Mean Field Theory

Look at one dipole ①
(all others frozen)

$$E_\uparrow = -\epsilon \sum_{\substack{k \\ \text{nearest} \\ \text{neighbors}}} (+1)\, s_k \;\overset{s_1}{\swarrow} = -\epsilon\, \underset{\uparrow}{n}\, \bar{s} \leftarrow +\epsilon \text{ above.}$$

n ↑ coordination number.

$$E_\downarrow = -\epsilon \sum_{\substack{k \\ \text{n.n.}}} (-1)\, s_k \;\overset{s_1}{\swarrow} = +\epsilon n \bar{s}$$

$$Z_1 = e^{-\beta E_\uparrow} + e^{-\beta E_\downarrow} = e^{+\beta\epsilon n \bar{s}} + e^{-\beta \epsilon n \bar{s}}$$

$$= 2\cosh(\beta \epsilon n \bar{s})$$

$$\bar{s}_1 = \frac{1}{Z_1}\Big[(+1)\, e^{\overbrace{-\beta E_\uparrow}^{\beta\epsilon n \bar{s}}} + (-1)\, e^{\overbrace{-\beta E_\downarrow}^{-\beta\epsilon n \bar{s}}}\Big]$$

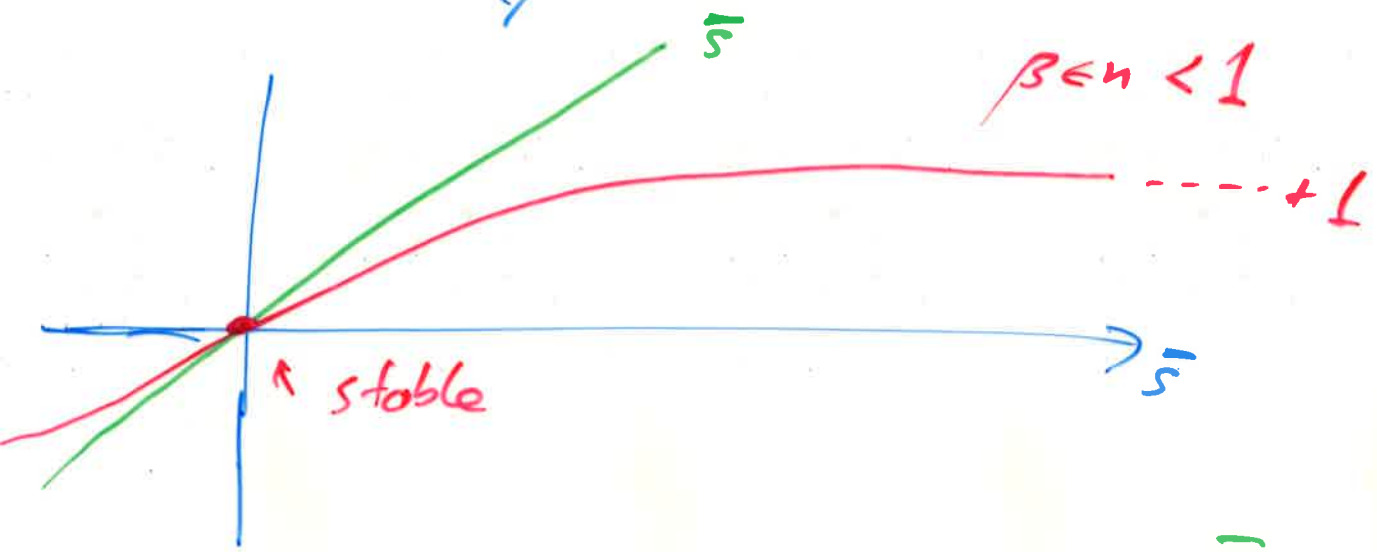
$s_1 = +1$ ↑ , $s_1 = -1$ ↓

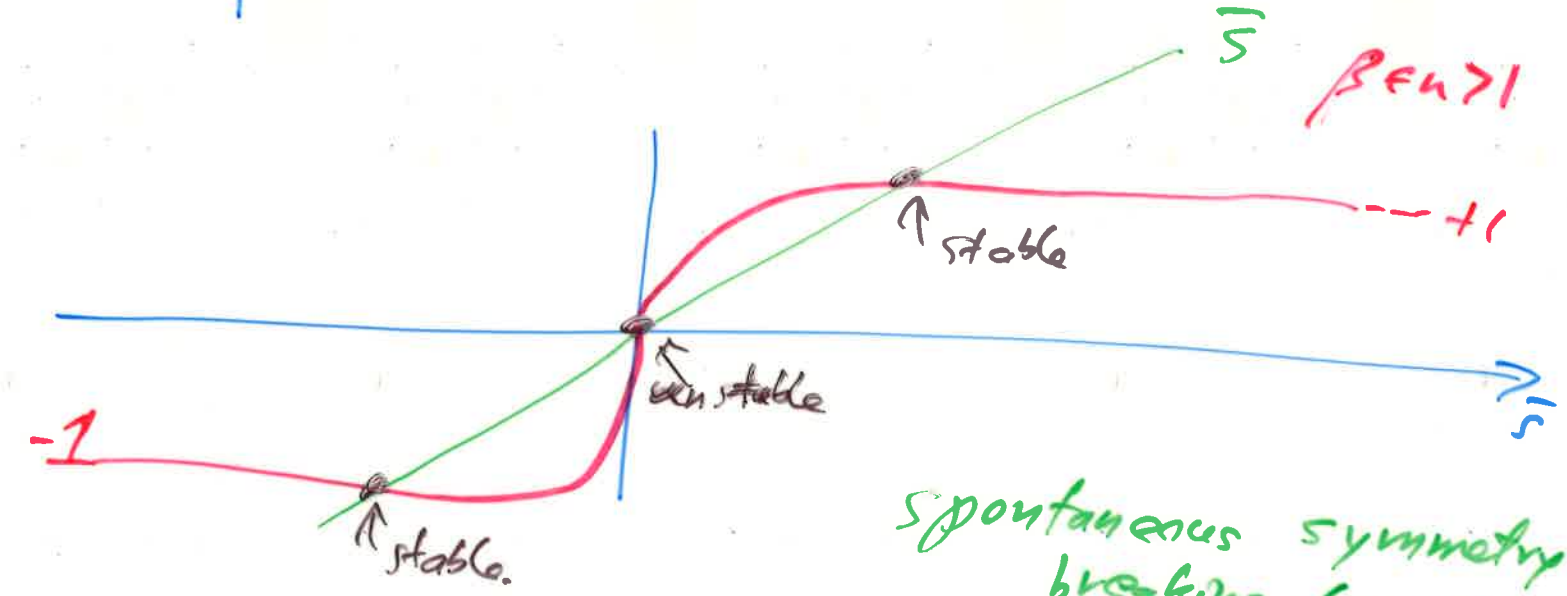
$$= \frac{2\sinh(\beta \epsilon n \bar{s})}{2\cosh(\beta \epsilon n \bar{s})} = \tanh(\beta \epsilon n \bar{s})$$

Mean Field Theory $\bar{s}_1 = \bar{s}$

$\bar{s} = \tanh(\beta \epsilon n \bar{s})$

slope $\tanh(mx) = m$





spontaneous symmetry breaking (SSB)

$k_B T_c = n \bar{s}$

1-dimension ($n = 2$) $\Rightarrow T_c = \cancel{\frac{2\epsilon}{k_B}}$ $\nexists$ many phase transition

$n = 2$ not high enough

2-dimension ($n = 4$) $\Rightarrow T_c = \frac{4\epsilon}{k_B}$

Energy required to flip large number of spins

2 broken bonds ($+4\epsilon$)

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$\frac{N(N-1)}{2}$ degeneracy

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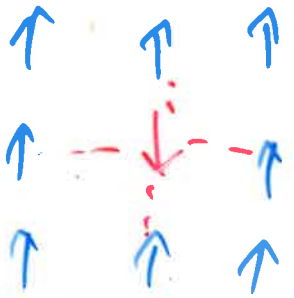
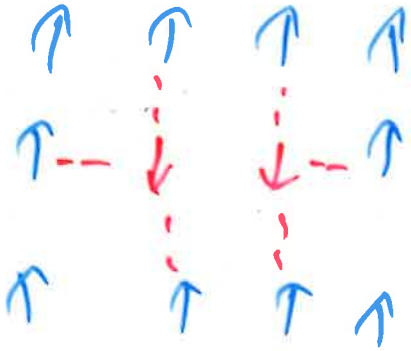
2 dimensions



4 broken bonds

$+4\epsilon$ energy cost

N degeneracy

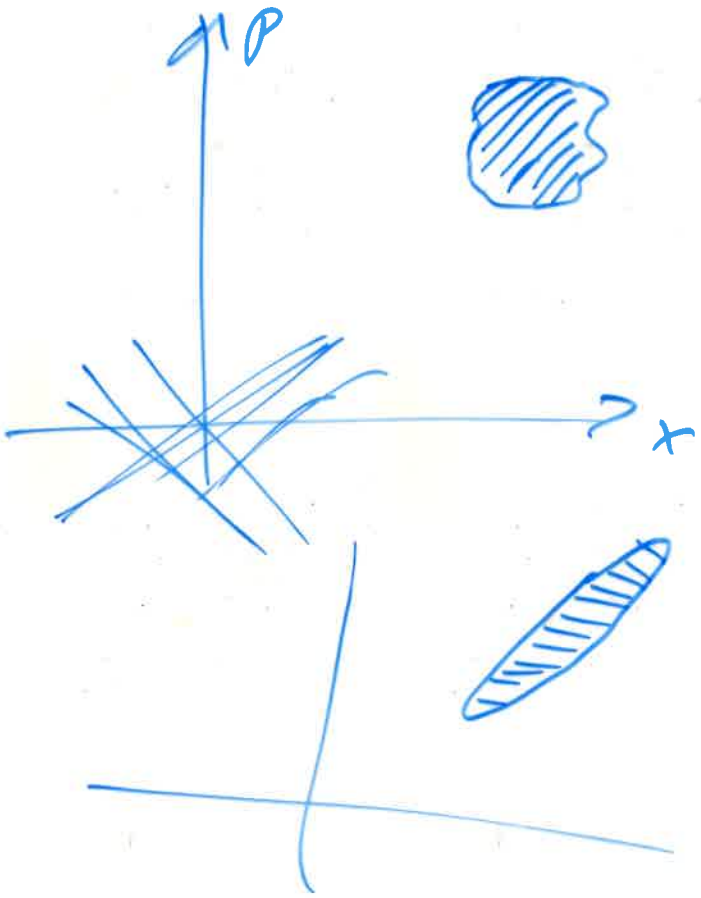


6 broken bonds

$+12\epsilon$

$2N$ degeneracy ?

Phase Space

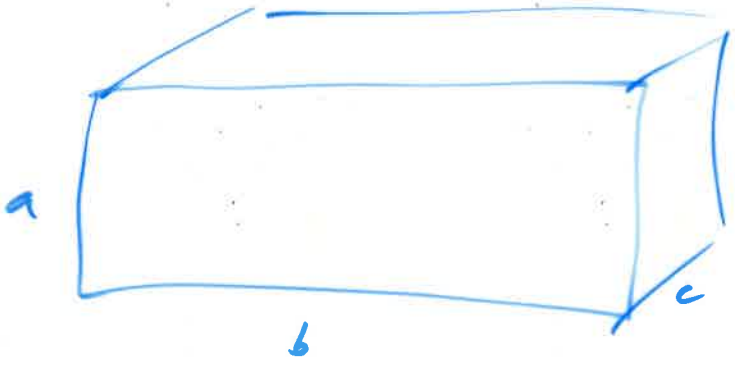


Liouville's Theorem

Hamilton's Equations of Dynamics

Fine-graining

Coarse-graining



mass m

$V = abc$

Particle in a box

$$V(x,y,z) = \begin{cases} 0, & 0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c \\ \infty, & \text{elsewhere} \end{cases}$$

$n, m, p = 1, 2, 3, \ldots$

$$\psi(x,y,z) = \sqrt{\frac{8}{abc}} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$$

$$\varepsilon_{n m_p} = \frac{\vec{p}^2}{2m} = \frac{\hbar^2 \vec{k}^2}{2m} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n^2}{a^2} + \frac{m^2}{b^2} + \frac{p^2}{c^2}\right)$$

$$k_i = \frac{2\pi}{\lambda_i} = \frac{2\pi n}{l_i 2} \qquad l_i = a, b, c$$