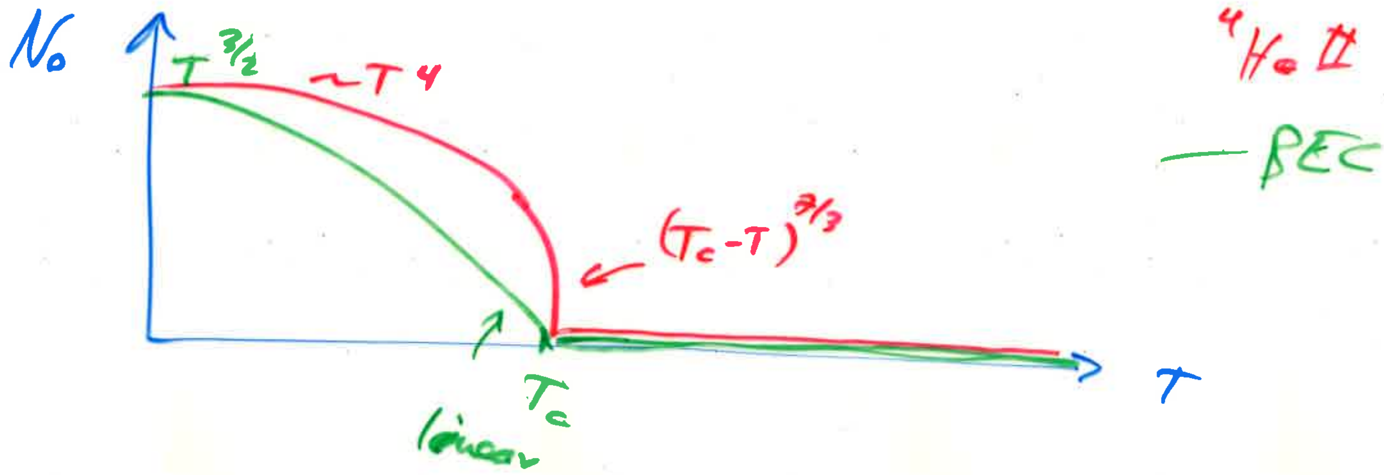
One last difference between ^4He II and BEC



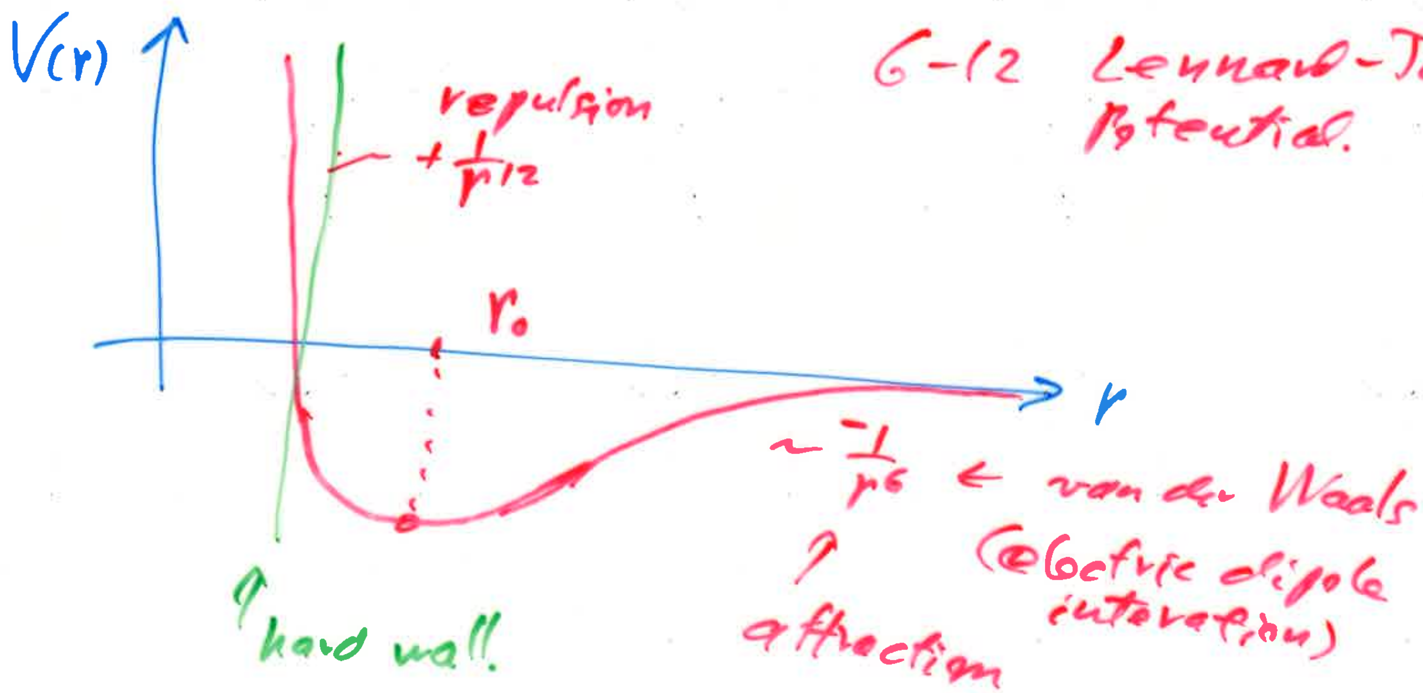
Interactions between molecules

① van der Waals, Cluster Expansion (gas)

② Ising Model (magnet)

Q Interacting (Non-Ideal) Gas

6-12 Lennard-Jones Potential.



van der Waals Equation of State (a, b)

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = Nk_B T$$

Ideal Gas $PV = Nk_B T$

attraction

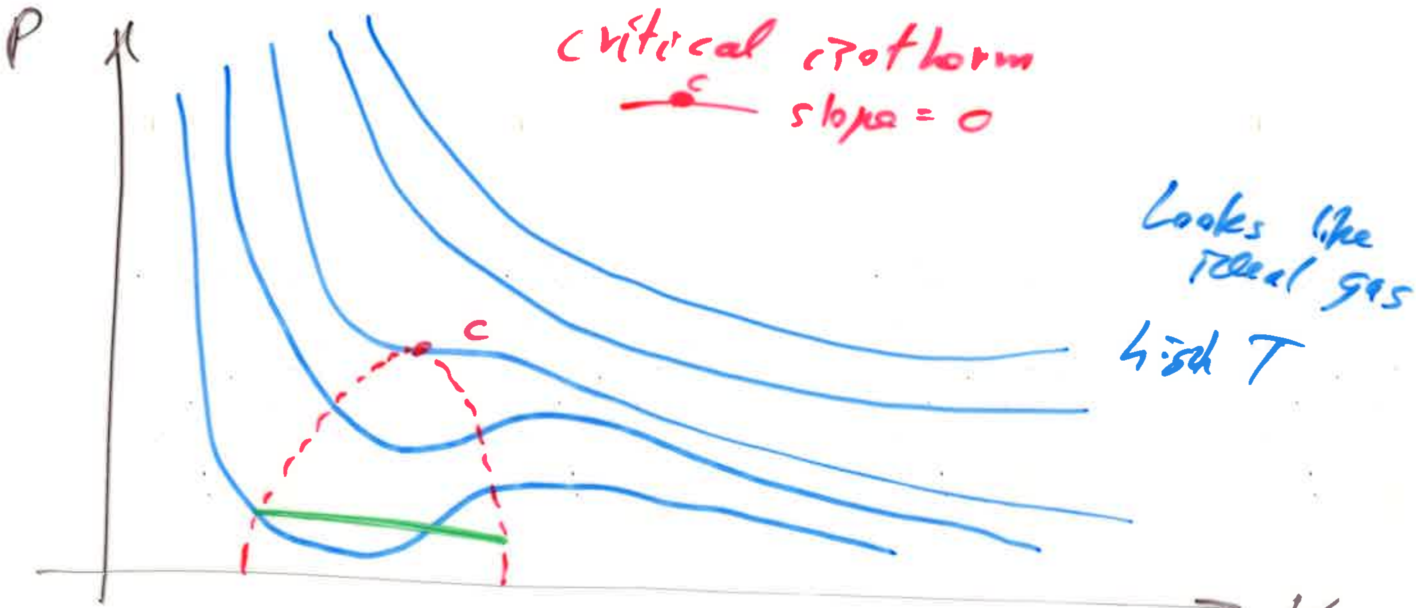
repulsion

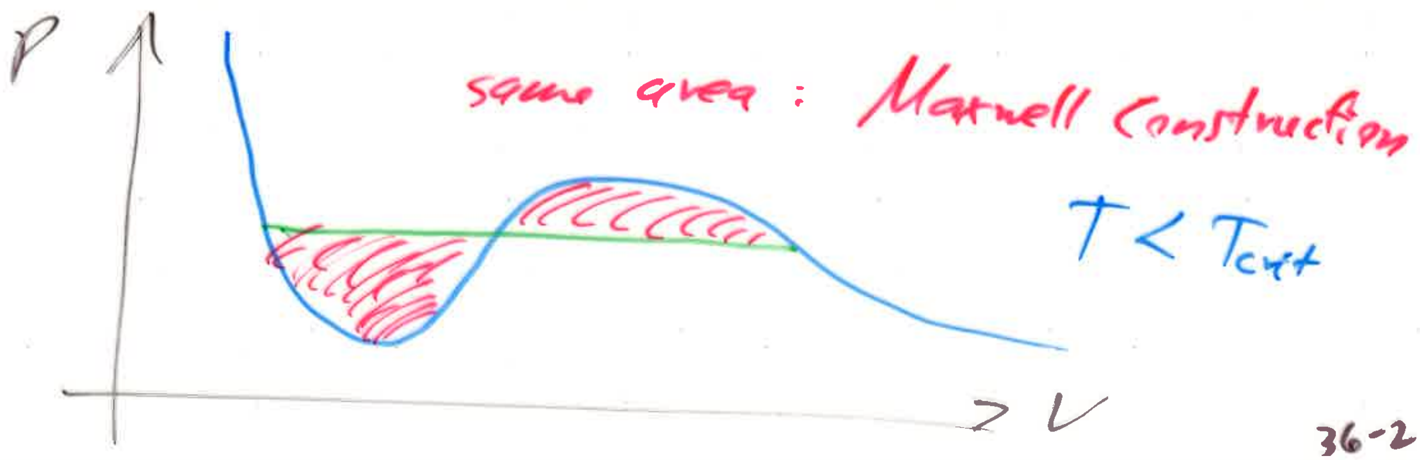
$b \sim$ volume of molecule.

$$P = \frac{Nk_B T}{V - Nb} - \frac{aN^2}{V^2}$$

Pressure is decreased!

pairs of molecules $\frac{N(N-1)}{2}$





Schroeder 5.3

Cluster Expansion

Partition Function

$$Z = \frac{1}{N!}\frac{1}{h^{3N}} \underbrace{\iiint d^3r_1 \cdots d^3r_N\, d^3p_1 \cdots d^3p_N}_{6N \text{ integrals}} e^{-\beta E_{TOT}}$$

phase space is 6N dimensional

Ideal Gas: $E = \frac{|\vec{p}_1|^2}{2m} + \cdots + \frac{|\vec{p}_N|^2}{2m}$.

$$|\vec{p}_1|^2 = p_{1x}^2 + p_{1y}^2 + p_{1z}^2$$

one integral $\displaystyle\int_{-\infty}^{+\infty} d^3p_j\, e^{-\beta \frac{|\vec{p}_j|^2}{2m}} = \left(\sqrt{2\pi m k_B T}\right)^3$

$\frac{h}{\sqrt{2\pi m k_B T}} = \lambda_Q$ quantum Thermal de Broglie wavelength

Now

$$E_{TOT} = \sum_i \frac{|\vec{p}_i|^2}{2m} + U$$ ← potential energy

Assume ① $U(\{\vec{r}_k\})$ and not momenta

excludes magnetic force, e.g.

$$\vec{F}_{mag} = q\,\vec{v}\times\vec{B}$$

$$Z = \frac{1}{N!} \underbrace{\left(\frac{\sqrt{2\pi m k_B T}}{h}\right)^{3N}}_{\frac{1}{v_Q^N}} \underbrace{\int d^3r_1 \cdots d^3r_N}_{3N \text{ integrals}} e^{-\beta U}$$

$$Z = Z_{\text{ideal gas}} \underbrace{\frac{1}{V^N} \int d^3r_1 \cdots d^3r_N\, e^{-\beta U}}_{Z_c}$$

configuration integral

$(x_1 y_1 z_1 \cdots x_N y_N z_N)$ = configuration space

$(p_{x_1} p_{y_1} p_{z_1} \cdots p_{z_N})$ = momentum space

$(x_1 p_{x_1}\, y_1 p_{y_1} \cdots z_N p_{z_N})$ = phase space.

② U can be written as a sum of potential energies due to pairs of molecules

$$U = u_{12} + u_{23} + u_{13} + u_{24} \;-\!-\!-\!-$$