwarfs made of ^{12}C and ^{16}O.
$5M_{sun} < M < 7M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O -> ^{20}Ne and ^{24}Mg.	Stars in this mass range are B on main sequence. End up as white dwarfs made of ^{20}Ne and ^{24}Mg.
$M > 7M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O -> ^{20}Ne and ^{24}Mg -> heavier elements .	Stars in this mass range are O on the main sequence. End up as neutron stars or black holes.

White dwarf

- Core of solar mass star
- Degenerate gas of oxygen and carbon
- No energy produced from fusion or gravitational contraction

Hot white dwarf NGC 2440. The white dwarf is surrounded by a "cocoons" of the gas ejected in the collapse toward the white dwarf stage of stellar evolution.

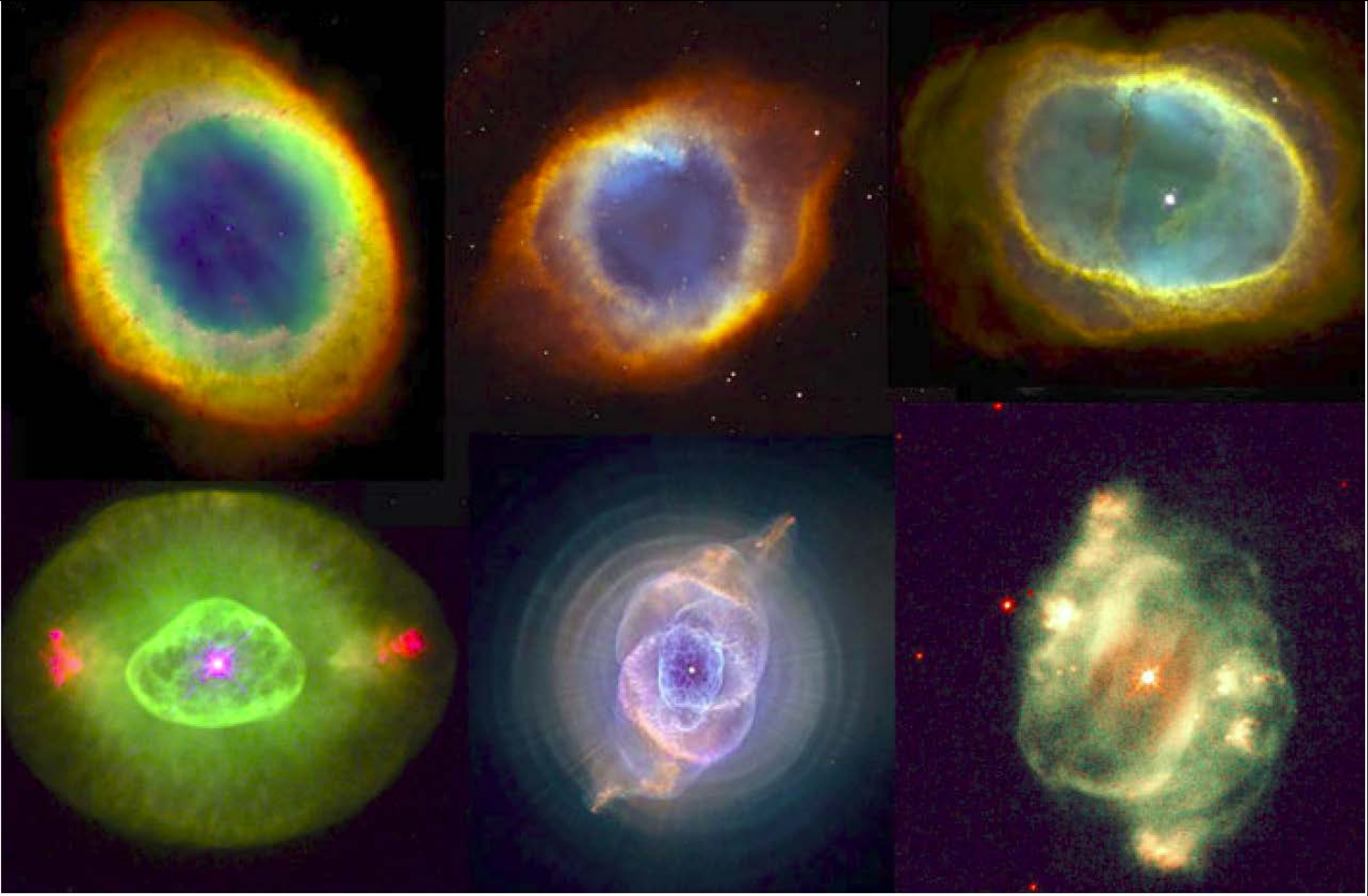
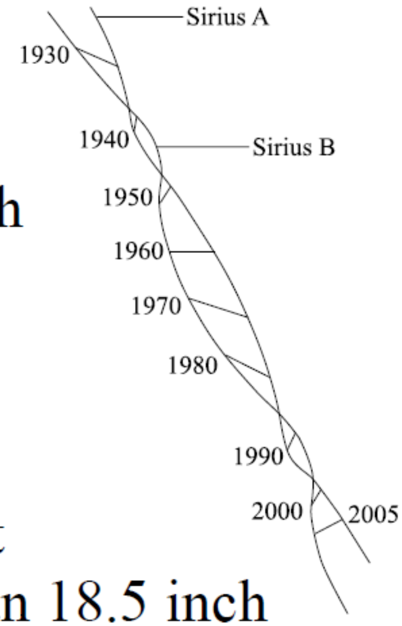
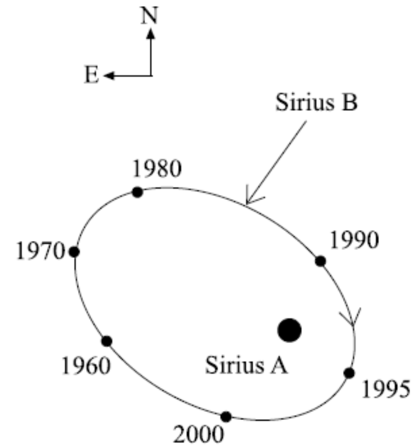


Figure 4.2 Several examples of planetary nebulae, newly formed white dwarfs that irradiate the shells of gas that were previously shed in the final stages of stellar evolution. The shells have diameters of $\approx 0.2 - 1$ pc. Photo credits: M. Meixner, T.A. Rector, B. Balick et al., H. Bond, R. Ciardullo, NASA, NOAO, ESA, and the Hubble Heritage Team

White Dwarfs

Sirius B is a white dwarf companion to Sirius A.





In 1844 German astronomer Friedrich Bessel deduced the existence of a companion star from changes in the proper motion of Sirius.

In 1862, astronomer Alvan Clark first observed the faint companion using an 18.5 inch refractor telescope at the Dearborn Observatory.

In 1915 Walter Adams observed the spectrum of the star, determining it was a faint whitish star. This lead astronomers to conclude it was a white dwarf.

Matter at Quantum Densities

As stars evolve, their cores contract and the core density increases. At some point the distance between the atoms is smaller than their de Broglie wavelengths and classical assumptions can no longer be used.

Recall: **de Broglie Wavelength**

$$\lambda = \frac{h}{p} = \frac{h}{(2mE)^{1/2}} \approx \frac{h}{(3mkT)^{1/2}}$$

Since,

$$p = mv$$

$$p = \sqrt{2mE}$$

$$E_K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$E \sim \frac{3kT}{2}$$

mean energy of a particle

Question: Which will reach the quantum domain first, electrons or protons?

Although both electrons and protons share the same energy, electrons have smaller mass and longer wavelengths. The electron density will reach the quantum domain first.

When the inter particle spacing is of order 1/2 a de Broglie wavelength, quantum effects will become important.

$$\rho_q \approx \frac{m_p}{(\lambda/2)^3} = \frac{8m_p(3m_e kT)^{3/2}}{h^3}$$

Calculate the quantum density at the center of the sun (T = 15 x 10^6 K).

$$\rho_q \approx \frac{8 \times 1.7 \times 10^{-24}\ \mathrm{g}\ (3 \times 9 \times 10^{-28}\ \mathrm{g} \times 1.4 \times 10^{-16}\ \mathrm{erg\ K^{-1}} \times 15 \times 10^6 \mathrm{K})^{3/2}}{(6.6 \times 10^{-27}\ \mathrm{erg\ s})^3}$$

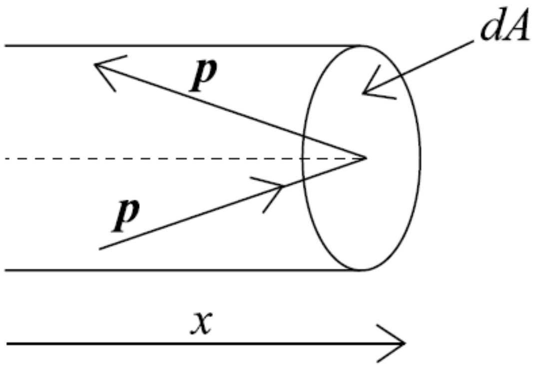
$$p_q = 640\ g\ cm^{-3}$$

The core density of the sun is 150 g cm^{-3}. Much below the quantum regime.

Pressure Exerted by Ideal Gas

Consider ideal gas particles hitting the sides of a container.

Recall, that particles with momentum p_x impart $2p_x$ to the surface with each reflection.



The force per unit area imparted is then

$$\frac{dF_x}{dA} = \frac{2p_x}{dAdt} = \frac{2p_x v_x}{dAdx} = \frac{2p_x v_x}{dV}$$

where we used:

$$v_x = \frac{dx}{dt}$$

To get the pressure, we sum forces due to particles of all momenta.

$$P = \int_0^\infty dN(p) \frac{p_x v_x}{dV} dp$$

Note: half the particles are not moving towards walls.

$$P = \int_0^\infty dN(p) \frac{p_x v_x}{dV} dp$$

Simplify:

$$p_x v_x = m v_x^2 = \frac{1}{3} m v^2 = \frac{1}{3} p v$$

If we assume the velocities are isotropic: $v_x^2 = v_y^2 = v_z^2$

Substitute:

$$P = \frac{1}{3} \int_0^\infty n(p) p v dp$$

where we used: $dN/dV \equiv n$

For a non-relativistic degenerate gas:

$$n_e(p)dp = \begin{cases} 8\pi p^2 \frac{dp}{h^3} & \text{if } |\mathbf{p}| \le p_f \\ 0 & \text{if } |\mathbf{p}| > p_f \end{cases}$$

$$v = p/m_e$$

$$n_e = \frac{8\pi}{3h^3} p_f^3$$

$$P_e = \frac{1}{3} \int_0^{p_f} \frac{8\pi}{h^3} \frac{p^4}{m_e} dp = \frac{8\pi}{3h^3 m_e} \frac{p_f^5}{5}$$

$$= \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{5m_e} n_e^{5/3}$$

Finally, noting $n_e = Zn_+ = Z\rho/Am_p$

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

Re-derive scaling relations between mass and radius with an index (4 + ε)/3.

$$P \sim b\rho^{5/3} \longrightarrow P \sim \rho^{(4+\epsilon)/3} = \frac{M^{(4+\epsilon)/3}}{r^{(4+\epsilon)}}$$

Equating with pressure from our stellar equations.

$$P \sim \frac{GM\rho}{r} \sim \frac{M^2}{r^4}$$

$$\frac{M^{4/3}M^{\epsilon/3}}{r^4 r^\epsilon} = \frac{M^{(4+\epsilon)/3}}{r^{4+\epsilon}} \sim \frac{M^2}{r^4}$$

$$r^\epsilon \sim M^{(\epsilon-2)/3}$$

$$r \sim M^{(\epsilon-2)/3\epsilon}$$

When $\epsilon \longrightarrow 0$

$$r \rightarrow M^{-\infty} = 0$$

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

Comments:

The electron pressure does not depend on temperature.

For a typical white dwarf, $\rho \sim 10^6$ g cm^{-3} and T ~ 10^7 K.
Their Z/A ~ 0.5.

$$P_e \sim \frac{(6.6 \times 10^{-27} \text{ erg s})^2}{20 \times 9 \times 10^{-28} \text{ g } (1.7 \times 10^{-24} \text{ g})^{5/3}} 0.5^{5/3} (10^6 \text{ g cm}^{-3})^{5/3} = 3 \times 10^{22} \text{ dyne cm}^{-2}$$

Compare to the thermal pressure of nuclei at this temperature.

$$P = n \;\; kT = 2 \times 10^{20} \; dyne \; cm^{-2}$$

Thus, degenerate electron pressure completely dominates the pressure in these stars.

Properties of White Dwarfs

Mass-Radius Relationship:

Recall the EOS for a degenerate non-relativistic electron gas:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

The scaling relation for this equation is

$$P \sim b\rho^{5/3} \sim b\frac{M^{5/3}}{r^5}$$

where b is a constant

Recall our scaling relations from the equations of stellar structure:

$$P \sim \frac{GM\rho}{r} \sim \frac{GM^2}{r^4}$$

Equating these pressures yields:

$$r \sim \frac{b}{G} M^{-1/3}$$

What?

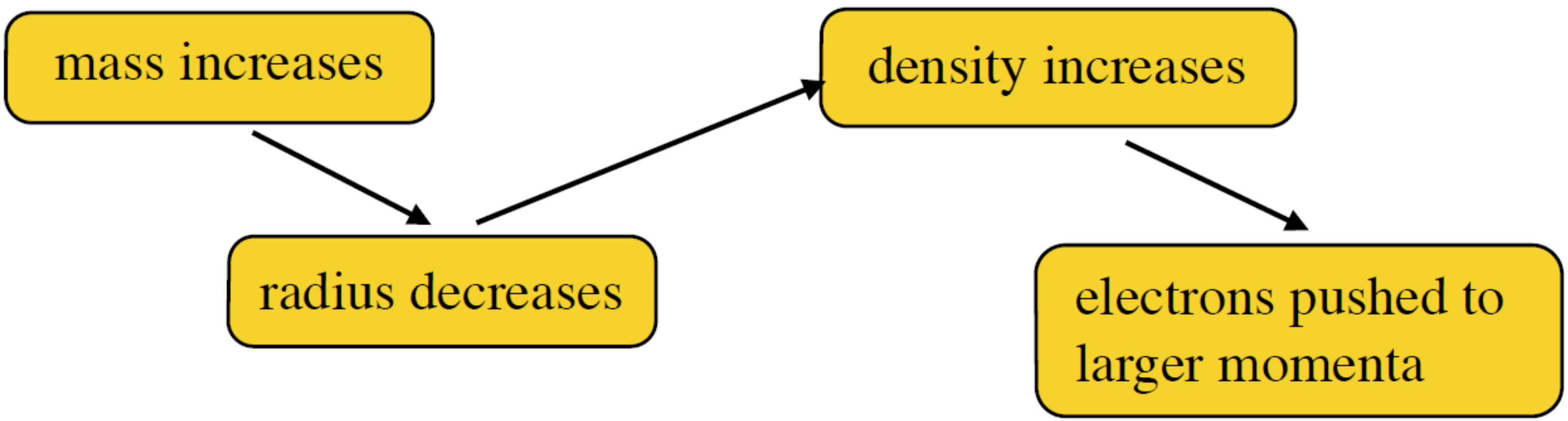
Notice: The radius decreases with increasing mass!

$$r \sim \frac{b}{G} M^{-1/3}$$

A white dwarf with Z/A = 0.5 and M = 1M_{sun} has a radius of ~ 4000 km.

Fully working out the equations of stellar structure gives an equation for radius of

$$r_{\rm wd} = 2.3 \times 10^9 {\rm cm} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3}$$



Mass/radius relation for degenerate star

- Stellar mass = *M;* radius = *R*
- Gravitational potential energy:

$$Egr = -\frac{3GM^2}{5R}$$

- Heisenberg uncertainty:

$$\Delta x \Delta p \geq \hbar$$

- Electron density:

$$n = \frac{3N}{4\pi R^3} \approx \frac{M}{m_p R^3}$$

$$\Delta x \approx n^{-1/3} \qquad \Delta p \approx \frac{\hbar}{\Delta x} \approx \hbar n^{1/3}$$

- Kinetic energy:

$$\varepsilon = \frac{p^2}{2m_e} \qquad K = N\varepsilon = \frac{M}{m_p}\varepsilon \approx \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2}$$

Mass/radius relation for degenerate star

- Total energy:

$$E = K + U \approx \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2} - \frac{GM^2}{R}$$

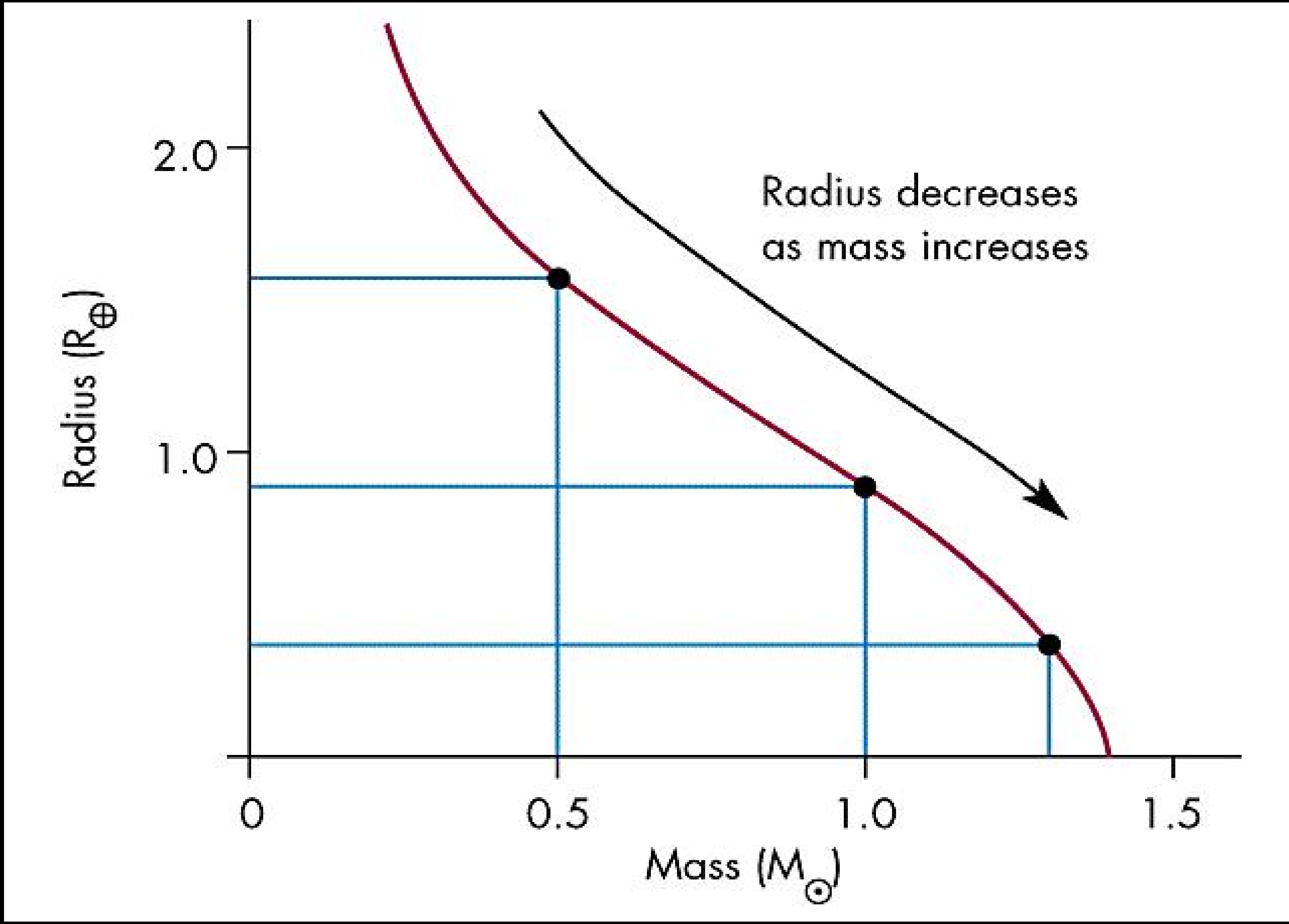
- Find *R* by minimizing *E:*

$$\frac{dE}{dR} \approx -\frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^3} + \frac{GM^2}{R^2} = 0$$

- Radius decreases as mass increases:

$$R \approx \frac{\hbar^2 M^{-1/3}}{G m_e m_p^{5/3}}$$

Mass vs radius relation



$$r \to M^{-\infty} = 0$$

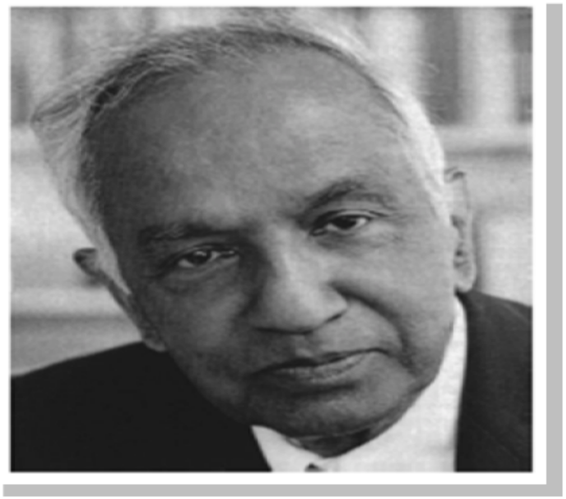
What does it mean?

At masses so high that electrons become ultra-relativistic, the electron pressure is unable to support the star against gravity.

If the density is high enough, degeneracy pressure due to protons and neutrons begins to operate. Stops collapse and produces a neutron star.

Chandrasekhar Mass:

The maximum stellar mass that can be supported by electron degeneracy pressure.



Subrahmanyan Chandrasekhar (1910–1995)

Estimate Chandrasekhar Mass

Start with virial theorem

$$\bar{P}V = -\frac{1}{3}E_{\rm gr}$$

Substitute the ultra-relativistic electron degeneracy pressure and self gravity

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

$$\left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3} V \sim \frac{1}{3}\frac{GM^2}{r}$$

$$\rho \sim \frac{M}{V}$$

$$V = \frac{4\pi}{3}r^3$$

Simplify:

$$M \sim 0.11 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

Full Solution using Equations of Stellar Structure:

$$M_{\rm ch} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

Accurately calculated value is 1.4 M_{sun}.

As electron velocities increase, the rates at which momentum transfers approaches c. So, we need to modify the EOS for degenerate electron gas.

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}$$

← **EOS for an ultra-relativistic degenerate spin-1/2 fermion gas**

Compare to non-relativistic case:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}$$

← **EOS for a degenerate non-relativistic electron gas**

Notes: The power index changes.

The electron mass disappears.

For ultra-relativistic particles, the rest mass is negligible.

As we go from small to large white dwarf masses, we transition gradually from non-relativistic to ultra-relativistic.

Notes:

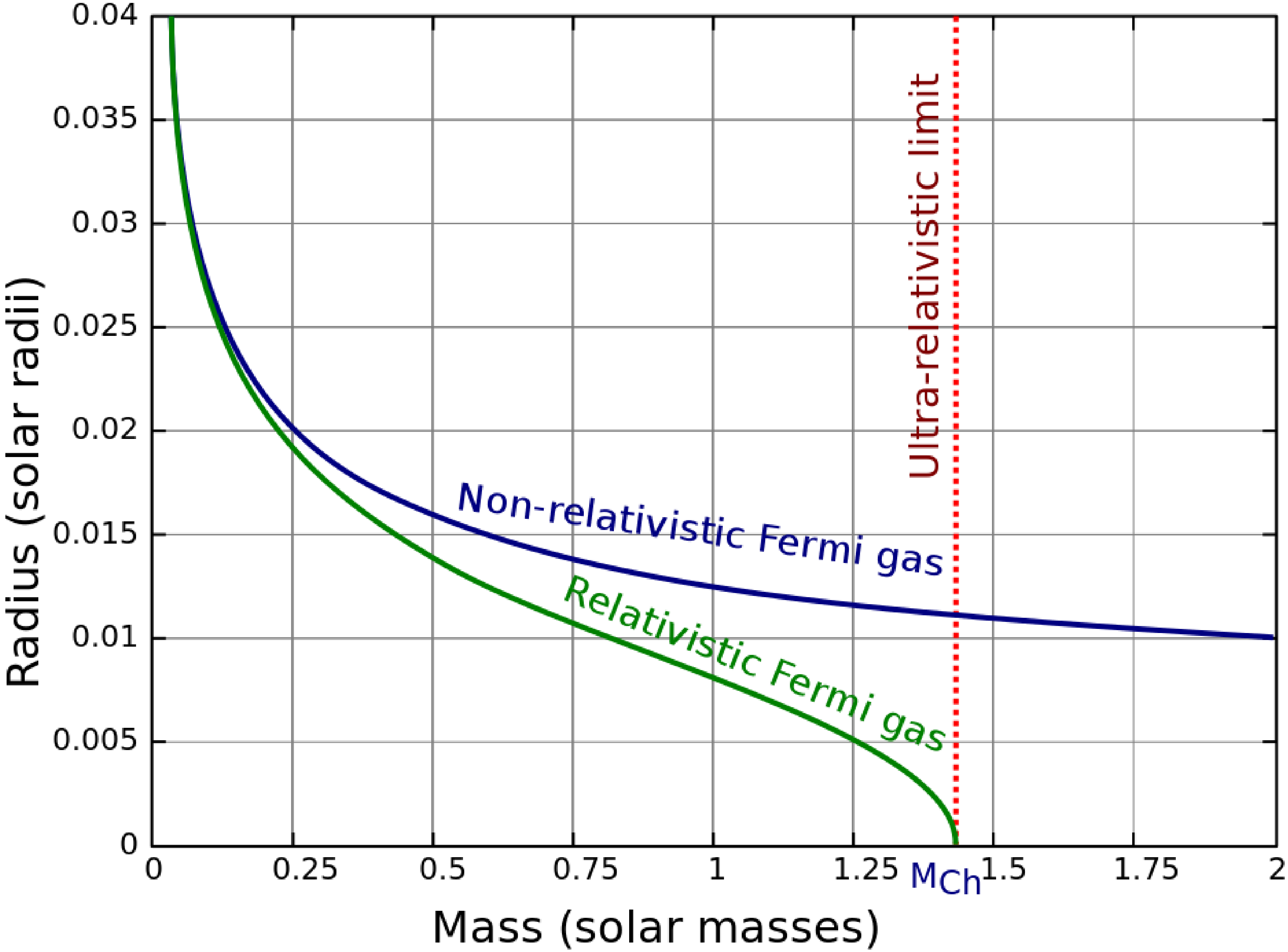
The accurately calculated Chandrashaker mass is 1.4 M_{sun}.

No white dwarfs with masses greater than M_{ch} have ever been found.

The lower bound of isolated white dwarfs found is 0.25 M_{sun}. Why is there a lower bound?

The universe is too young! Stars that have mass < 0.8 M_{sun} could produce smaller white dwarfs. However, even if they were formed in the early universe, they have not yet gone though their main sequence lifetime.

Mass vs radius relation



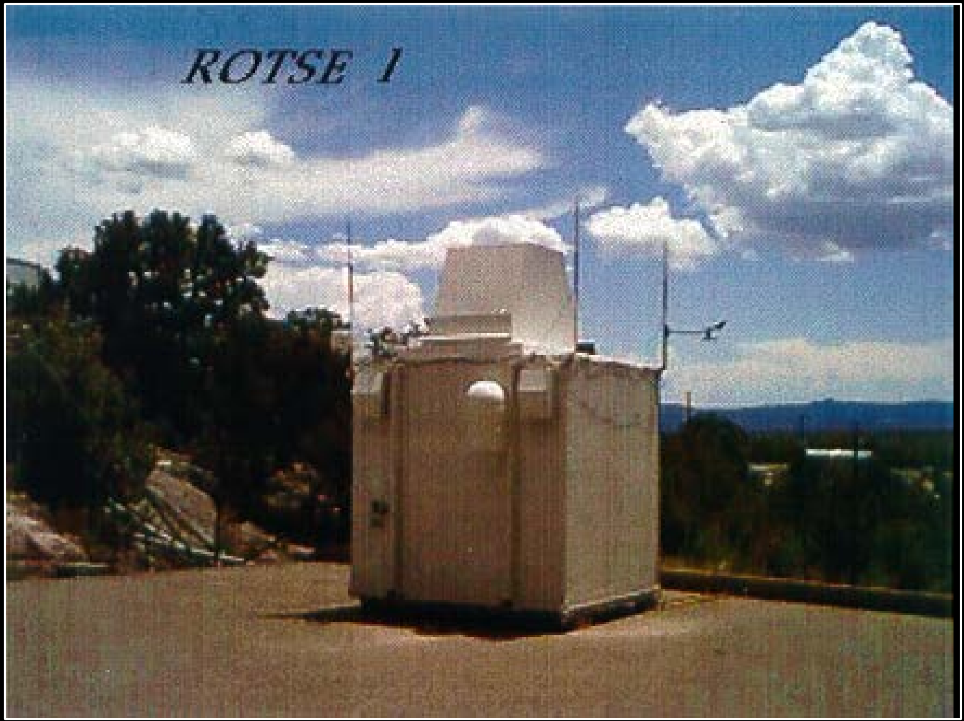
ROTSE

- Robotic Optical Transient Search Experiment
- Original purpose: Observe GRB optical counterpart (“afterglow”)
- Observation & detection of optical transients (seconds to days)
- Robotic operating system
  - Automated interacting Linux daemons
  - Sensitivity to short time-scale variation
  - Efficient analysis of large data stream
  - Recognition of rare signals
- Current research:
  - GRB response
  - SNe search (RSVP)
  - Variable star search
  - Other transients: AGN, CV (dwarf novae), flare stars, novae, variable stars, X-ray binaries


ROTSE-IIIa
Australian National Observatory

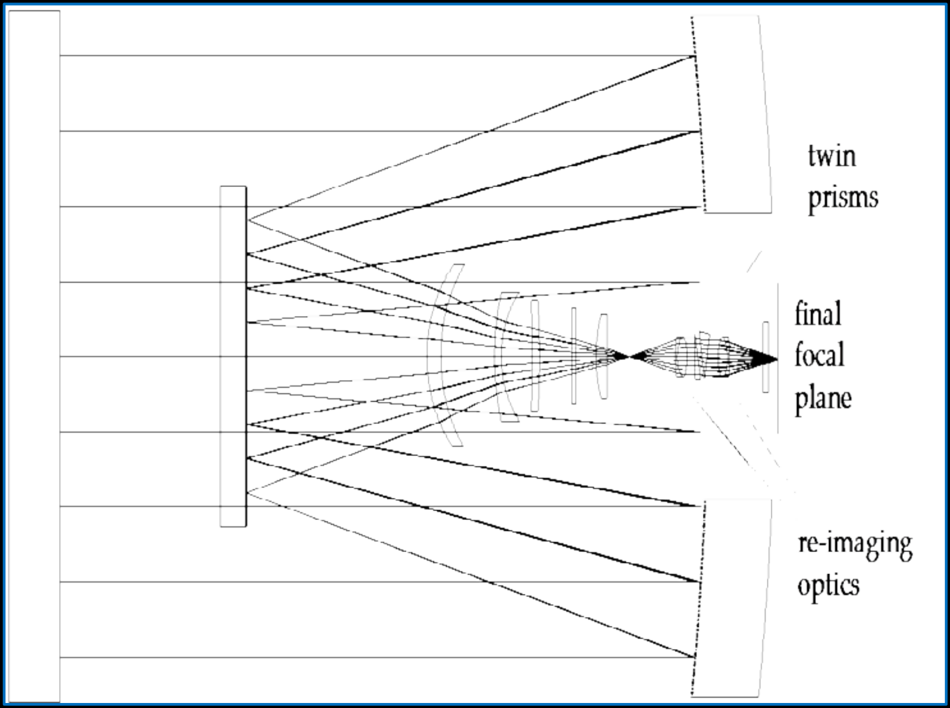
ROTSE-I

- 1st successful robotic telescope
- 1997-2000; Los Alamos, NM
- Co-mounted, 4-fold telephoto array (Cannon 200 mm lenses)
- CCD
  - 2k x 2k Thomson
  - "Thick"
  - Front illuminated
  - Red sensitive
  - R-band equivalent
  - Operated "clear" (unfiltered)
- Optics
  - Aperture (cm): 11.1
  - f-ratio: 1.8
  - FOV: 16°×16°
- Sensitivity (magnitude): 14-15
  - Best: 15.7
- Slew time (90°): 2.8 s
- 990123: Observed 1st GRB afterglow in progress
  - Landmark event
  - Proof of concept

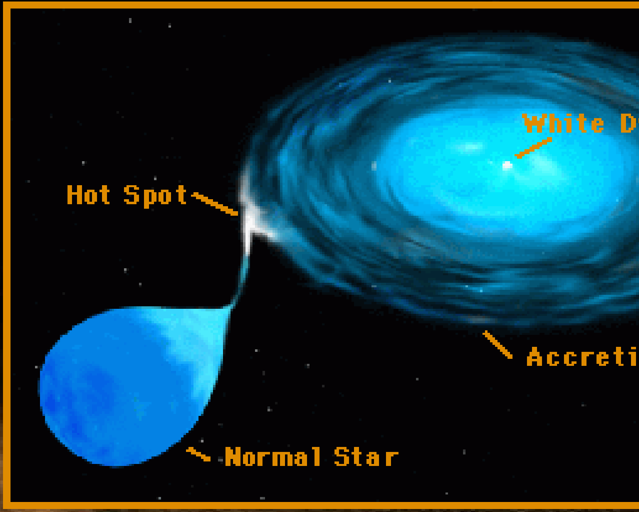


ROTSE-III

- 2003 – present
- 4 Cassegrain telescopes
- CCD
  - "Thin"
  - Back illuminated
  - Blue-sensitive
  - High QE (UBVRI bands)
  - Default photometry calibrated to R-band
- Optics
  - Aperture (cm): 45
  - f-ratio: 1.9
  - FOV: 1.85°×1.85°
- Sensitivity (magnitude): 19-20
- Slew time: < 10 s



Dwarf Novae

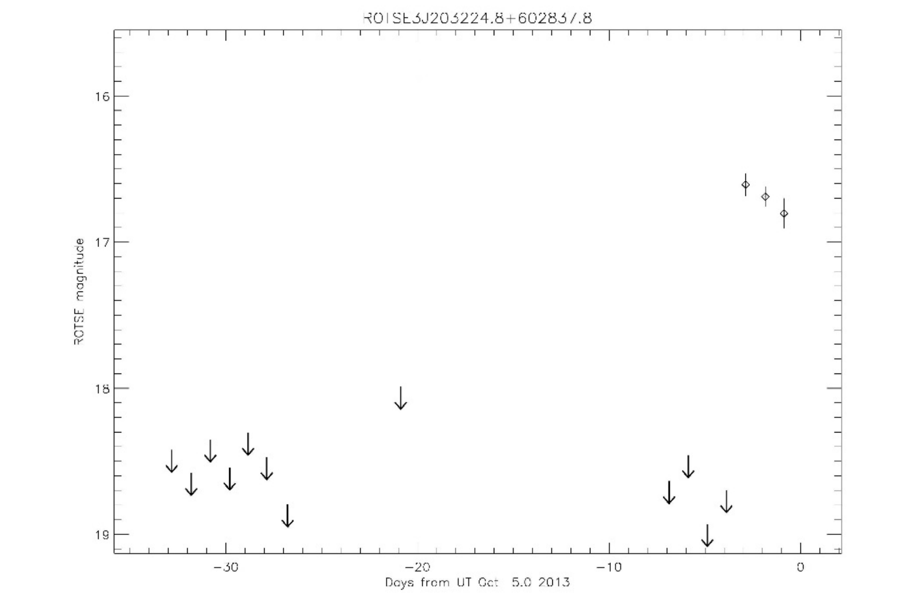


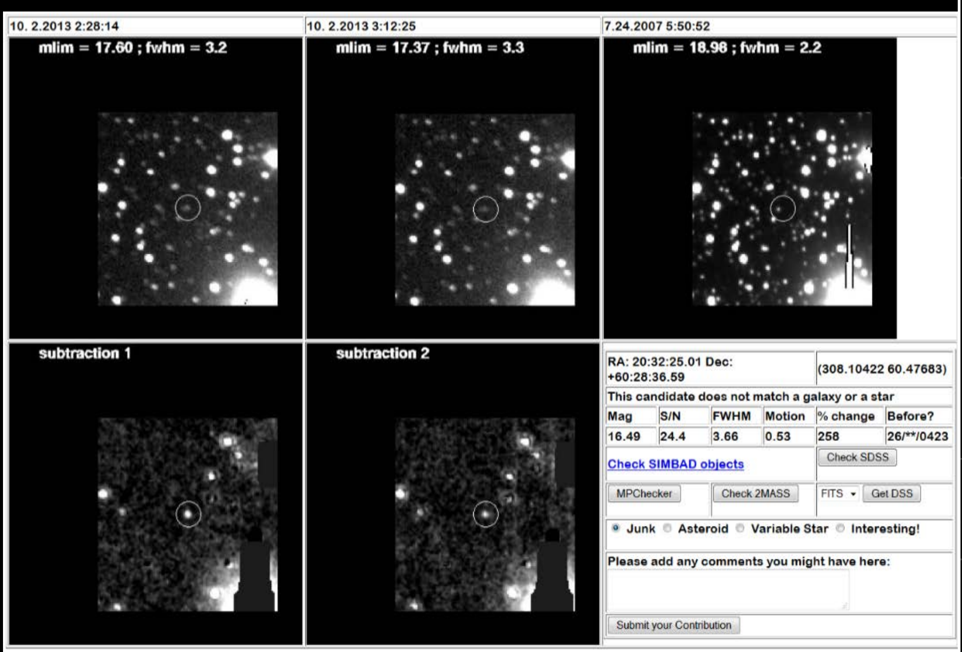
An artist's concept of the accretion disk around the binary star WZ Sge. Using data from Kitt Peak National Observatory and N Spitzer Space Telescope, a new picture of this system has emer which includes an asymmetric outer disk of dark matter.

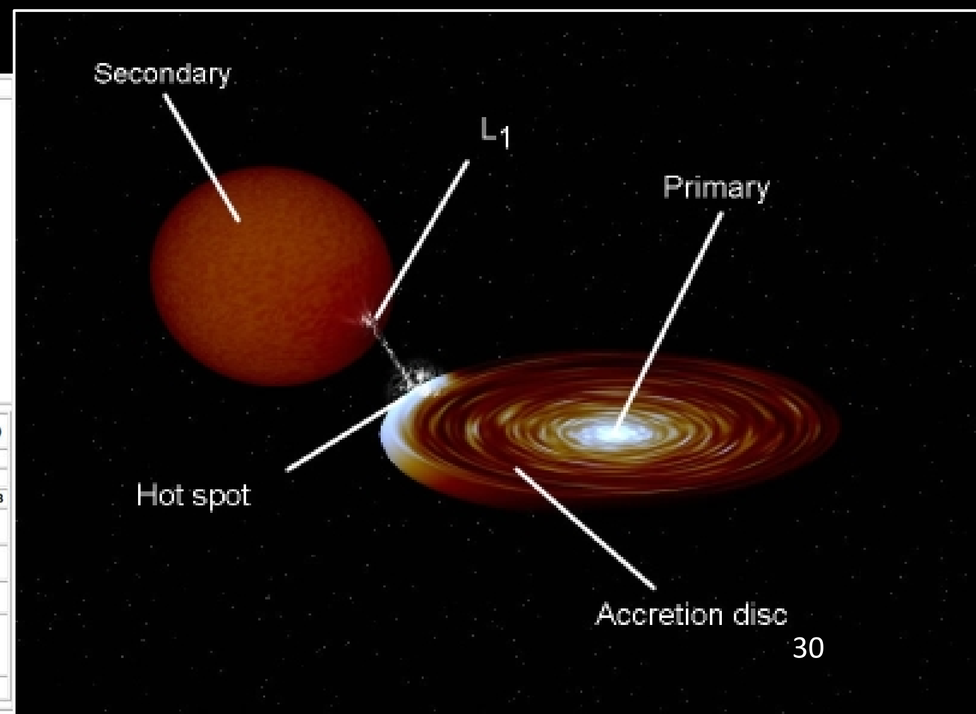
ROTSE3 J203224.8+602837.8

- 1st detection (110706):
  - ROTSE-IIIb & ROTSE-IIId
  - ATel #2126
- Outburst (131002 – 131004):
  - ROTSE-IIIb
  - ATel #5449
- Magnitude (max): 16.6
- (RA, Dec) = (20:32:25.01, +60:28:36.59)
- UG Dwarf Nova
  - Close binary system consisting of a red dwarf, a white dwarf, & an accretion disk surrounding the white dwarf
  - Brightening by 2 - 6 magnitudes caused by instability in the disk
  - Disk material infalls onto white dwarf

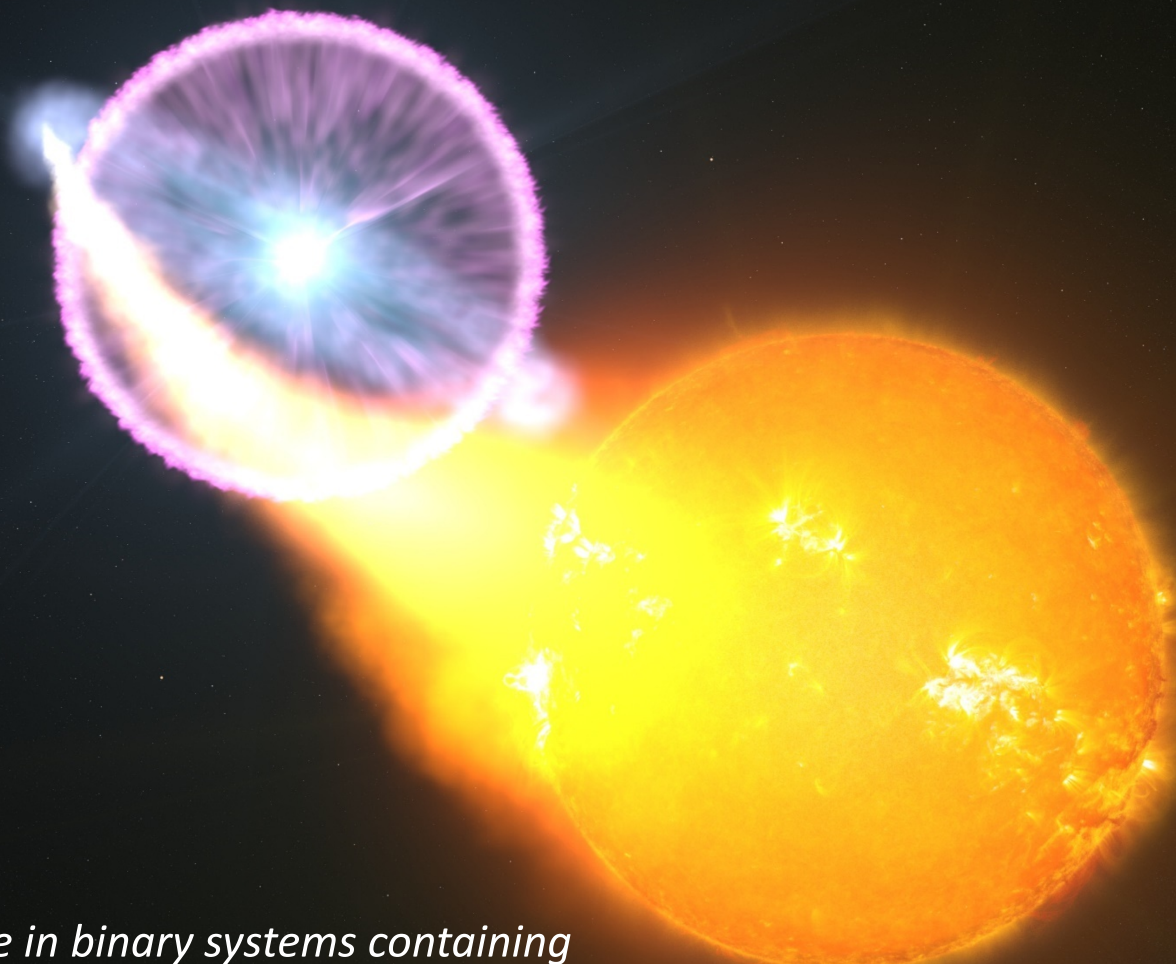
"Damien"







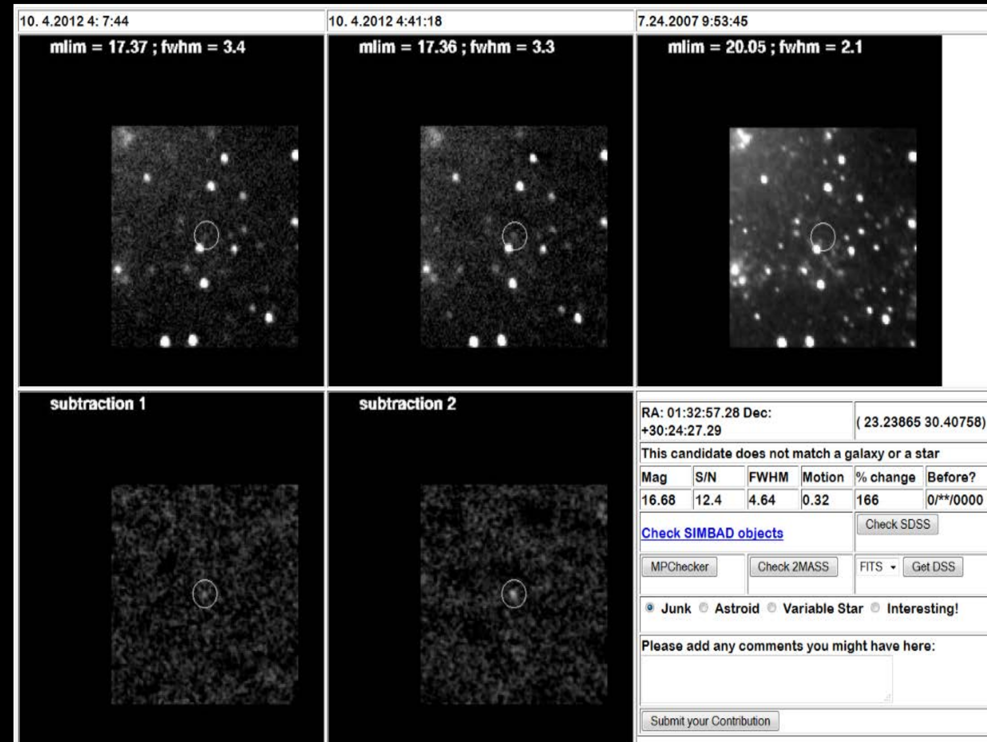
Novae (classical)

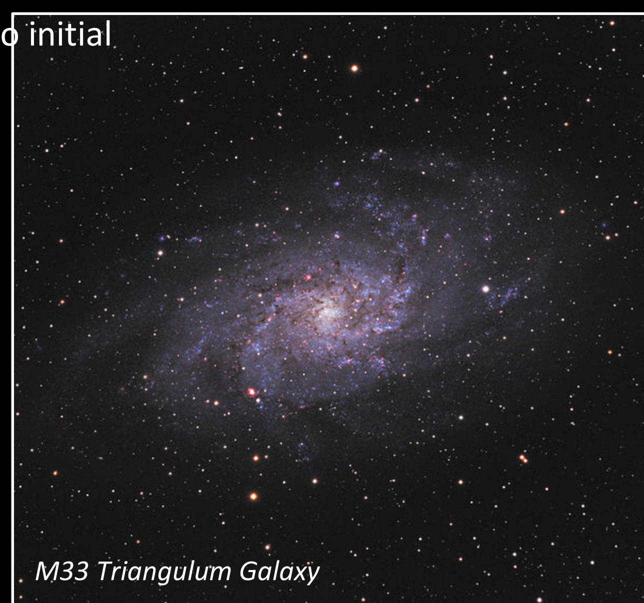


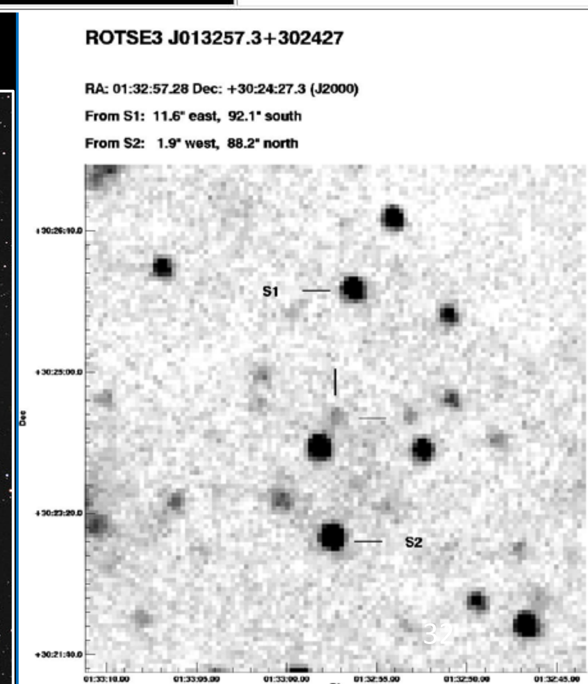
Novae typically originate in binary systems containing sun-like stars, as shown in this artist's rendering.

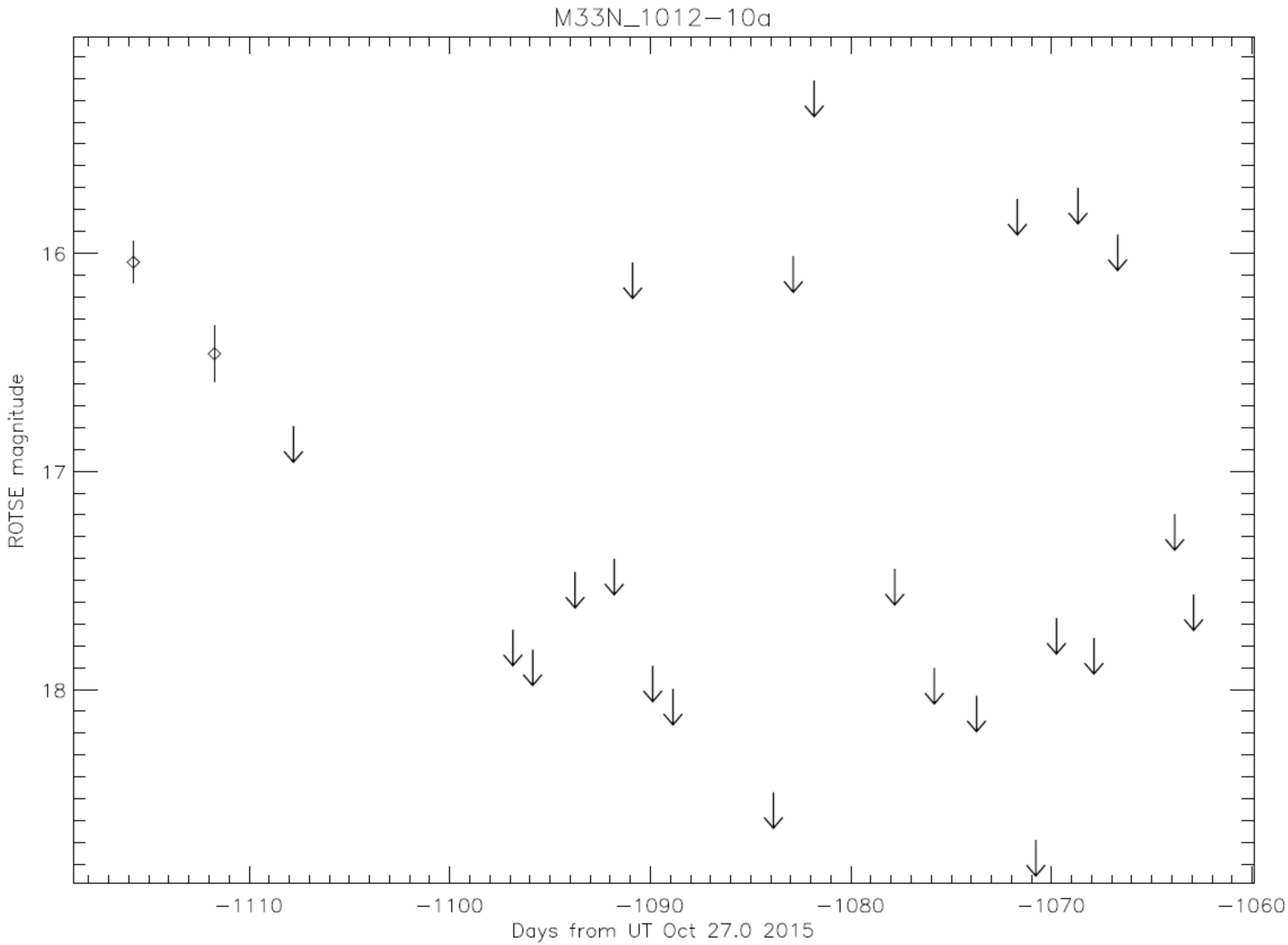
M33N 2012-10a

- 1st detection: 121004 (ROTSE-IIIb)
- (RA, Dec) = (01:32:57.3, +30:24:27)
- Constellation: Triangulum
- Host galaxy: M33
- Magnitude (max): 16.6
- z = 0.0002 (~0.85 Mpc, ~2.7 Mly)
- Classical nova
    - o Explosive nuclear burning of white dwarf surface from accumulated material from the secondary
    - o Causes binary system to brighten 7 - 16 magnitudes in a matter of 1 to 100s days
    - o After outburst, star fades slowly to initial brightness over years or decades
- CBET 3250




M33 Triangulum Galaxy

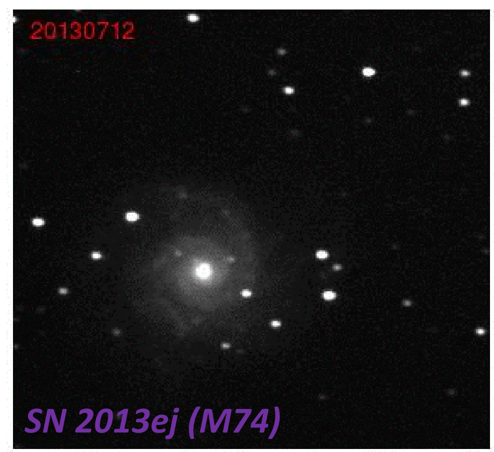




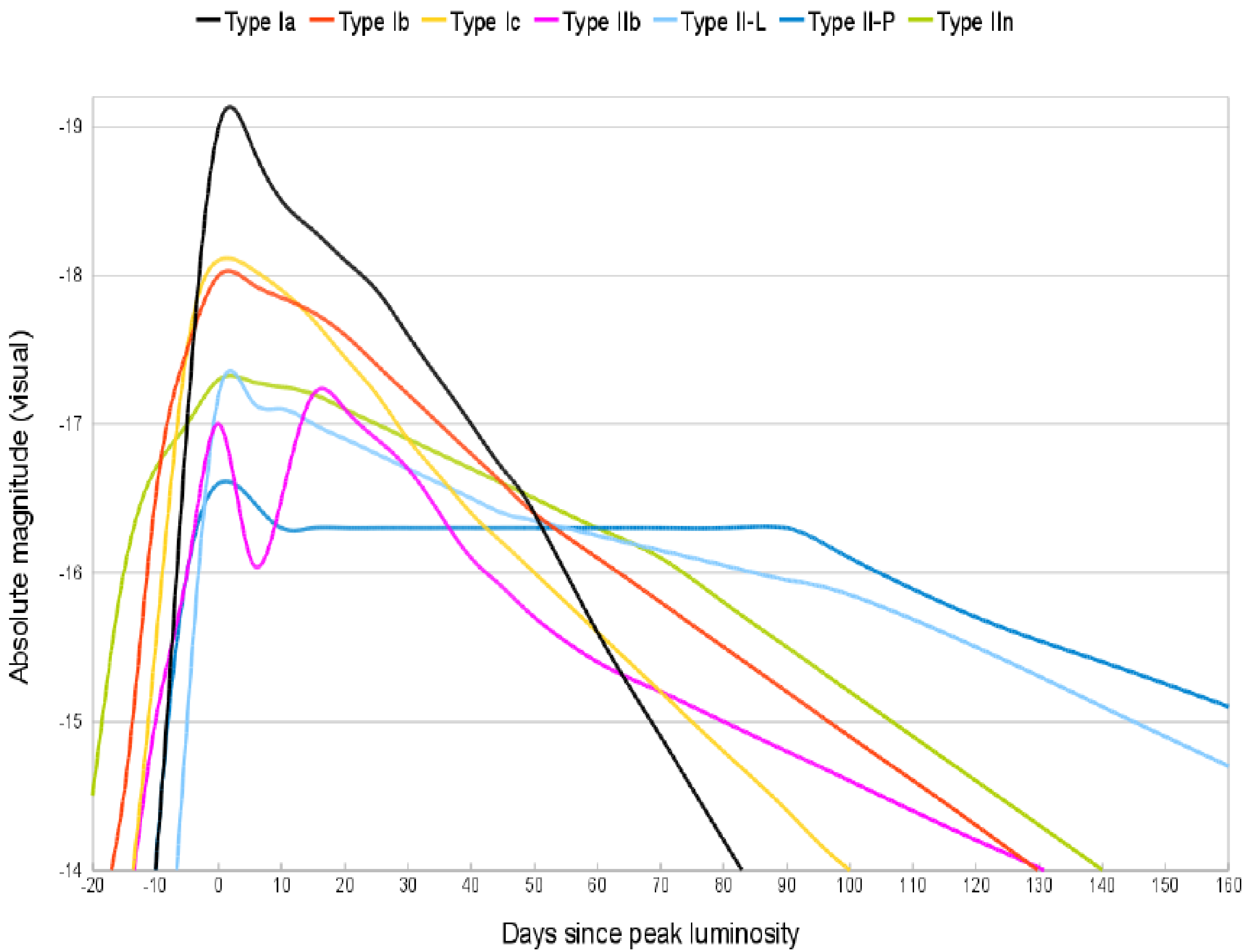
M33N_1012-10a
ROTSE magnitude
16
17
18
−1110
−1100
−1090
−1080
−1070
−1060
Days from UT Oct 27.0 2015

Supernovae

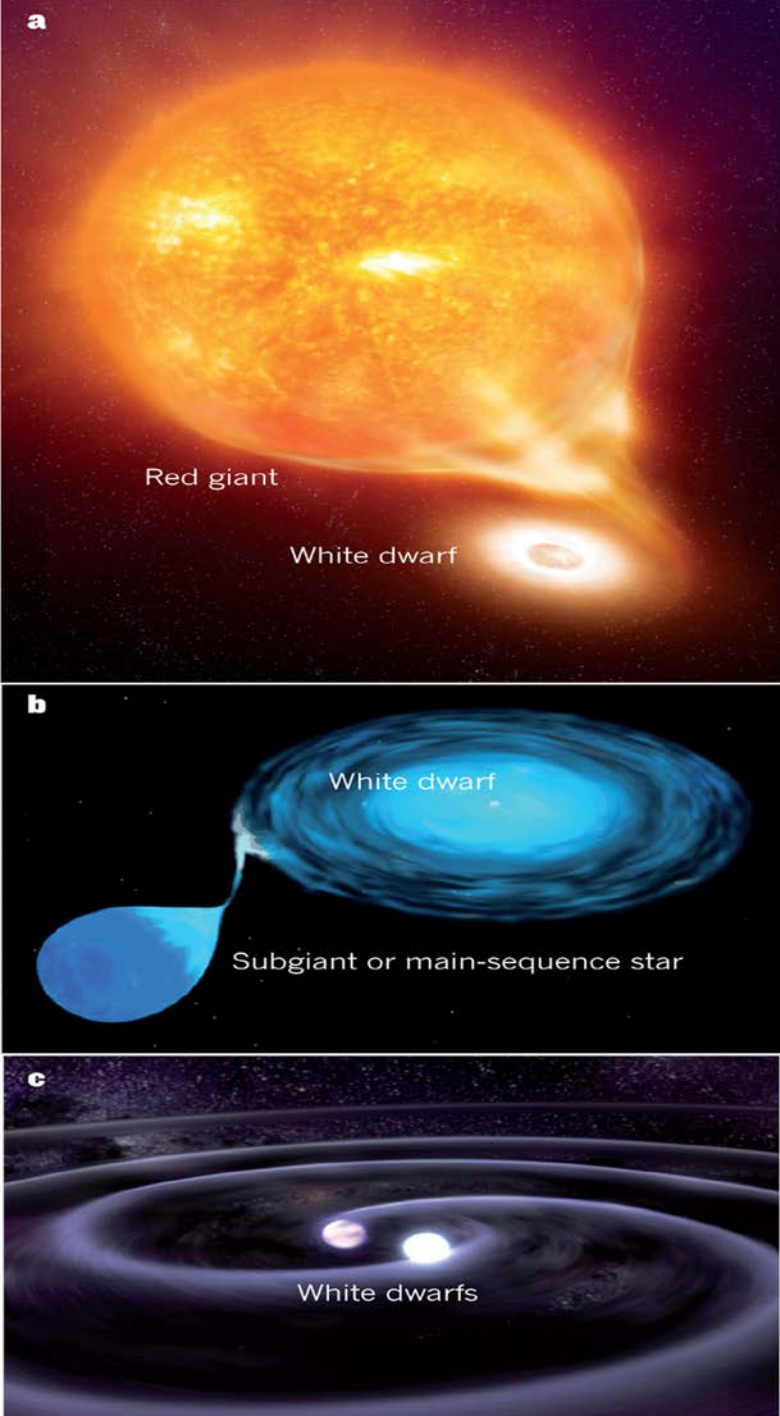
SN 1994D (NGC 4526)

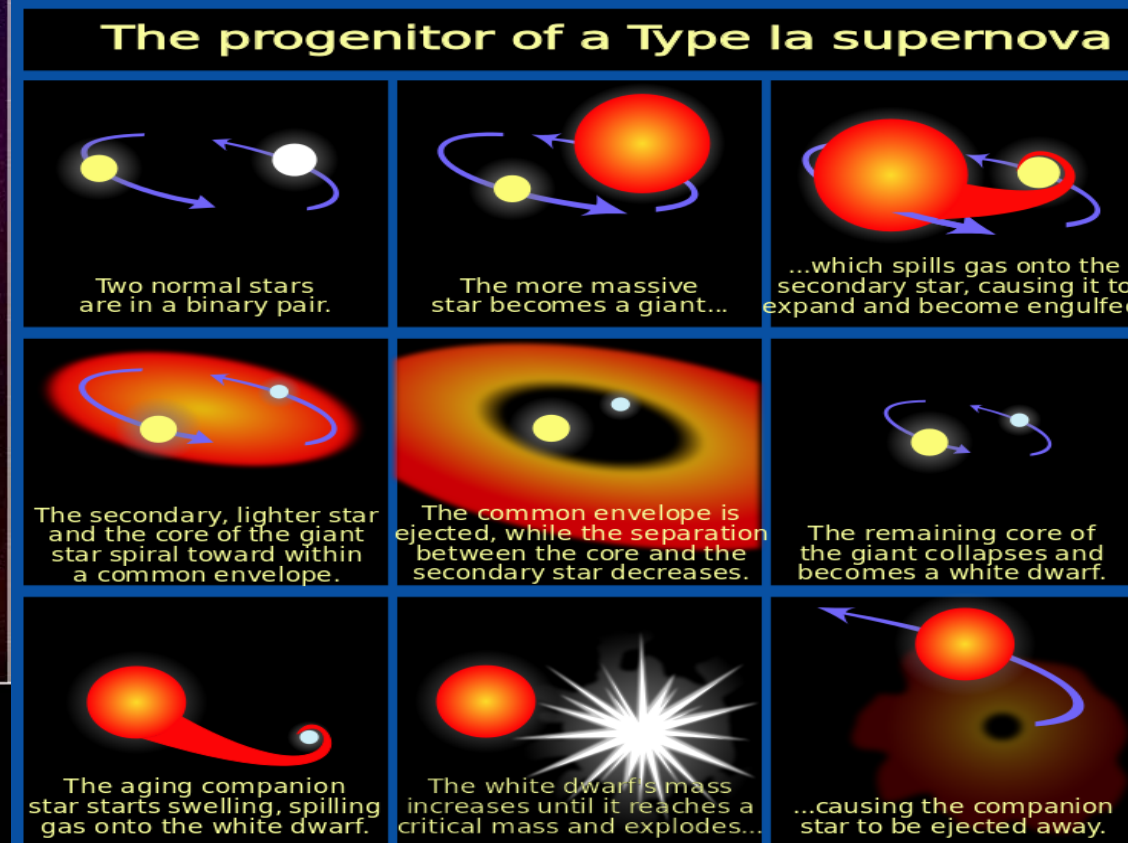


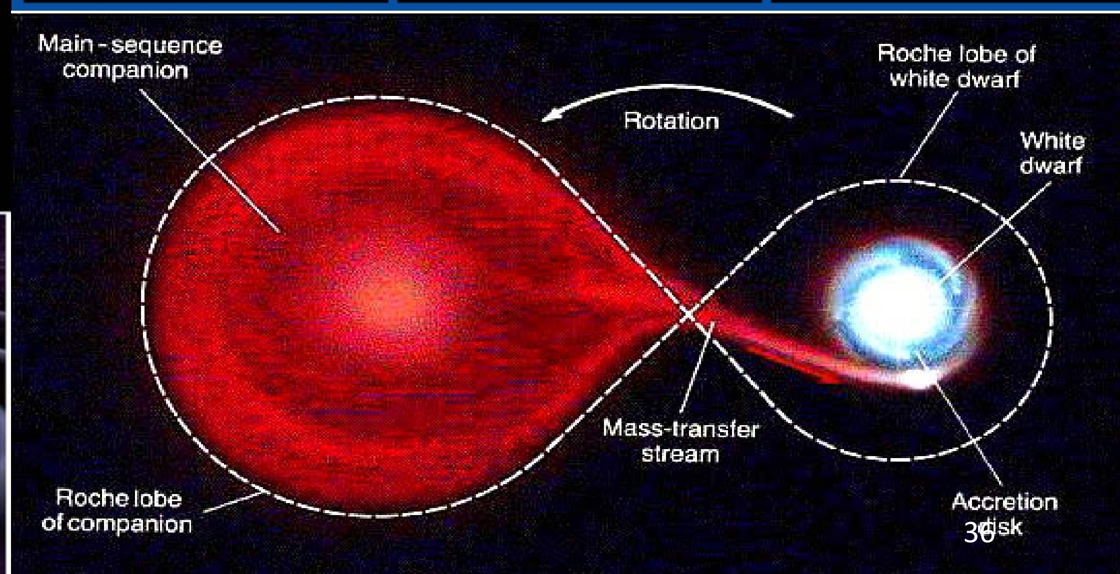
SN 2013ej (M74)

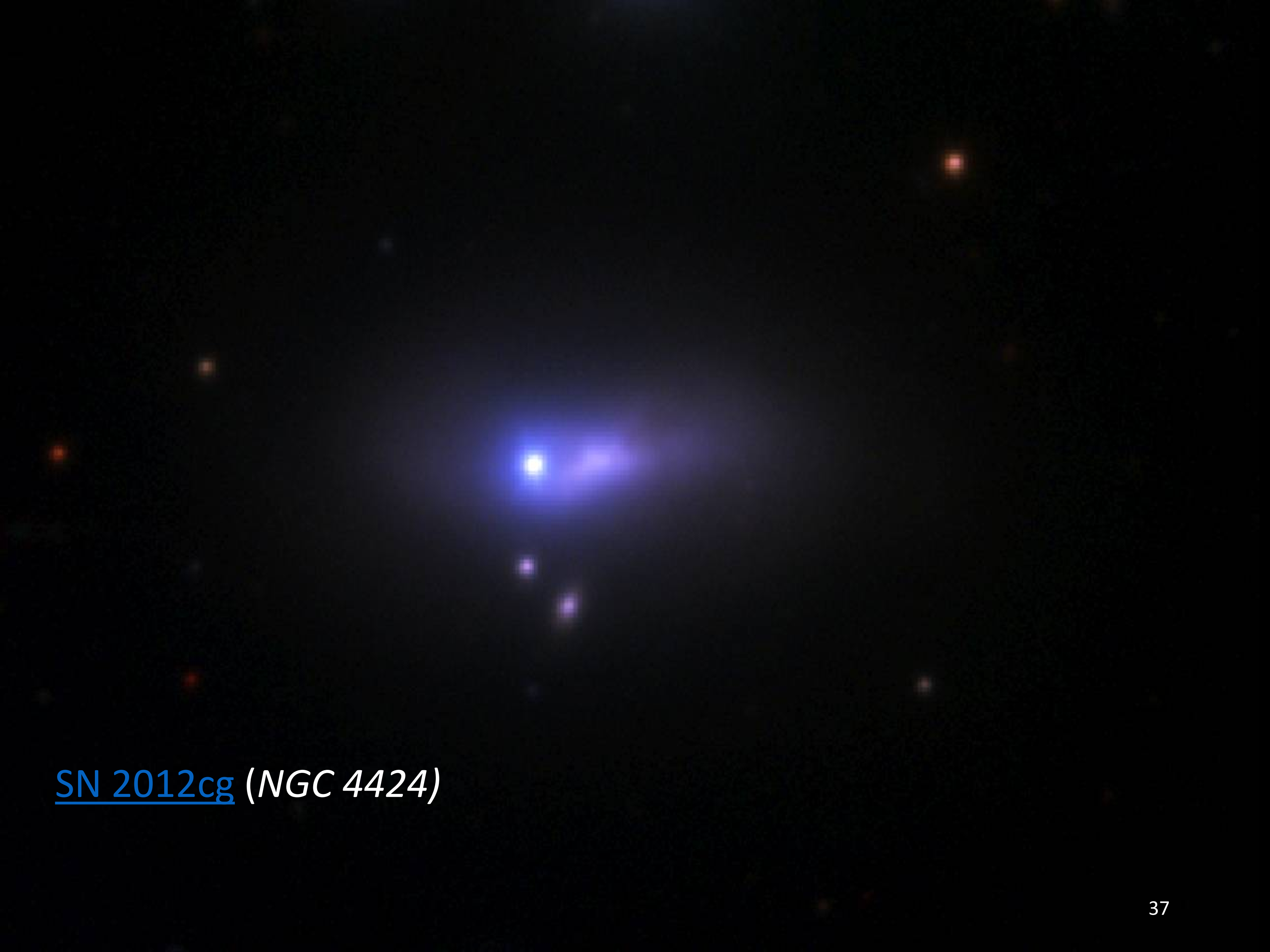


Type Ia
Type Ib
Type Ic
Type IIb
Type II-L
Type II-P
Type IIn
Absolute magnitude (visual)
-19
-18
-17
-16
-15
-14
-20
-10
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
Days since peak luminosity


a
Red giant
White dwarf
b
White dwarf
Subgiant or main-sequence star
c
White dwarfs


The progenitor of a Type Ia supernova
Two normal stars are in a binary pair.
The more massive star becomes a giant...
...which spills gas onto the secondary star, causing it to expand and become engulfed.
The secondary, lighter star and the core of the giant star spiral toward within a common envelope.
The common envelope is ejected, while the separation between the core and the secondary star decreases.
The remaining core of the giant collapses and becomes a white dwarf.
The aging companion star starts swelling, spilling gas onto the white dwarf.
The white dwarf's mass increases until it reaches a critical mass and explodes...
...causing the companion star to be ejected away.

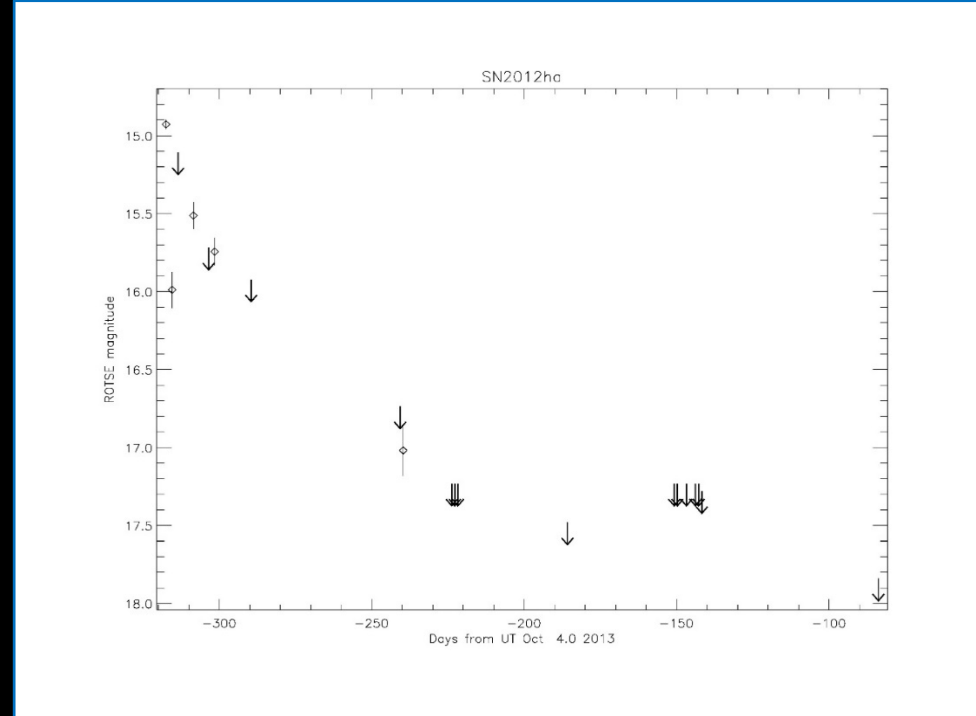

Main-sequence companion
Rotation
Roche lobe of white dwarf
White dwarf
Mass-transfer stream
Roche lobe of companion
Accretion disk

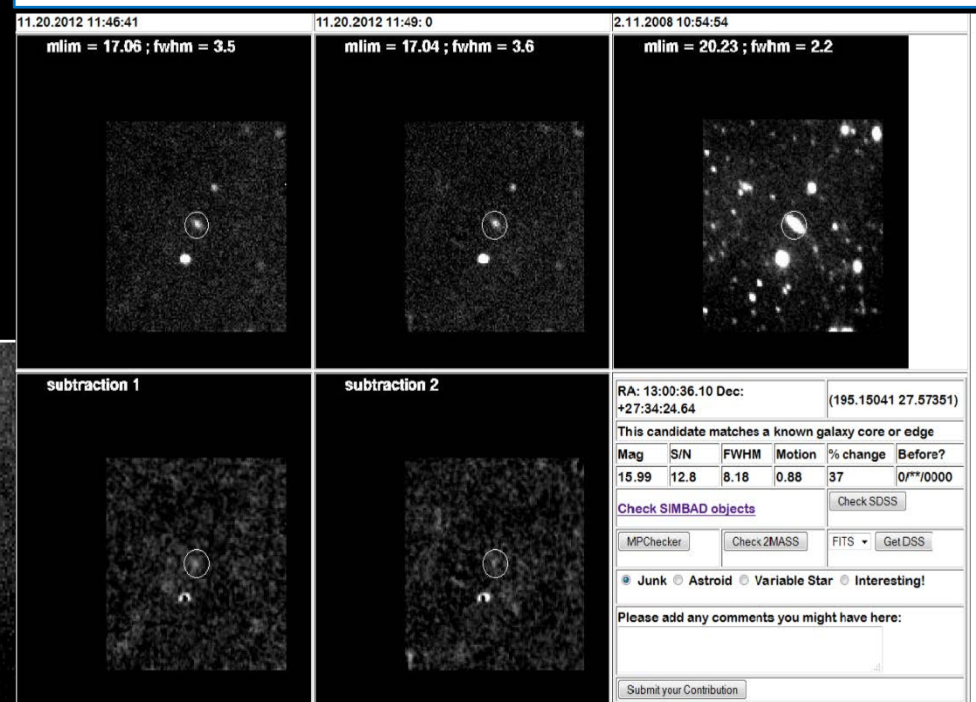


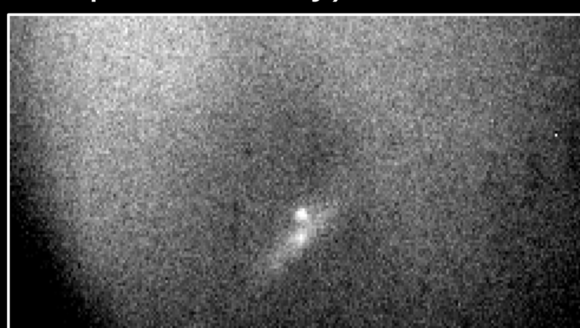
SN 2012cg (*NGC 4424)*

SN 2012ha *("Sherpa")*

- 1st detection: 121120 (ROTSE-IIIb)
- Type: Ia-normal
  - o Electron degeneracy prevents collapse to neutron star
  - o Single degenerate progenitor: C-O white dwarf in binary system accretes mass from companion (main sequence star)
  - o Mass → Chandrasekhar limit (1.44 $M_\odot$)
  - o Thermonuclear runaway
  - o Deflagration or detonation?
  - o Standardizable candles
    - ❑ acceleration of expansion
    - ❑ dark energy
- Magnitude (max): 15.0
- Observed 1 month past peak brightness
- (RA, Dec) = (13:00:36.10, +27:34:24.64)
- Constellation: Coma Berenices
- Host galaxy: PGC 44785
- *z* = 0.0170 (~75 Mpc; ~240 Mly)
- CBET 3319



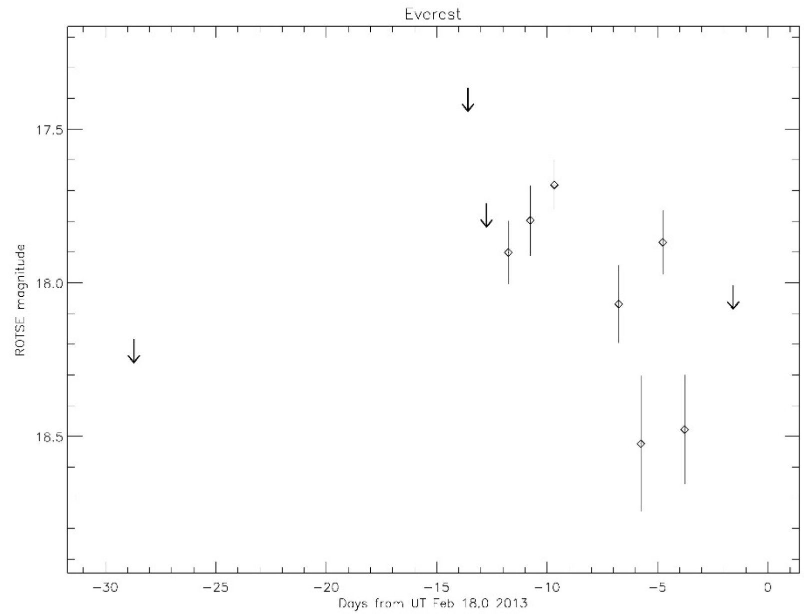


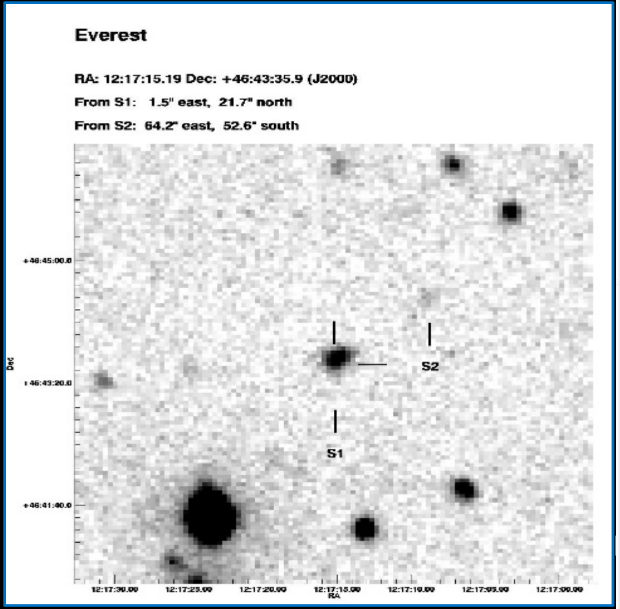


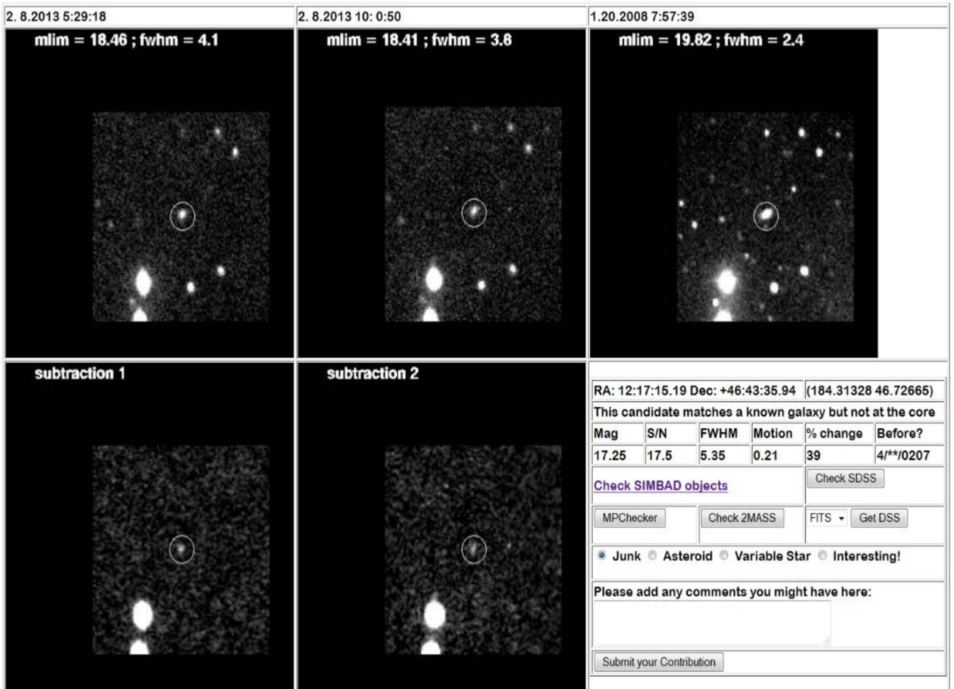
SN 2012ha: HET finder scope

SN 2013X *("Everest")*

- Discovered 130206 (ROTSE-IIIb)
- Type Ia 91T-like
  - o Overluminous
  - o White dwarf merger?
    - o Double degenerate progenitor?
- Magnitude (max): 17.7
- Observed 10 days past maximum brightness
- (RA, Dec) = (12:17:15.19, +46:43:35.94)
- Constellation: Ursa Major
- Host galaxy: PGC 2286144
- *z* = 0.03260 (~140 Mpc; ~450 Mly)
- CBET 3413





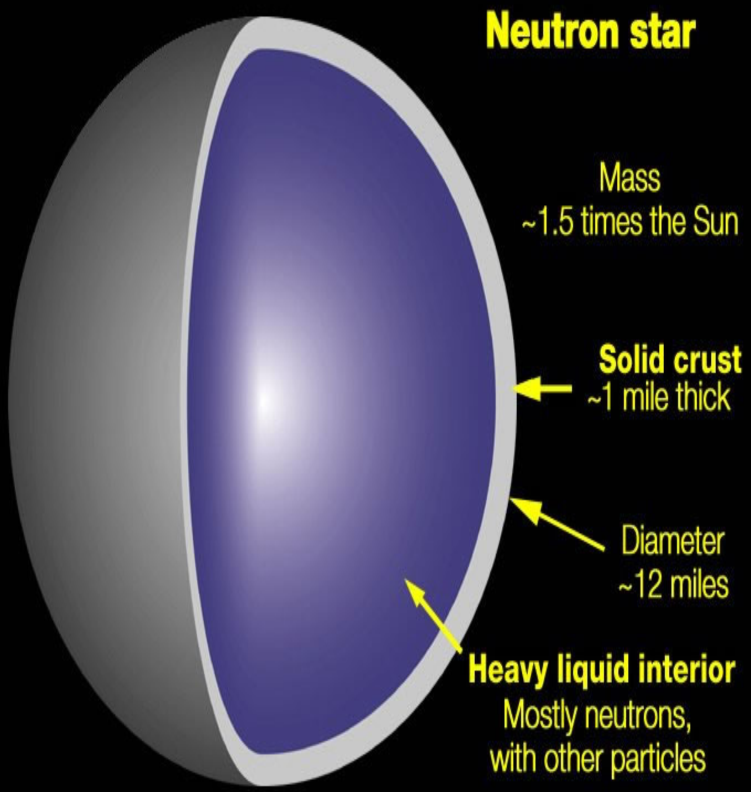


What happens to a stellar core more massive than 1.4 solar masses?

1. There aren't any
2. They shrink to zero size
3. They explode
4. They become something else

Neutron Stars

- Very compact – about 10 km radius
- Very dense – one teaspoon of neutron star material weighs as much as all the buildings in Manhattan
- Spin rapidly – up to 600 *rev/s*
- High magnetic fields – compressed from magnetic field of progenitor star



Neutron Stars

Similar to white dwarfs - basic physics is degenerate fermion gas. However, we have neutrons, not electrons. Replace m_e with m_p.

$$r_{ns} \approx 2.3 \times 10^9 \text{ cm} \frac{m_e}{m_n} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3} \approx 14 \text{ km} \left(\frac{M}{1.4 M_\odot}\right)^{-1/3}$$

Note: the Z/A factor is one, since almost all nucleons are neutrons.

Important Effects (we neglected):

1. Nuclear interactions play an important role in the EOS. The EOS is poorly known due to our poor understanding of details of the strong interaction.
2. The star is so compact that the effects of GR must be taken into account.

Compare gravitational and rest mass energies of a test particle of mass m.

$$E_{gr} = \frac{GMm}{2r} \qquad \text{and} \qquad E = mc^2$$

$$\frac{E_{\rm gr}}{mc^2} = \frac{GM}{rc^2} \approx \frac{6.7 \times 10^{-8}\ \text{cgs} \times 1.4 \times 2 \times 10^{33}\ \text{g}}{10 \times 10^5\ \text{cm}\ (3 \times 10^{10}\ \text{cm s}^{-1})^2} \approx 20\%$$

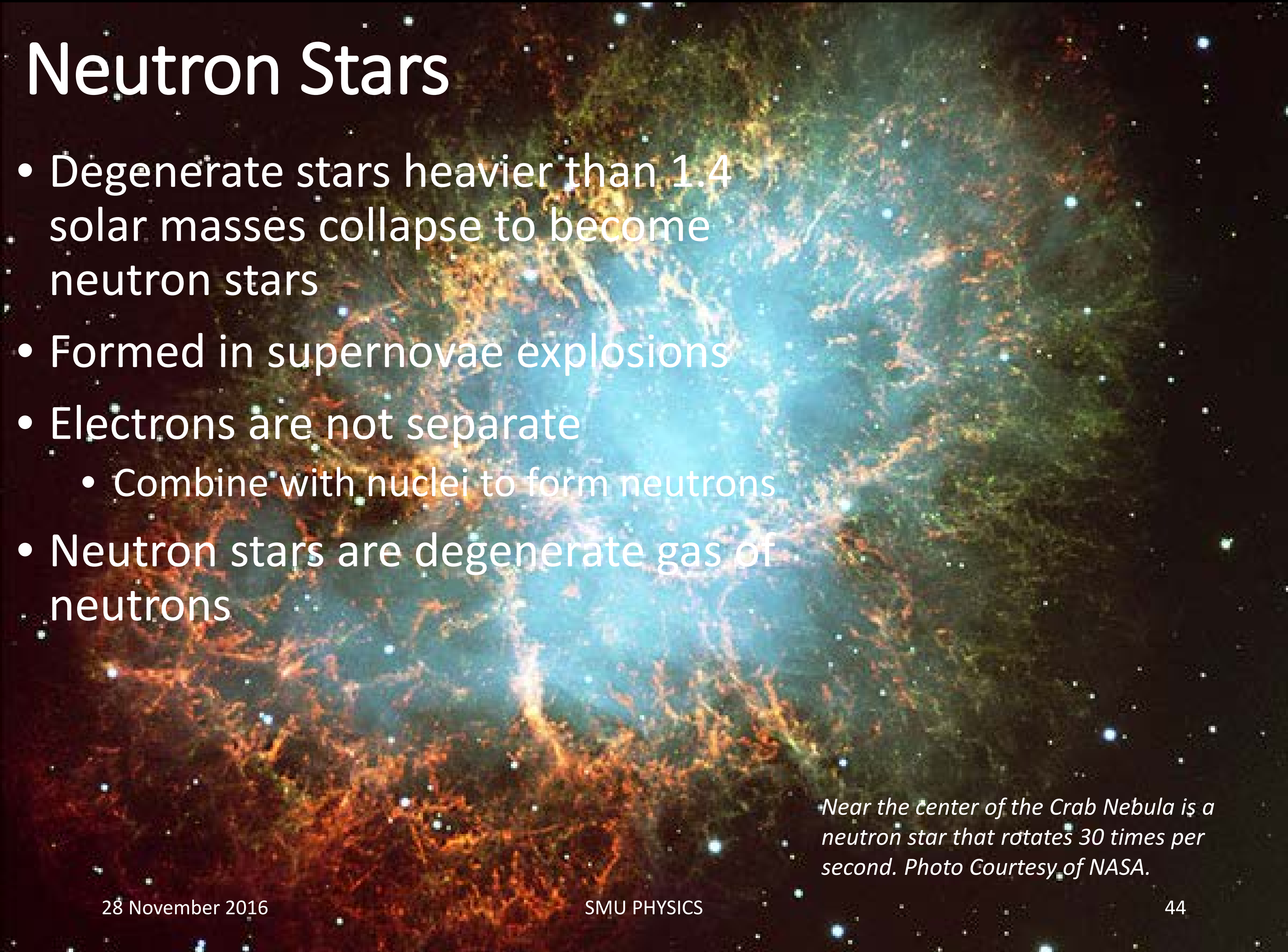
Matter falling onto a neutron star loses 20% of its rest mass and the mass of the star as measured via Kepler's law is 20% smaller than the total mass that composed it!

Detailed calculations that take into account GR and nuclear interactions give a radius of 10 km for a neutron star of $1.4M_{sun}$.

Limiting mass of a neutron star is not accurately known. The value is between $2M_{sun}$ and $3.2M_{sun}$.

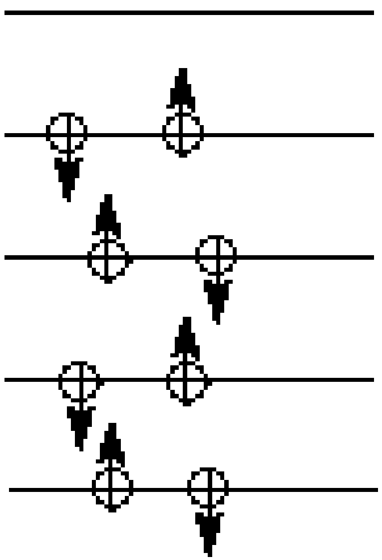
Neutron Stars

- Degenerate stars heavier than 1.4 solar masses collapse to become neutron stars
- Formed in supernovae explosions
- Electrons are not separate
  - Combine with nuclei to form neutrons
- Neutron stars are degenerate gas of neutrons



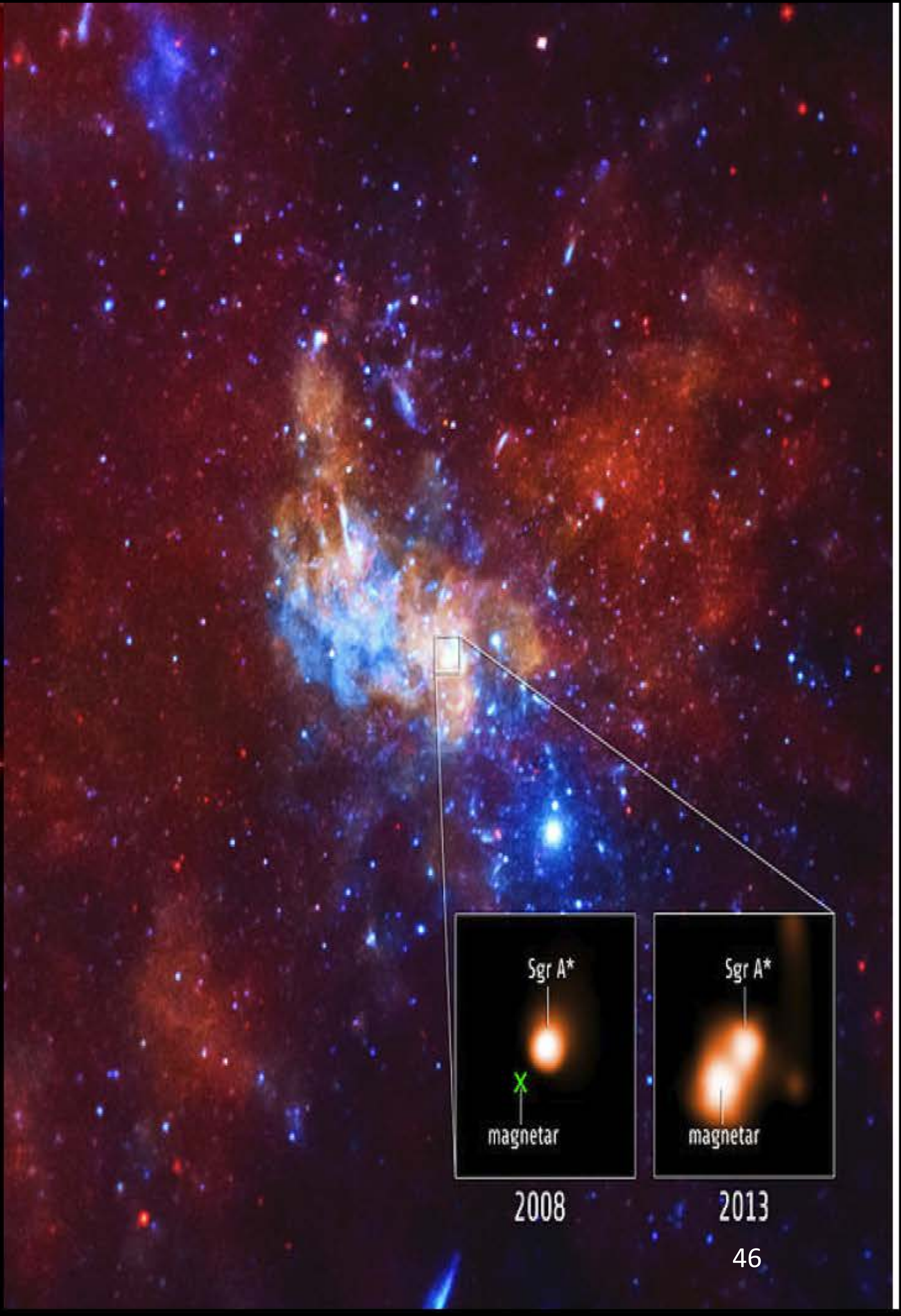
Near the center of the Crab Nebula is a neutron star that rotates 30 times per second. Photo Courtesy of NASA.

Neutron energy levels



Degenerate gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

- Only two neutrons (one up, one down) can go into each energy level
- In a degenerate gas, all low energy levels are filled
- Neutrons have kinetic energy, and therefore are in motion and exert pressure even if temperature is zero
- Neutron stars are supported by neutron degeneracy


Sgr A*
magnetar
2008
Sgr A*
magnetar
2013


BBC TWO