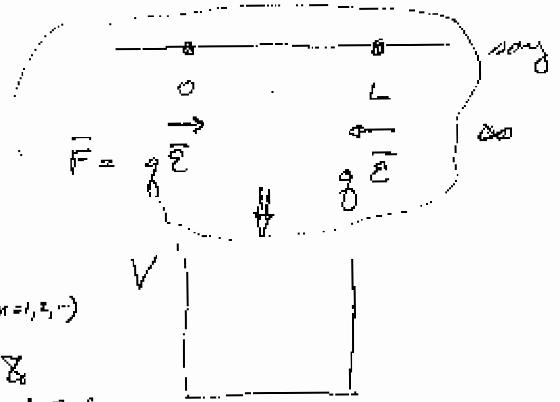
# CH 2 STATISTICAL DESCR. OF SYSTEMS OF PARTICLE

building from grnd up; *ex* sps want to know ave pressure when have a gas in box: (a) how can they be arranged (what p (b) how likely is each arrangement (c) what does each config contrib'e

## MICROSTATE: QM

BOX OF $e^-$'s: start simple, work up

(1) *ex* 1 $e^-$ IN BOX (1 d)




TO SPECIFY:

(a) SOLVE FOR STATES OF DEFINITE ENERGY $E_n$, $\psi_n(x)$ $(n=1,2,\ldots)$

$\uparrow$ soln to $t$ ind Schr eqn

$\Rightarrow$ GIVES ALLOWED $E_n$'s

– $t$ DEP: $\Psi_n(x,t) = \psi_n(x)e^{-iE_n t/\hbar}$ (standing wave)

(b) INT $n$ SPECIFIES STATE EXACTLY

– IF START IN STATE OF DEF. $E_n$, STAYS THERE

– $P(x) = |\Psi_n(x,t)|^2 = |\psi_n(x)|^2$ IND. OF $t$

/ save for problems:

– IF START IN STATE OF MIXED $E_n$

$$\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t) + \cdots$$

FIND

$$\bar{E} = |c_1|^2 E_1 + |c_2|^2 E_2 + \cdots$$

$$\overline{E^2} = |c_1|^2 E_1^2 + |c_2|^2 E_2^2 + \cdots$$

(2) (MORE REALISTIC)

1 $e^-$ IN 3d BOX

RECALL

$$\psi_{n_x n_y n_z}(\vec{r}) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z) \quad (t \text{ ind})$$

$\equiv \vec{n}$ (3 COMP. "VECT")

THEN $E_{\vec{n}} = E_{n_x} + E_{n_y} + E_{n_z} \Rightarrow \Psi_{\vec{n}}(\vec{r},t) = \psi_{\vec{n}}(\vec{r})e^{-iE_{\vec{n}}t/\hbar}$

IF INCLUDE SPIN: (assume doesn't play role in E for now)

$S = \frac{1}{2}$ FOR $e^-$, $S^2 = s(s+1)\hbar^2$

$S_z = \pm \frac{1}{2}\hbar$ (~UP OR DN)

TO SPECIFY STATE: $\vec{n}, S_z \equiv (n_x, n_y, n_z, S_z)$

"QUANTUM NUMBERS" (can be discrete or continuous)(ex $p_{x,y,z}$ for free particle)
(depend on details of problem)

CAN ENUMERATE: ASSIGN INT $\ell$ TO EACH

| STATE | $\ell$ |
|---|---|
| $(1,1,1,+\frac{1}{2}\hbar)$ | 1 |
| $(1,1,1,-\frac{1}{2}\hbar)$ | 2 |
| $(1,1,2,+)$ | 3 |
| $(1,1,2,-)$ | 4 |
| $(1,2,1,+)$ | 5 |
| ⋮ | ⋮ |

} LISTS 1 PARTICLE STATES

(3) $N$ $e^-$'s

TO SPECIFY:

LET $\nu_\ell$ = # $e^-$'s IN STATE $\ell$

FERMIONS: $\nu_\ell = 0$ OR $1$ (Pauli exclusion principle)

(BOSONS → ANY #)

ex

| | | | | | | |
|---|---|---|---|---|---|---|
| $\ell$: | 1 | 2 | 3 | 4 | 5 | ... |
| $\nu_\ell$: | 0 | 1 | 1 | 0 | 1 | .... |

$\left\{ \sum_{\ell=1}^{\infty} \nu_\ell = N \right.$

COMPLETELY SPECIFIES $N$-$e^-$ STATE

CAN BE VERY PERVERSE & ASSIGN LABEL $r$ FOR EVERY N-PARTICLE STATE : ( eg $N = 3$ )

| $\ell$: | 1 | 2 | 3 | 4 | 5 | ... | $r$ |
|---|---|---|---|---|---|---|---|
| $\nu_\ell$'s: | 1 | 1 | 1 | 0 | 0 | ... | 1 |
| | 1 | 1 | 0 | 1 | 0 | ... | 2 |
| | 1 | 1 | 0 | 0 | 1 | ... | 3 |
| | | | ⋮ | | | | ⋮ |

TOTAL STATE LABEL

UNPACK: MICROSTATE $r = 2$ :

1 $e^-$ IN $\ell = 1$ $\Rightarrow$ $(\vec{n}, S_z) = (1,1,1,+)$

1 $e^-$ IN $\ell = 2$ $\Rightarrow$ $(1,1,1,-)$

" " $\ell = 4$ $(1,1,2,-)$

( $\therefore$ TO SPECIFY STATE W/ $N = 3$ $\Rightarrow$ 12 QM #S )

$E_{r=2} = E_{111} + E_{111} + E_{112}$

$\psi_{r=2}$ NOW FN OF $\vec{r}_1, \vec{r}_2, \vec{r}_3$

(4) EVEN MORE REALISTIC: (than we want to be)

INCLUDE $V'(\vec{r}_i)$ WHICH GIVES COUL. REPULSION

FELT BY $e^-$'s: $V'(\vec{r}_1-\vec{r}_2)+V'(\vec{r}_1-\vec{r}_3)+\cdots = \frac{e^2}{|\vec{r}_1-\vec{r}_2|}+\cdots$

— ~~RELATIVELY~~ WEAK COMPARED TO WALL, ESP IF $e^-$'s NOT DENSE

- IF WORK HARD, CAN FIND NEW SOLNS TO SCHR. EQN w/ $V + V'$

USUALLY OVERKILL (& TOO HARD) (& NEVER HOPE TO INCLUDE ALL $V'$'s)

INSTEAD:

(a) USE PREVIOUS STATES AS BASIC DESCRIPTION
ie FIND SIMPLE approx SOLN w/ BASIC FEATURES
(if solved exact would get $\psi = \sum_n c_n \psi_n$!)

(b) INCLUDE EFFECT OF $V'$ AS CORRECTION (PERTURBATION

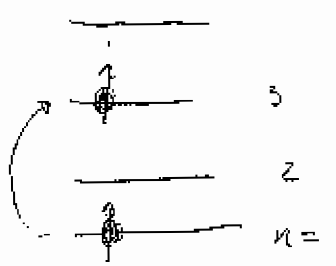
— EXIST STD TOOLS TO DO

— ALL WE NEED TO KNOW:

(i) PERTS. CAN SHIFT $E_{\vec{n}}$ LEVELS (& ∴ $E_r$) {here? Coul. repulsion → E's higher

(ii) CAN CAUSE TRANSITIONS BETWEEN STATES
→ NO LONGER STA. STATES

1 $e^-$:
id




(NOTE: IN ATOM, MEASURE E LEVELS BY INT. OF $e^-$ w/ $\gamma$'s → EMIT AT CERTAIN $\omega$ → MEAS.)

(WILL GET TO EVERY POSSIBLE STATE eventually)

{other perturbations ⇒
• wall moves
• radiation from $e^-$'s

N - PARTICLE: TRANSITIONS FROM $r_1$ TO $r_2$

ie → NO LONGER STATIONARY STATES;
IF START IN $r_1$ AND COMPUTE t DEVEL. EXACTLY,
FIND AT LATER t SOME PROB. OF BEING IN OTHER STATES
⇒ PERTS. MAKE SYSTEM MOVE AROUND → CAN SPEND TIME IN ALL STATES!

## CLASSICAL STATES:

SOMETIMES OK (QM ALWAYS OK)

*ex* N $e^-$'S IN BOX BUT DIFFUSE, w/ $E \gg \Delta E$'s

- CAN DESCRIBE PARTICLES AS LOCAL WAVE FNS WHICH MOVE AS "CLASSICAL" (almost)
- DON'T RUN INTO PROBLEM w/ EXCLUSION PRINCIPLE OR UNCERTAINTY RELN ~~$\Delta p \Delta x \gtrsim \hbar$~~ IF DIFFUSE

(OFTEN HARDER TO USE; LEADS TO PARADOXES → FUND. INCONSISTENC
(3-BODY; GIBBS PARADOX)

(1) 1 PARTICLE, 1d

TO SPECIFY, NEED

(a) POSITION $x$

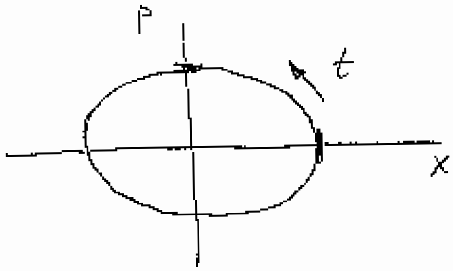
(b) MOMENTUM $p = m\dot{x}$ (NON REL)

} given this, Newton's laws tell what happens next

1 "DEGREE OF FREEDOM" = # $(x,p)$ PAIRS NEED TO SPECIFY = # DIRECTIONS SYS CAN MOVE

{= # QM #'S NEEDED FOR QM SYS.}

CAN REP. STATE AS PT MOVING IN $(x,p)$ SPACE ≡ "PHASE SPACE"




*ex* HARM OSC (MASS ON SPRING)

say $\begin{cases} x = x_{MAX} & p = 0 \\ x = 0 & p = p_{MAX} \end{cases}$

$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$ ⇒ ELLIPSE

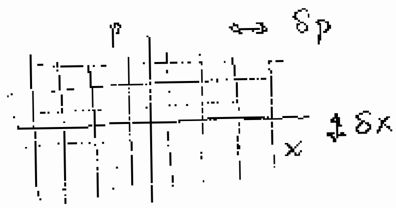
(CONST)

AT TIME $t$, "POSITION" SPECIFIES STATE OF SYSTEM

ENUMERATE STATES:

- CHOP INTO CELLS

(PRACTICAL: NEVER MEAS. $x, p$ w/ $\infty$ PRECISION)




AREA

$\delta p \, \delta x \equiv h_0$

(UNITS: ANG. MOM)

- FUND. LIMIT: $h_0 > \hbar$ (no pt. going smaller)
  $\Rightarrow$ HEIS. UNC. RELN.

$\rightarrow$ CAN NOW <u>ENUMERATE</u>: LABEL CELLS $r = 1, 2, \dots$
STATE $\equiv$ WHICH CELL

7)

(2) <u>1 PARTICLE, 3 d</u>

SPECIFY $\vec{r} = (x, y, z)$, $\vec{p} = (p_x, p_y, p_z)$ } 3 DOF.

PHASE SPACE: 6 D     STATE = 1 PT MOVING THRU 6 D SPACE

CELLS: VOL = $\delta x\ \delta y\ \delta z\ \delta p_x\ \delta p_y\ \delta p_z$

(3) <u>N PARTICLES:</u>

$\vec{r}_1 \cdots \vec{r}_N, \ \vec{p}_1 \cdots \vec{p}_N$     3N DOF

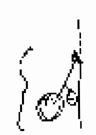
STATE $\equiv$ 1 PT. IN 6N DIM PHASE SPACE
(ie SPECIFY STATE AT $t$ BY PT. IN ")

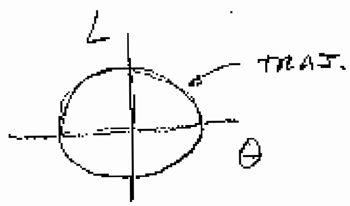
<u>GENERALIZED COORDS</u> – (cf LAGRANGIAN MECHANICS)

OFTEN CONVENIENT TO USE COORDS OTHER THAN CARTESIAN $\vec{r}$; (EX $\theta$ FOR PENDULUM)

FOR EACH, HAVE CORRESPONDING GEN. p
(EX FOR $\theta$, ANG. MOM  CORRESP. TO ROT. IN $\theta$)

NOTATION: $q_i, \ p_i$




PHASE SPACE FOR PENDULUM 


<u>f DOF</u>

STATE: $(q_1, \dots, q_f, p_1, \cdots p_f)$

$\left\{ E \sim \frac{L^2}{2I} + \frac{mgl}{2}\theta^2 \right\}_{\theta \text{ small}}$

## STATISTICAL ENSEMBLE:

{instead of specifying exactly what system is doing, list all things it could be doing, what's prob. of each, use to predict aves}

LARGE # OF PARTICLES / MANY DOF ⇒

- MICRO DESCRIPTION NOT PRACTICAL, USEFUL
- WILL DESCRIBE W/ AVE MACRO VARIABLES ($E$, VOL $V$, PRESSURE $p$, MAGN. MOMENT $M$, DENSITY $\rho$ ...)

SETTLE FOR AVE VALUES

### COMPUTING AVES:

(1) IDENTIFY CONSTRAINTS or KNOWN CONDITIONS: MACRO VARS. WITH KNOWN VALUES

- MEAS'BLE OR CONTROLLABLE (ex $E$ or $V$)
- SMALL SUBSET OF WHAT'S KNOWABLE

ACCESSIBLE STATE: MICRO STATE $r$ CONSISTENT WITH CONSTRAINTS ⇒ LOTS

(2) CREATE STATISTICAL ENSEMBLE: (mental construct)

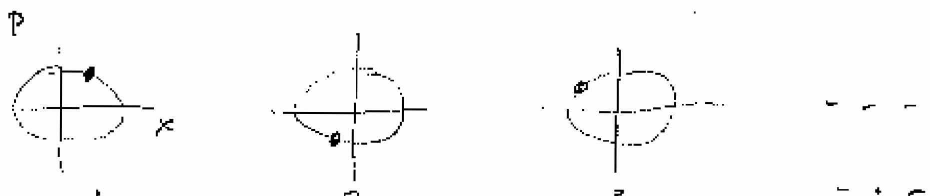
LARGE SET OF SIMILAR SYSTEMS IN ACCESS. STATES {ie prepared same way}

(ie MAKE COPIES OF SYST. ~~OF INTEREST~~, IF JUST IN HEAD)

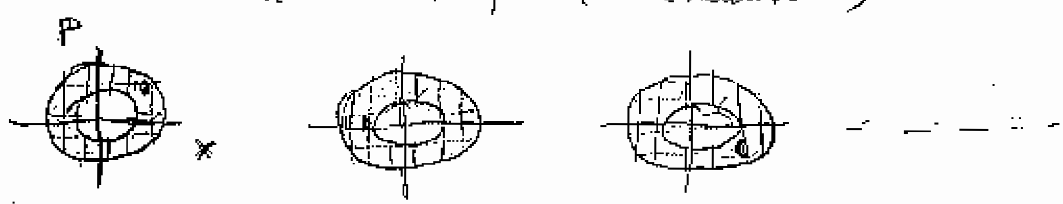
more often same system looked at repeatedly

ex HARM OSC (mass on a spring) (1d) (CLASSICAL)

(a) IF KNOW $E$, ALLOW/EXPECT




(b) IF KNOW $E_a < E < E_b$, (more realistic)




big enough so that all ~~acc~~ access states can occur few ti...

STATE FOR EACH: GIVEN BY CELL #

start them out randomly → let them go

} think about taking 100 harm. osc. randomly out of box (not careful about x, p), put on table ex MAGN MOMENTS — ILLUSTR. IN TXT pg 53

(a) KNOW 3 SPIN $\frac{1}{2}$ PARTICLES, IN PLACE $\Rightarrow$ ONLY ~~DOF~~ QM # IS SPIN DIR.
(DIFF PLACES $\rightarrow$ DON'T WORRY ABOUT EXCL. PRIN.)

IN MAGN FIELD $\vec{B} = B\hat{z}$

say { DEFINE SPIN COMP. ALONG $\hat{z}$ $\Rightarrow$ $S_z = \pm\frac{1}{2}\hbar$

MAGN MOM ALONG $\hat{z}$ $\propto S_z$ : $\mu_z = \pm\mu$ (takes 2 values)

$E = -M_z B$ { $M_z$ = TOTAL MOMENT } (constraint: 3 parts.)

ACCESSIBLE STATES : ENTIRE TABLE

(b) IF $E = -\mu B$ FIXED $\Rightarrow$ of TABLE

WITH $t$, EACH STATE WILL MOVE TO OTHER STATES (from small perts, $\rightarrow$ dipole ints)

(3) $P(r,t)$ = PROB OF BEING IN STATE $r$ AT $t$

$$\equiv \frac{\#\ \text{STATES IN } r}{N_{SYS}} \quad \text{IN ENSEMBLE AT } t$$

{ no assumption yet; just measure in ensemble

(4) AVE MAC. VARS. (ex $E$, case (a))

$$\bar{E}(t) = \sum_r P(r,t)\, E_r$$

(sometimes I know $E$, sometimes just as ave)

(5) EQUILIBRIUM :

$$P(r,t) = P(r) \quad (\text{IND. OF } t)$$

$$\bar{E},\ \bar{p},\ \ldots \quad \text{"} \quad \text{"}$$

$\Rightarrow$ EVEN THO INDIVID. SYSTEMS BOUNCE AMONG ACCES. STATES

Can now do measurements of aves.

$\Rightarrow$ MACRO SYSTEM, AVES $\Rightarrow$ ALWAYS HAVE IN MIND ENS. OF SIMILAR SYSTEMS

conf

2.8.5



| State index $r$ | Quantum numbers $m_1, m_2, m_3$ | Total magnetic moment | Total energy |
|---|---|---|---|
| 1 | + + + | $3\mu$ | $-3\mu H$ |
| 2 | + + − | $\mu$ | $-\mu H$ |
| 3 | + − + | $\mu$ | $-\mu H$ |
| 4 | − + + | $\mu$ | $-\mu H$ |
| 5 | + − − | $-\mu$ | $\mu H$ |
| 6 | − + − | $-\mu$ | $\mu H$ |
| 7 | − − + | $-\mu$ | $\mu H$ |
| 8 | − − − | $-3\mu$ | $3\mu H$ |
| r | QM #'s | M | E |

$(H \equiv B)$

One usually has available some partial knowledge about the system under consideration. (For example, one might know the total energy and the volume of a gas.) The system can then only be in any of its states which are compatible with the available information about the system. These states will be called the "states accessible to the system." In a statistical description the representative ensemble thus contains only systems all of which are consistent with the specified available knowledge about the system; i.e., the systems in the ensemble must all be distributed over the various accessible states.

***Example*** Suppose that in the previous example of a system consisting of three spins the total energy of the system is known to be equal to $-\mu H$. If this is the only information available, then the system can be in only one of the following three states:

$$(+ + -), \quad (+ - +) \quad (- + +)$$

Of course, we do not know in which of these states the system may actually be, nor do we necessarily know the relative probability of finding the system in any one of these states.

Can we make predictions? (sure, could compute motion of each system; crazy)

BASIC POSTULATE:

$$\boxed{\text{ISOLATED SYSTEM IN EQUIL : ALL ACCESS. STATES EQUALLY LIKELY}}$$

- REASONABLE, DEMOCRATIC

(FOR BOTH CLASSICAL & QM)

- THM: IF $P(r)$ EQUAL FOR ALL $r$, STAYS EQUAL (ie IF IN EQUIL., STAYS THERE)

QM: USES RATE AT WHICH STATE GOES FROM $r \to r'$
= " " " " $r' \to r$
(detailed balance)

CLASSICAL: REQUIRES USING CELLS IN $q_i$ & $p_i$ (NOT $v_i$) TO SPECIFY STATE; THEN IF ENS. IS UNIF. SPREAD IN PHASE SP., STAYS UNIF. (LIOUVILLE THM)

(cf APPENDICES)

ex SPIN SYSTEM, $E = -\mu B$, $N = 3$

STATES: $+ + -$ $\Rightarrow$ $P = 1/3$
$+ - +$ $1/3$
$- + +$ $1/3$

* INSERT NEXT PG

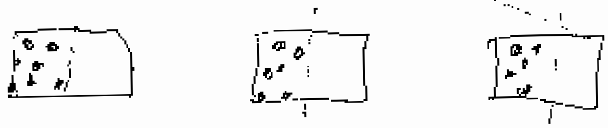
skip

APPROACH TO EQUIL:

SPS START WITH ONLY SUBSET OF POSSIBLE STATES REP. IN ENSEMBLE

- MORE PROBABLE THAT INDIVID. SYSTEMS IN ENS. WILL LEAVE SUBSET THAN ENTER

ex.



$t = 0$

LET GO: $L \to R$ MORE PROB THAN $R \to L$

CAN MAKE PREDICTIONS:

*ex* SPIN SYSTEM (cf OVERHEAD)

IN EQUIL, WHAT'S PROB SPIN 1 IS + ?

(a) $N=3$, $E=-\mu B$

STATES: + + − $P=\frac{1}{3}$ (all equal)

\+ − + $\frac{1}{3}$

− + + $\frac{1}{3}$ $\Rightarrow$ $P(1 \text{ is } +) = \frac{2}{3}$

ALSO $\overline{S_z(1)} = \frac{1}{3}(+\frac{\hbar}{2}) + \frac{1}{3}(+\frac{\hbar}{2}) + \frac{1}{3}(-\frac{\hbar}{2}) = \frac{1}{3}(\frac{\hbar}{2})$

(b) $N=3$, ANY $E$

say $\Big\{$ ⇒ EQUAL # MICRO STATES w/ + & −
EACH EQUALLY LIKELY (FUND POST) ⇒ $P(r)=\frac{1}{8}$

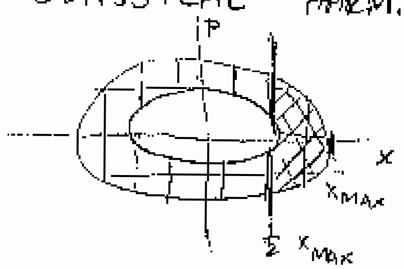
∴ $P(1 \text{ is } +) = \frac{1}{2}$

ALSO $\overline{E} = 0$

IN GENERAL: IN EQUIL

$$\boxed{P(\text{condition } y) = \frac{\#\text{ ACCES. STATES w/ } y}{\#\text{ ACCES. STATES}}}$$

*ex* CLASSICAL HARM. OSC:




PROB $x > \frac{1}{2}x_{MAX}$ ?

$$= \frac{\#\text{ CELLS w/ } x > \frac{1}{2}x_{MAX}}{\#\text{ CELLS}}$$

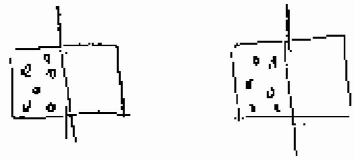
$$= \frac{\text{SHADED AREA}}{\text{ENTIRE AREA}}$$

Ensemble defines P
think how P behaves for
sys. not in equil:

say

WHY "IN EQUIL"?

CONSIDER



⇒ RELEASE WALL

(1) NOT ALL ACCESS STATES OCCUPIED
w/ EQUAL # IN ENS.

(2) $P(r,t)$ CHANGES w/ $t$

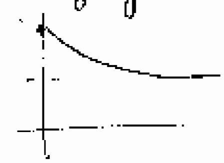
{ STILL DEFINE SAME WAY, IN TERMS OF ENSEMBLE

BUT

$t > t_{RELAX}$

$P(r,t) \rightarrow P_{EQUIL}(r)$

{ could use to compute curve g(t)
ex: DENSITY $\rho(t)$



POST: SAME FOR ALL $r$ (∴ CONST)

COMPLICATED

⇒ FUND. POSTULATE WHICH MAKES ENSEMB. USEFUL:
TURNS COMPUTING P INTO SIMPLER PROB. OF
ENUM. MICRO STATES.

WILL RESTRICT TO EQUIL. (EVEN WHEN CHG. → DO IT SLOWLY)

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say

ENERGY:

- MOST COMMON CONSTRAINT
- CONSERVED: LIMITS WHAT'S POSSIBLE → BIG ROLE IN ENUM. MICROSYS'S
- WILL STUDY GEN. INT'S AMONG SYS'S ⇒ INVOLVE EXCH. OF E (this is, after all, thermody
- PRACTICAL: HOW TO GET E FROM MACRO SYS

⇒ VERY IMP. TO KNOW HOW ACCESS. STATES CHG w/ E

{ WILL BE VERY INTERESTED IN HOW P, AVE QTYS CHANGE w/ E. FIRST OBS: # ACCESS STATES GROWS RAPIDLY w/ E }

## GROWTH IN # STATES w/ E:

- LOOKED AT LOW END IN HW w/ HARM. OSC
- MANY PARTICLES, DOF $f$ → EXTREME GROWTH
  (MACRO SYSTEM: $f \sim 10^{24}$)

ASSUME KNOW SYS. HAS $E$ TO $E + \delta E$
≈ ACC OF MEAS. { SMALL ON MACRO SCALE, LARGE ON MICRO

$\Omega(E) \equiv$ # STATES FROM $E$ TO $E + \delta E$
(ie # CONFIGS CONSIST. w/ THIS $E$)

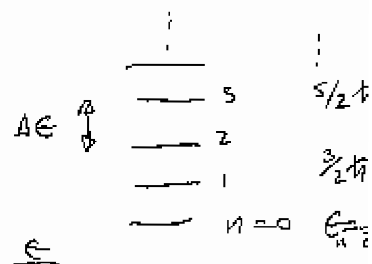
ESTIMATE (CRUDE) (QM)

1ST COUNT $\Phi(E) \equiv$ # CONFIGS w/ $E$ OR LESS } easier

ex HARM OSC.

1 OSC, 1 D $\qquad \epsilon_n = (n + \frac{1}{2})\hbar\omega$

$\Delta\epsilon = 1 \cdot \hbar\omega$




IF HAVE $\epsilon$ OR LESS, # STATES $= \frac{\epsilon}{\Delta\epsilon}$

$f$ OSCILLATORS: (ie $f$ PARTICLES ON 1d SPRINGS
or $f/3$ " " 3d " )
{MACRO: $f \sim 10^{24}$}

$E$ OR LESS:

PER OSC (≈ PER DOF):

NEGLECTING CASES WHERE MOST E CONCENTRATED IN FEW OSCILLATORS

$\epsilon \sim \frac{E}{f}$ (could be more, but starts to severely limit other oscillators, doesn't add much more to total; ie most states occur with E ~ equally distributed

⇒ # STATES PER OSC: $\frac{\epsilon}{\Delta\epsilon}$

TOTAL:

$$\phi(E) \sim \left(\frac{\epsilon}{\Delta\epsilon}\right)^f \sim \left(\frac{E}{f\Delta\epsilon}\right)^f$$

$$\Omega(E) = \phi(E+\delta E) - \phi(E)$$

$\delta E \ll E\}$

$$\sim \frac{\partial\phi}{\partial E}\delta E = f\, E^{f-1}\left(\frac{1}{f\Delta\epsilon}\right)^f \delta E$$

$$= \left(\frac{E}{f\Delta\epsilon}\right)^{f-1}\left(\frac{\delta E}{\Delta\epsilon}\right)$$

GROWS RAPIDLY w/ f

$\Rightarrow$ USE LOG:

$$\ln \Omega(E) = (f-1)\ln\left(\frac{E}{f\Delta\epsilon}\right) + \ln\left(\frac{\delta E}{\Delta\epsilon}\right)$$

EST: $\frac{\delta E}{\Delta\epsilon}$ CAN BE LARGE ($\delta E$ is macro)

CERTAINLY $\lesssim f$

($= f \Rightarrow$ resol. $\delta E$ would be equiv. to all osc. jumping one level together $\to$ big) (ex H atom: 13eV $\times 10^{24}$)

$\frac{E}{f\Delta\epsilon} \sim \frac{\epsilon}{\Delta\epsilon}$ LEVEL EACH CAN REACH

$\sim$ FEW; (CAN BE LARGE, BUT NOT $f$)

(in typical system at normal temps, only a small fraction excited)

NOTE $f \ggg \ln f$ ($10^{24}$ vs $55$)

ex

Reif uses 3 of these

$$\Rightarrow \ln\Omega(E) \approx f\ln\left(\frac{E}{f\Delta\epsilon}\right)$$

OR $\boxed{\Omega(E) \propto E^f}$ EXTREMELY USEFUL RULE OF THUMB $\to$ USE REPEATEDLY

(DOESN'T DEP. ON HARM. OSC) want E dependence

{ NOTE: FOR SMALL $\delta E$

$\Omega(E) \propto \delta E$ (i.e. ~~$\Omega(E)$ APPROX CONST FROM $E$ TO $E+\delta E$~~)

MAKE EXPLICIT:

$$\boxed{\Omega(E) \equiv \omega(E)\,\delta E}$$

(both are omegas)

$\omega(E)$ $\equiv$ DENSITY OF STATES

SO $\omega(E) \propto E^f$ ALSO }

## REPEAT FOR CLASSICAL SYSTEM : MONATOMIC IDEAL GAS

N PARTICLES

IN GENERAL :

$$E = K + \mathcal{U}(\bar{r}_1, \bar{r}_2 \cdots \bar{r}_N) + E_{INTERNAL}$$

KINETIC :

1 PARTICLE : $\frac{1}{2} m \bar{v}^2 = \frac{\bar{p}^2}{2m} = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$

N " : $\frac{1}{2m} \sum_{i=1}^{N} \bar{p}_i^2 = \frac{1}{2m} \sum_{i=1}^{N} (p_{ix}^2 + p_{iy}^2 + p_{iz}^2)$

3N TERMS

POTENTIAL:

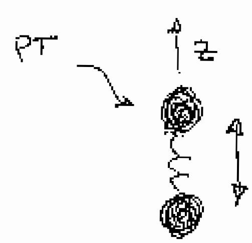
BETWEEN PARTICLES

ex COULOMB : $\frac{e^2}{|\bar{r}_1 - \bar{r}_2|} + \cdots$ FOR EACH PAIR

HERE IDEAL GAS : IGNORE (DIFFUSE)

INTERNAL:

ex DIATOMIC




$$E_{INT} = \frac{1}{2} I (\omega_x^2 + \omega_y^2) + \frac{p^2}{2\mu} + \mathcal{U}(z)$$

rel. momentum

(3 DOF)

HERE: MONATOMIC $\rightarrow$ NO $E_{INT}$

REPEAT FOR CLASSICAL SYSTEM : MONATOMIC IDEAL GAS

N PARTICLES

IN GENERAL :

$$E = K + \mathcal{U}(\bar{r}_1, \bar{r}_2 \cdots \bar{r}_N) + E_{INTERNAL}$$

KINETIC :

1 PARTICLE : $\frac{1}{2} m \bar{v}^2 = \frac{\bar{p}^2}{2m} = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$

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3N TERMS

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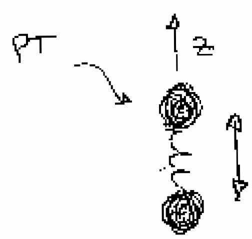
HERE IDEAL GAS : IGNORE (DIFFUSE)

INTERNAL :

ex DIATOMIC




rel. momentum

$$E_{INT} = \frac{1}{2} I (\omega_x^2 + \omega_y^2) + \frac{p^2}{2\mu} + \mathcal{U}(z)$$

(3 DOF)

HERE : MONATOMIC → NO $E_{INT}$

COUNT STATES w/ $E$ TO $E + \delta E$

$$\Omega(E) = \#\ \text{CELLS}$$

$$= \frac{1}{h_0^{3N}} \int_{E \text{ TO } E+\delta E} d^3x_1 \cdots d^3x_N \; d^3p_1 \cdots d^3p_N$$

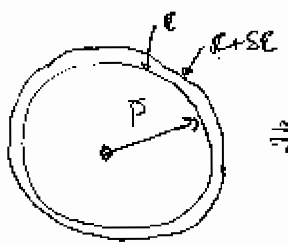
($\cancel{f = 3N}$ ; $h_0 = \delta x \, \delta p$ ; $h_0^{3N}$ = CELL SIZE IN 3N-DIM SPACE)

1. $E$ RESTRICTS $p$, NOT $x$ (BECAUSE $U = 0$)

$$\Omega(E) = \frac{1}{h_0^{3N}} V^N \int_{\delta E} d^3p_1 \cdots d^3p_N$$

2. $$E = \frac{1}{2m} \underbrace{\sum_{i=1}^{N} \left( p_{ix}^2 + p_{iy}^2 + p_{iz}^2 \right)}$$

$(\text{MAG})^2$ OF N 3D VECTS = $(\text{MAGN.})^2$ OF 1 3N-DIM. VECTOR $\bar{P}$ (abstract)




$$\Rightarrow \quad |\bar{P}| = (2mE)^{\frac{1}{2}}$$

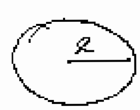
EASIER: (as before)

$$\phi(E) = \# < E$$

$$= \frac{1}{h_0^{3N}} V^N \left[ \text{VOL. OF "SPHERE" IN 3N-DIM w/ RAD. } |\bar{P}| = (2mE)^{\frac{1}{2}} \right]$$

NOTE:

1d  VOL = $R$

2d  $\pi R^2$

3d  $\frac{4\pi}{3} R^3$ - - -

(∃ simple way to get $C(D)$ for any $D$ by eq. gaussian int. in spher. vs cartesian coords)

GENERAL: $C(D)R^D$ IN D-DIM.

$$\Rightarrow \quad \Phi(E) \propto V^N |\bar{p}|_{MAX}^{3N} \propto V^N E^{(3N/2)}$$

THEN

$$\Omega(E) \sim \frac{\partial \Phi}{\partial E}\,\delta E \propto \left(\frac{3N}{2}\right) V^N E^{(3N/2)-1}\,\delta E$$

$N \gg 1$

$$\boxed{\Omega_{IG}(E) \propto V^N E^{3N/2}}$$

WILL SEE – ALL OF IDEAL GAS LAW HERE + MORE

RECALL $f = 3N$, SO $\boxed{\Omega(E) \propto E^{\frac{1}{2}f}}$

again ($\frac{1}{2}$ not important)