# Entropy

$$S = -\left(\frac{\partial F}{\partial T}\right)_{BN} = +\frac{\partial}{\partial T}\left[k_B T \ln(Z)\right]_{BN}$$

$$= \frac{\partial}{\partial T}\left\{N k_B T \ln\left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right]\right\}_{BN}$$

$$= N k_B \ln\left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right]$$

$$+ N k_B T \frac{\cancel{2}\sinh\left(\frac{\mu B}{k_B T}\right)}{\cancel{2}\cosh\left(\frac{\mu B}{k_B T}\right)} \frac{\mu B}{k_B}\left(-\frac{1}{T^2}\right)$$

$$= N k_B \left\{\underbrace{\ln\left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right]}_{-\frac{F}{T}} - \underbrace{\frac{\mu B}{k_B T}\tanh\left(\frac{\mu B}{k_B T}\right)}_{\frac{U}{T}}\right\}$$

$$S = \frac{U - F}{T}$$

We have entropy $S$ in terms of $T$, want $S$ in terms of $U$.

$$U = -N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$$

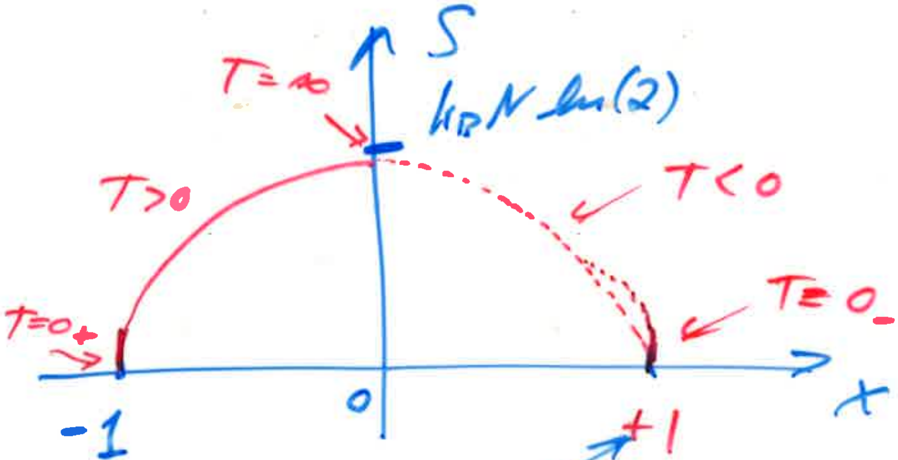
$$T = -\frac{\mu B}{k_B \operatorname{arctanh}\left(\frac{U}{N\mu B}\right)}$$

$$S = -\frac{U k_B}{\mu B} \operatorname{arctanh}\left(\frac{U}{N\mu B}\right) + k_B N \ln\left[\frac{2}{\sqrt{1-\left(\frac{U}{N\mu B}\right)^2}}\right]$$

define $x \equiv \frac{U}{N\mu B}$ $\qquad -1 \le x \le +1$

low T $\qquad 0 = x$ high T limit.

$$S = k_B N \left[ -x \operatorname{arctanh}(x) + \ln\left(\frac{2}{\sqrt{1-x^2}}\right)\right]$$



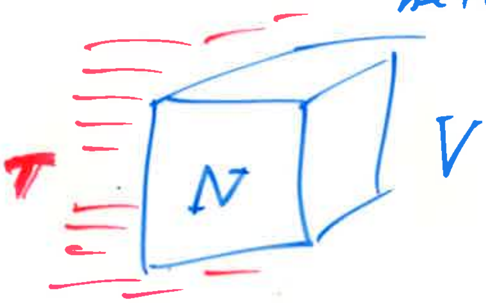

$$\frac{\partial S}{\partial U} = \frac{1}{T}$$

$x = +1 \Longrightarrow U = +N\mu B$ $\qquad$ all N spins are down.

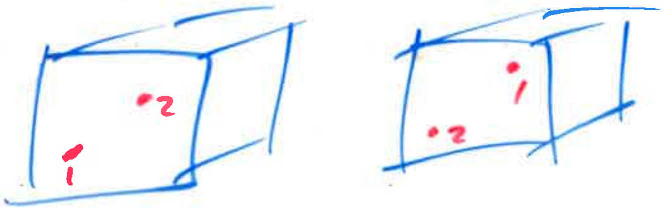
$U = -N\mu B$ - all spins up
lowest energy

# Partition Function for a Classical (not Quantum Mechanical) Ideal Gas. (Monatomic)

Approximation: Dilute - no or rare interaction between molecules. (mass m)




$$Z = \frac{1}{N!} z^N$$




← identical particles (indistinguishable)

Last time, the spins were distinguishable.

& the states were labelled by $i \equiv$ # spins down

$\longrightarrow$ $(N+1)$ macrostates, $E_0, E_1, E_2 \ldots E_N$

$2^N$ microstates

Now states are labelled by coordinates $\{x_i\}$ and momenta $\{p_i\}$.

e.g. molecule #5 is at $\{x_5, y_5, z_5\}$ and has $\{p_{x_5}, p_{y_5}, p_{z_5}\}$.

Need $3N$ $\{x_i\text{'s}\}$ + $3N$ $\{p_i\text{'s}\}$

The phase space is $6N$-dimensional.

one point in there.

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$$Z = \sum_i e^{-\beta E_i} \longrightarrow \frac{1}{N!} \int \cdots \int dx^{3N} dp^{3N} \, e^{-\beta \sum_{j=1}^{3N} \frac{p_j^2}{2m}}$$

sum over $2^N$ microstates

$6N$