D_2 spin 1 ⊗ spin 1 (3 states) + (3 states) = 9

Three multiplets

	$m_s=+2$	$m_s=+1$	$m_s=0$	$m_s=-1$	$m_s=-2$
spin 2 sym 5 states	$\uparrow_1\uparrow_2$	$\frac{(\uparrow_1\cdot_2 + \cdot_1\uparrow_2)}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}\uparrow\downarrow + \sqrt{\frac{2}{3}}\cdot\cdot - \frac{1}{\sqrt{6}}\downarrow\uparrow$	$\frac{(\downarrow\cdot + \cdot\downarrow)}{\sqrt{2}}$	$\downarrow\downarrow$
spin 1 anti 3 states	X	$\frac{(\uparrow_1\cdot_2 - \cdot_1\uparrow_2)}{\sqrt{2}}$	$\frac{(\uparrow\downarrow - \downarrow\uparrow)}{\sqrt{2}}$	$\frac{(\downarrow\cdot - \cdot\downarrow)}{\sqrt{2}}$	X
spin 0 sym 1 state	X	X	$\frac{(\uparrow\downarrow - \cdot\cdot + \downarrow\uparrow)}{\sqrt{3}}$	X	X

$$\psi_{overall} = \psi_{spin} \cdot \psi_{orbital}$$

$$(+1) = \begin{cases} (+1)_{S=0}^{S=2} & (+1) \text{ even } \ell \text{ in sum} \\ (+1)_{S=1} & (-1) \text{ odd } \ell \text{ in sum} \end{cases}$$

$$\hat{S}^2 |S, m_s\rangle = \underbrace{S(S+1)\hbar^2}_{\text{eigenvalue}} |S, m_s\rangle$$

$$\hat{S}_\pm |S, m_s\rangle =$$

(1) Black body Radiation

(2) C_V for diatomic molecule

(3) Heat capacity for a crystal solid. (metal)
e.g. Si e.g. Cu

"Particle in a cage"

3-dim SHO

$$V(\vec{r}) = V_0 + \frac{1}{2} k_s \vec{r}\cdot\vec{r}$$
$$= V_0 + \frac{1}{2} k_s (x^2+y^2+z^2)$$

Einstein-Debye

3N oscillators

$$E_j = \left(\frac{1}{2}+j\right)\hbar\omega \qquad k_s = m\omega^2$$

Partition function

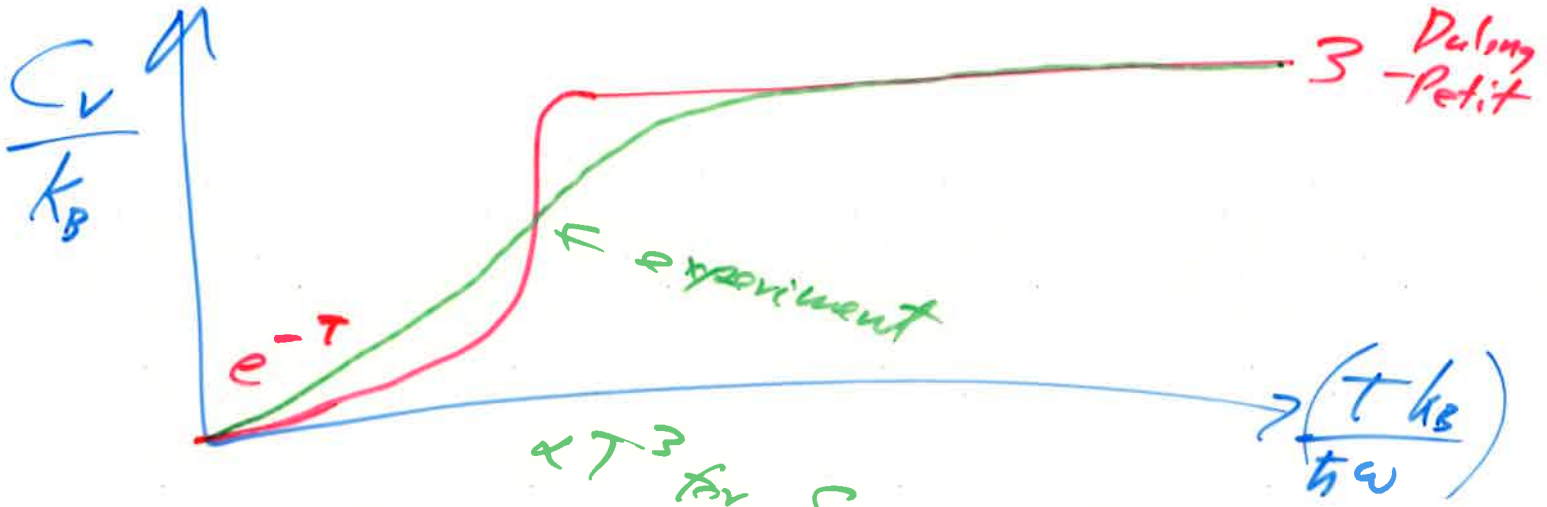
$$Z = \sum_{j=0}^{\infty} e^{-\beta E_j} = \sum_{j=0}^{\infty} e^{-\beta\hbar\omega j} = \frac{1}{1-e^{-\beta\hbar\omega}}$$

$$\ln(Z) = -\ln\left(1-e^{-\beta\hbar\omega}\right)$$

energy $U = -\dfrac{\partial \ln(Z)}{\partial \beta} = \dfrac{\hbar\omega\, e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}} = \dfrac{\hbar\omega}{e^{+\beta\hbar\omega}-1}$

$\beta = \dfrac{1}{k_B T}$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{VN} = k_B \left(\frac{\hbar\omega}{k_B T}\right)^2 \left(\frac{1}{e^{\beta\hbar\omega}-1}\right)$$

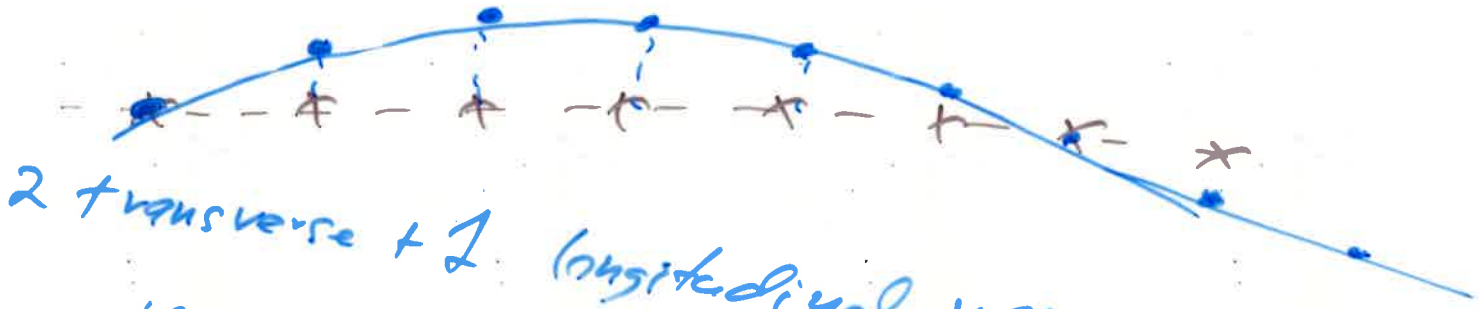


$\propto T^3$ for Si

$\propto aT + bT^3$ for Cu, Al

↑ electrons ↑ phonons

By concentrating on one site in crystal, we have ignored long-wavelength oscillations



2 transverse + 1 longitudinal wave

100s or 1000s of sites