Right way

\[ U(x, y, z) \]

\[ \int dU = U(x, 1, 2) - U(0, 0, 0) = \frac{y + x^3y^2 - x^2z^2}{\text{constant}} \]

\[ U(0, 0, 0) \]

Any path

\[ (x, 0, 0) \]

\[ \int (3x^2y^2 - 2x^2) dx \to 0 \]

\[ (0, 0, 0) \]

\[ dy = 0 \]
\[ dz = 0 \]
\[ y = 0 \]
\[ z = 0 \]

\[ + \int (1 + 3x^2y^2) dy \to y + x^3y^2 \]

\[ (x, 0, 0) \]

\[ dx = 0 \]
\[ dz = 0 \]
\[ z = 0 \]

\[ + \int (-2x^2z) dz \to -x^2z^2 \]

\[ (x, y, 0) \]

\[ dx = 0 \]
\[ dz = 0 \]

\[ U(x, y, z) = y + x^3y^2 - x^2z^2 + \text{constant} \]

\[ \int - \text{not exact} \quad \frac{d\theta}{T} = dS = \text{exact} \]

\[ S(x, y, z) \]
Imagine a system with discrete energy levels (e.g. Einstein solid)

Create an ensemble of $N$ copies of the system

Put boxes in thermal contact, can exchange energy.

Let $N_i$ be the number of boxes with energy $E_i$

$i$ labels the energy states, $n_i$ the boxes

$N_i = 0, 1, 2, \ldots, N$

Constraint #1: $\sum_{i=1}^{\infty} N_i = N$

How many ways are there to arrange the energies?

$$\Omega = \frac{N!}{N_1! N_2! \ldots} = \frac{N!}{\prod_{i=1}^{\infty} (N_i)!}$$

E.g. How many ways are there to put all the boxes in state $E_1$?

$N_1 = N$

$$\frac{N!}{N! 0! 0! \ldots} = 1$$
E.9. How many ways to fill 5 = N boxes, 3 in state $E_2$, 2 in state $E_4$? 10

\[
\begin{align*}
2 & 2 4 4 \quad 2 & 2 4 2 4 \quad 2 & 4 2 2 2
\end{align*}
\]

eff post net. $n_1 = 0$, $n_2 = 3$, $n_3 = 0$, $n_4 = 2$

\[
\sum_{i=1}^{\infty} n_i = 2 + 3 = 5 = N
\]

---

Constraint #2 Fix the total energy of the $N$ systems $\Leftrightarrow$ Fix average energy

\[
E_{\text{tot}} = N \langle E \rangle = N U = N \langle E \rangle
\]

Probability that a random box has energy $E_i \Leftrightarrow P_i = \frac{n_i}{N}

---

Constraint #1 $\leq n_i = \frac{N}{N} \Rightarrow \leq P_i = 1$

Constraint #2 $\sum_{i=1}^{\infty} \frac{n_i E_i}{N} = \frac{E_{\text{tot}}}{N} \Rightarrow \sum_{i} P_i E_i = \bar{E} = 0$
Maximize $\Omega$, but subject to both constraint
functions and microstates
by varying $\beta, \epsilon, \Omega, \epsilon_i$ etc.

$$S = \frac{1}{k_B} \ln \Omega - \beta (\epsilon \rho \xi - 1) - \frac{\beta}{\xi} (\xi \rho \xi - U)$$

Could have been any monotonically
increasing function