# First Law: Conservation of Energy

Noether's theorem

Continuous symmetry → Conserved "charge"

Time translation invariance → Conserved Energy.

If we do not allow heat exchange, nor particle exchange

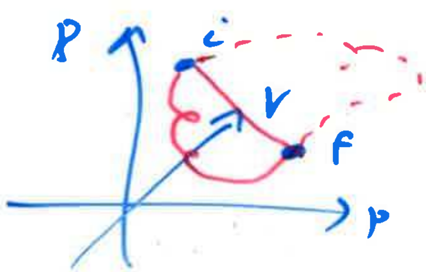
$$dU = dW \longrightarrow U = W + \text{const}$$

mechanical work

$$U = U(J_i, \xi_i)$$

$$= U(P, V, F_{tension}, x, \dots)$$

independent of path → depends only on endpoints




$$\Delta U = U(P_f, V_f, F_{t_f}, x_{f}\dots) - U(P_i, V_i, F_{t_i}, x_i \dots)$$

With heat exchange

$$\underbrace{dU}_{\text{depends on state}} = \underbrace{\underbrace{\text{đ}W}_{\text{depends on path}} + \underbrace{\text{đ}Q}_{\text{depends on path}}}_{\text{depends on state}} + \sum_i \mu_i dN_i$$

$dW$ can take many forms:

In general $dW = \sum_i J_i \, d\xi_i$ ← generalized forces ($J_i$); generalized displacements ($d\xi_i$)

| | $J$ | $d\xi$ | |
|---|---|---|---|
| spring | $F_{\text{tension}}$ | $dx, ds, dl$ | |
| surface | $\sigma$ (surface tension) | $dA$ (area) | |
| gas | $-P$ | $dV$ ← work done on the gas | |
| magnet | $B$ ← magnetic field | $dM$ ← magnetization | |
| chemical | $\mu$ | $dN$ | — "chemical work" |
| thermo | $T$ | $? = dS$ | "thermal work" heat $dQ$ |

not our book: $\boxed{\text{some books } dW = PdV, \ dU = -dW + dQ}$

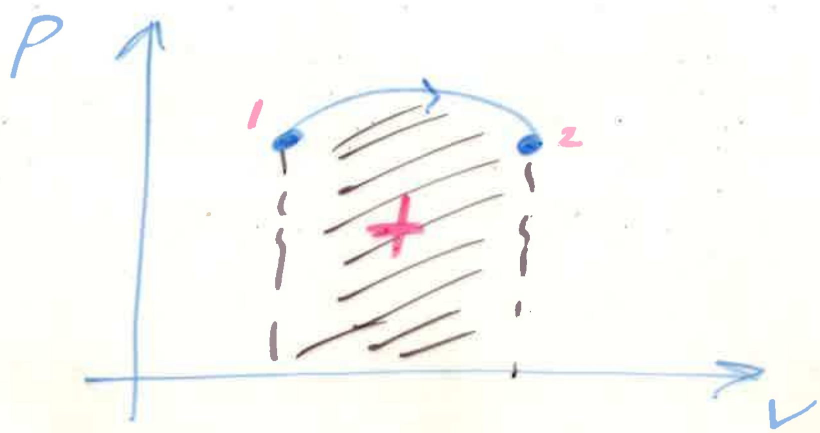
intensive ($J$): constant if the system gets 2x big.

extensive ($d\xi$): double if the system gets twice as large.

$\frac{ext_1}{ext_2} = int$ | $ext_1 \cdot int = ext_2$ | $ext_1 \cdot ext_2 = \underline{\text{wrong!}}$

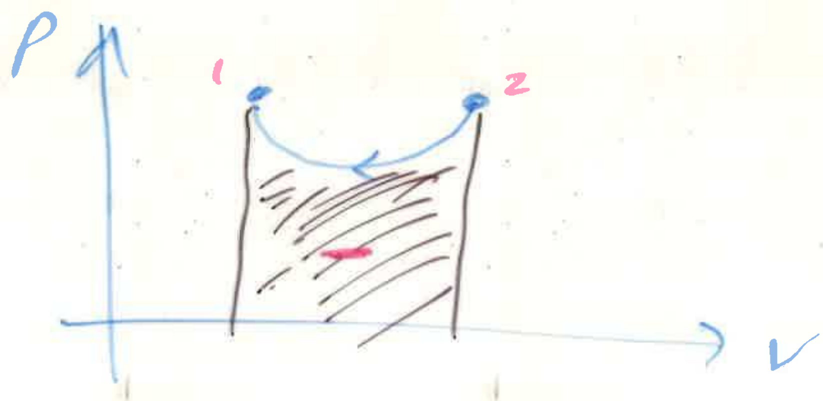
e.g. $\frac{M}{V} = \rho$

# Cycle for engine — 2 strokes

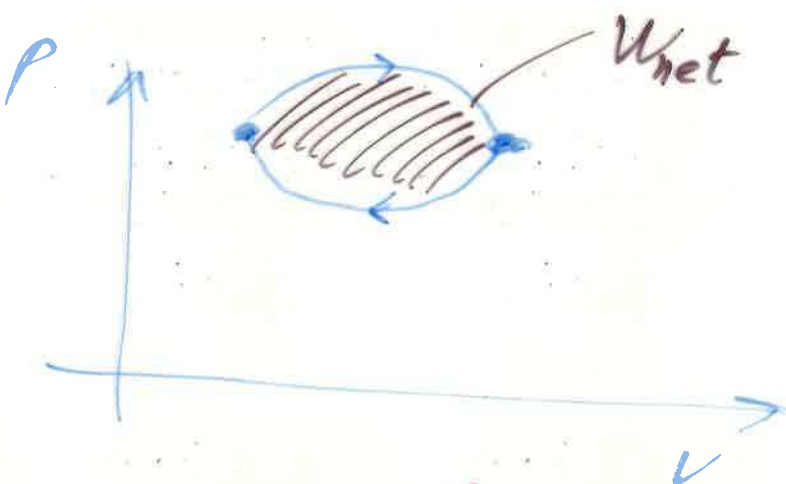



$$\int_1^2 dW = \int_1^2 P\,dV = W_{12}$$

$$\int_1^2 dU = U(2) - U(1)$$




$$\int_2^1 dW = \int_2^1 P\,dV = W_{21}$$




$$\oint dW \neq 0$$

In contrast $\oint_{1\to2\to1} dU = U_{(1)} - U_{(1)} = 0$

U is a <u>state</u> function (The system <u>has</u> internal energy U).

e.g. $W(A,B) = A^2B^2 + 2A$

↑ state function

$$dW = \left(\frac{\partial W}{\partial A}\right)_B dA + \left(\frac{\partial W}{\partial B}\right)_A dB$$

$$= (2AB^2 + 2)\,dA + 2A^2B\,dB$$

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Inexact 

$$đW = \underset{\uparrow}{A}\,dA + \underset{\uparrow}{A^2}\,dB$$

$$\left(\frac{\partial}{\partial B}\left(\frac{\partial W}{\partial A}\right)_B\right)_A \overset{?}{=} \left(\frac{\partial}{\partial A}\left(\frac{\partial W}{\partial B}\right)_A\right)_B$$

$$\left(\frac{\partial A}{\partial B}\right)_A = 0 \neq \left(\frac{\partial A^2}{\partial A}\right)_B = 2A$$

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$$4AB = 4AB \checkmark$$

$$\frac{\partial^2 W}{\partial A \partial B} = \frac{\partial^2 W}{\partial B \partial A}$$