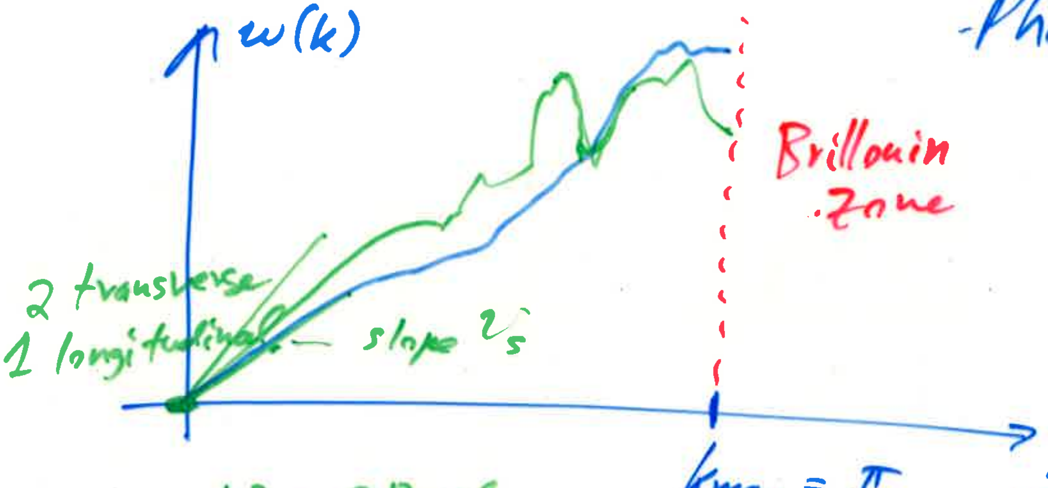
Phonons



v_s = speed of sound

$$k_{max} = \frac{\pi}{a} = \frac{2\pi}{\lambda_{min}} = \frac{2\pi}{2a}$$

At low T, the dispersion relation is $\omega = v_s k$

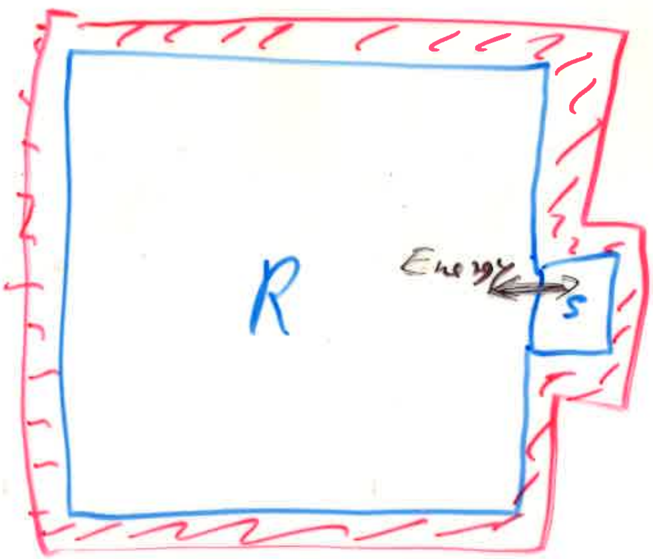
See notes for Blackbody radiation

Replace speed light c by v_s.

$U \propto T^4$ $\qquad C_V = \frac{dU}{dT} \propto T^3$ ✓

Previously

All states of an isolated system are equally likely.
Microcanonical ensemble.



S = system
R = reservoir

closed system ⟺ does not exchange particles with the reservoir.
Canonical Ensemble.

(R+S) is isolated ⟺ all states of (R+S) are equally likely.

Look at two states of the system s_1, s_2

Also look at multiplicities of the reservoir

$\Omega_R(s_1)$, $\Omega_R(s_2)$ ← # accessible states of the reservoir.

$$\frac{P(s_2)}{P(s_1)} = \frac{\Omega_R(s_2)}{\Omega_R(s_1)} = \frac{e^{\ln \Omega_R(s_2)}}{e^{\ln \Omega_R(s_1)}} = \frac{e^{S_R(s_2)/k_B}}{e^{S_R(s_1)/k_B}}$$

$$\frac{P(s_2)}{P(s_1)} = \exp\left\{\frac{1}{k_B}\left[S_R(s_2) - S_R(s_1)\right]\right\}$$

1st Law $\quad dU_R = T\,dS_R - P\,dV_R + \mu\,dN_R$

$$dS_R = \frac{1}{T}\left[dU_R + \underbrace{P\,dV_R}_{\text{small}} - \cancel{\mu\,dN_R}\right] \rightarrow 0 \text{ for canonical ensemble.}$$

$\Rightarrow$ ignore.

$\Delta U_R = -\Delta U_s \leftarrow$

$\Delta N_R = -\Delta N_s$

$$S_R(s_2) - S_R(s_1) = \frac{1}{T}\left[U_R(s_2) - U_R(s_1)\right]$$

$$= -\frac{1}{T}\left[E(s_2) - E(s_1)\right]$$

for system

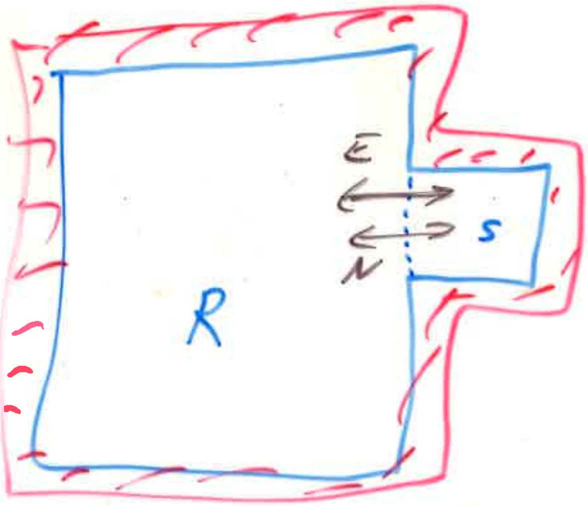
$$\frac{P(s_2)}{P(s_1)} = \exp\left\{-\frac{1}{k_B T}\left[E(s_2) - E(s_1)\right]\right\} = \frac{e^{-\beta E_2}}{e^{-\beta E_1}}$$

$e^{-\beta E_i} \propto P_i$ Boltzmann factor.

Unitarity $\sum P_i = 1 \longrightarrow P_i = \dfrac{e^{-\beta E_i}}{\left(\sum_n e^{-\beta E_n}\right)} \leftarrow Z$

Z is *the* partition function, canonical P.f., Boltzmann or Helmholtz P.f.

$$F = A = -k_B T \ln(Z)$$



$$S_R(s_2) - S_R(s_1)$$

$$= -\frac{1}{T}\left[E_2 - E_1 - \mu N_2 + \mu N_1\right]$$

for system

$$\frac{P_2}{P_1} = \frac{e^{-\beta[E_2 - \mu N_2]}}{e^{-\beta[E_1 - \mu N_1]}}$$

$$P_i \propto e^{-\beta[E_i - \mu N_i]} \leftarrow \text{Gibbs factor}$$

Unitarity $\sum P_i = 1 \Longrightarrow P_i = \dfrac{e^{-\beta(E_i - \mu N_i)}}{Q}$

$$Q = \sum_n e^{-\beta(E_n - \mu N_n)}$$ Grand Partition function, Gibbs sum

Gibbs partition function

$$\mathcal{Z} = \sum_n e^{-\beta \frac{H_n}{\tau}}$$ enthalpy

$$-k_B T \ln(\mathcal{Z}) = G$$

$$-k_B T \ln(Q) = \Phi$$ Grand Potential
London Pot.
The. therm. Pot