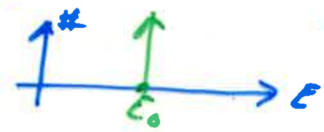
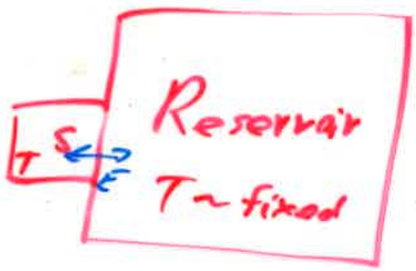
Energy E_0 of a system is fixed, the set of all possible states is called the "microcanonical ensemble." $\boxed{S}$

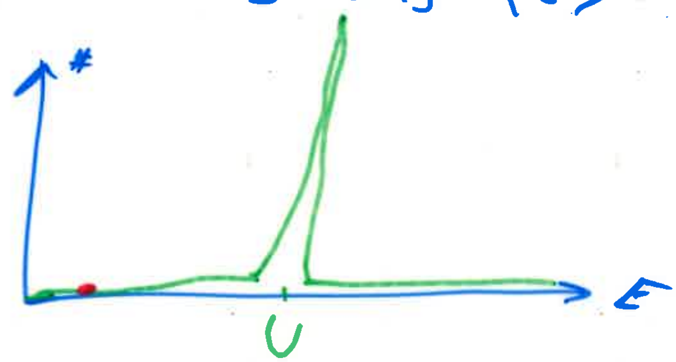
all states are equally likely.



If the temperature of the system is fixed T by putting the system in contact with a heat bath (reservoir), this is called a "canonical ensemble." E not fixed.



System S has $\langle E \rangle = U$



Probabilities are Boltzmann weighted

$$P_i \propto e^{-\beta E_i} \quad ; \quad P_i = \frac{e^{-\beta E_i}}{Z}$$

$$Z = \sum_k e^{-\beta E_k}$$

↑ <u>all</u> microstates

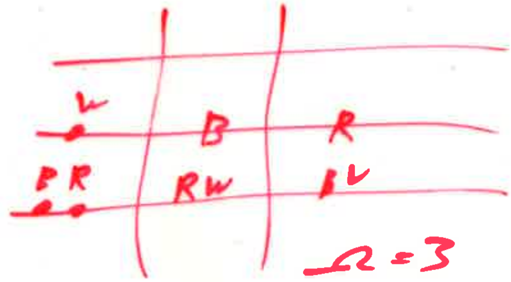
Exam problem w/ marbles, but now with temp T specified.

How many microstates? ∞

✗ Hard way

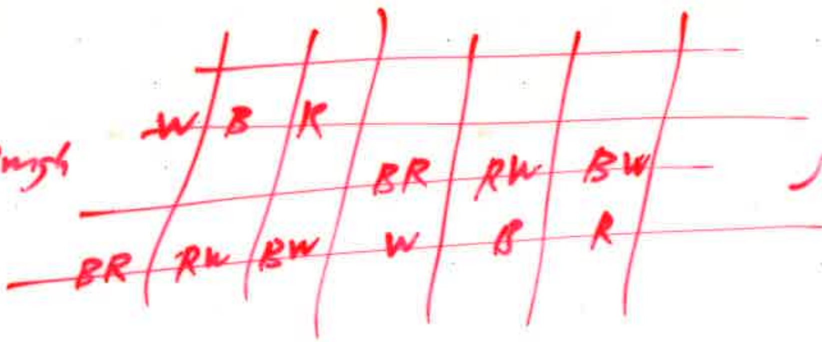
$\boxed{E=0}$ $\quad$ $\Omega = 1$

$\boxed{E = 1mgh}$



$\Omega = 3$

$E = 2mgh$



$\Omega = 6$

$$Z = \sum_k e^{-\beta E_k} = 1e^{-\beta 0} + 3e^{-\beta mgh} + 6e^{-2\beta mgh} + 10e^{-3\beta mgh} + \dots$$

✓ Easy way

$$z_1 = \text{partition function for one marble} = \sum_{n=0}^{\infty} e^{-\beta n mgh}$$

$\sum_k r^k = \frac{1}{1-r}$ Geometric series

$$z_1 = \frac{1}{1 - e^{-\beta mgh}}$$

$$Z = z^3 = \left(\frac{1}{1-e^{-\beta mgh}}\right)^3$$

Probability all 3 marbles will be on step #1?

$$P_{111} = \frac{e^{-\beta mgh3}}{Z} = e^{-\beta mgh3}\left(1-e^{-\beta mgh}\right)^3$$

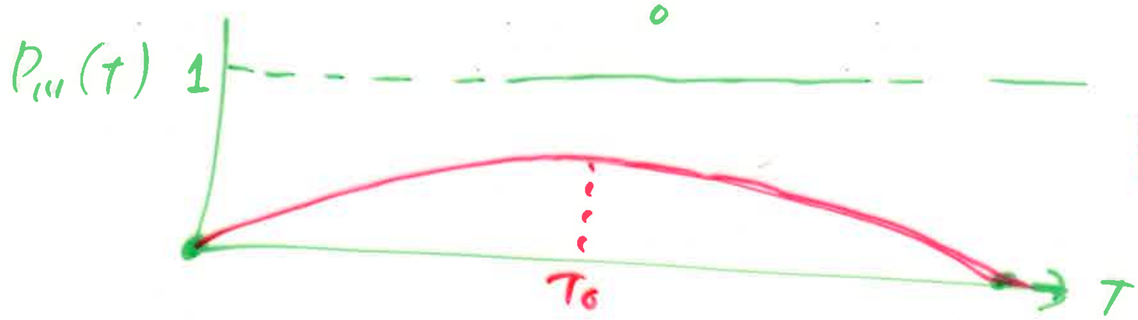
$$P_{npq} = \frac{e^{-\beta mgh[n+p+q]}}{Z}$$

What is P_{111} at low T? (High β)

$$P_{111} = \left[\underbrace{e^{-\beta mgh}}_{0}\left(1-e^{-\beta mgh}\right)\right]^3 \to 0$$

What is P_{111} at high T? (Low β)

$$P_{111} = \left[e^{-\beta mgh}\underbrace{\left(1-e^{-\beta mgh}\right)}_{0}\right]^3 \to 0$$



Find T_0 that maximizes P_{111}.

Given T, what is $\langle E \rangle = U$?

$$f_{AVG} = \bar{F} = \langle f \rangle = \sum_i f_i P_i = \int_u f(u)\, p(u)\, du$$

$$U = \sum_i E_i P_i = \sum_i E_i \frac{e^{-\beta E_i}}{Z} = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_k e^{-\beta E_k}}$$

$$= \frac{-\frac{\partial}{\partial \beta} Z}{Z} = -\frac{\partial}{\partial \beta} \ln(Z)$$

$$= -\frac{\partial}{\partial \beta} \ln\left[\left(\frac{1}{1-e^{-\beta mgh}}\right)^3\right] = +\frac{\partial}{\partial \beta} 3 \ln\left(1 - e^{-\beta mgh}\right)$$

$$= \frac{3\left(mgh\, e^{-\beta mgh}\right)}{1-e^{-\beta mgh}} \cdot \left(\frac{e^{+\beta mgh}}{e^{+\beta mgh}}\right)$$

$$= \boxed{\frac{3mgh}{e^{\beta mgh} - 1} = U}$$

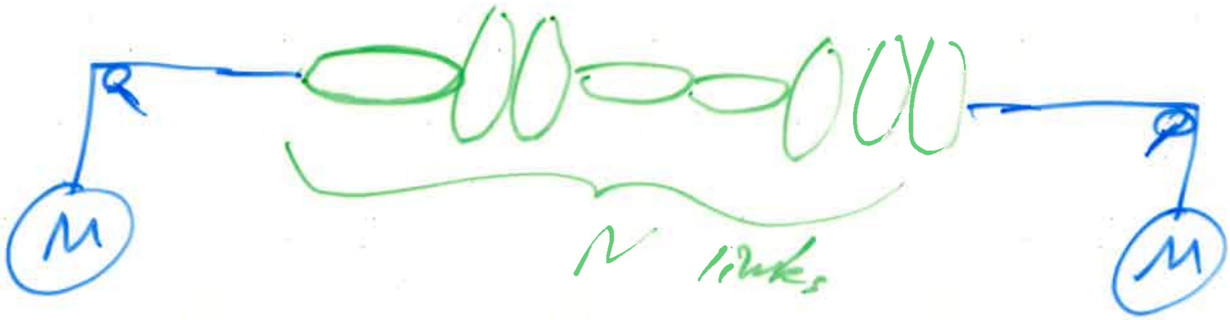
Low T, high β

$U \to 0$

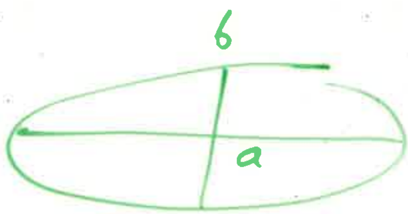
High T, low β

$U \to \infty$

One-dimensional chain w/ massless link



Tension in chain? $\tau = Mg$



At temp T, what is L?