Energy $E_0$ of a system is fixed, the set of all possible states is called the "microcanonical ensemble." In this ensemble, all states are equally likely.

If the temperature of the system is fixed $T$, by putting the system in contact with a heat bath (reservoir), this is called a "canonical ensemble." $E$ not fixed.

Probabilities are Boltzmann weighted:

\[ p_i = e^{-\beta E_i} \]

\[ \mathbb{Z} = \sum_{\text{all microstates}} e^{-\beta E} \]

System $S$ has $\langle E \rangle = U$.
Exam problem w/ marker, but now with temp T specified.

How many microstates?

\[ X \text{ Hard way} \]

\[ E = 0 \quad \sigma = 1 \quad \sigma = 3 \]

\[ E = 2 \text{mgh} \quad W \quad B \quad R \quad B \quad R \quad W \quad B \quad R \]

\[ \sigma = 6 \]

\[ Z = \sum e^{-\beta E_k} = e + 3e + e + 10e \ldots \]

\[ Z \text{ Easy way} \]

\[ Z_n = \text{partition function for one mark.} \]

\[ \sum_k k = \frac{1}{1-r} \quad \text{Geometric series} \]

\[ Z_n = \frac{1}{1 - e^{-\beta \text{mgh}}} \]
\[ Z = z^3 = \left( \frac{1}{1 - e^{-\beta mgh}} \right)^3 \]

Probability all 3 markber will be on step #1?

\[ P_{\text{all}} = \frac{e^{-\beta mgh}}{Z} = e^{-\beta mgh} \left( 1 - e^{-\beta mgh} \right)^3 \]

\[ P_{\text{all}} = \frac{e^{-\beta mgh \left[ n + p + q \right]}}{Z} \]

What is \( P_{\text{all}} \) at low \( T \)? (High \( \beta \))

\[ P_{\text{all}} = \left[ e^{-\beta mgh} \left( 1 - e^{-\beta mgh} \right) \right]^3 \rightarrow 0 \]

What is \( P_{\text{all}} \) at high \( T \)? (Low \( \beta \))

\[ P_{\text{all}} = \left[ e^{-\beta mgh} \left( 1 - e^{-\beta mgh} \right) \right]^3 \rightarrow 0 \]

\[ P_{\text{all}}(T) \sim 1 \]

Find \( T_0 \) that maximizes \( P_{\text{all}} \).
Given $T$, what is $\langle E \rangle = U$?

$$f_{\text{ave}} = \bar{f} = \langle f \rangle = \sum_i f_i P_i = \int f(w) P(w) dw$$

$$U = \sum_i E_i P_i = \sum_i E_i e^{-\beta E_i} = \frac{\sum_i E_i e^{-\beta E_i}}{Z} = \frac{\sum_i e^{-\beta E_i}}{Z}$$

$$= -\frac{\partial \beta}{\partial \beta} \frac{Z}{Z} = -\frac{\partial \ln(Z)}{\partial \beta}$$

$$= -\frac{\partial}{\partial \beta} \left[ \frac{1}{1-e^{-\beta \mu_H}} \right] = \frac{\partial}{\partial \beta} \left[ 3 \ln \left( 1-e^{-\beta \mu_H} \right) \right]$$

$$= 3 \left( \frac{m g H e^{-\beta m g h}}{1-e^{-\beta m g h}} \right) + \left( \frac{e^{-\beta m g h} + \mu_H}{e^{\beta m g h}} \right)$$

$$= \frac{3 m g h}{e^{-\beta m g h} - 1} = U$$

Low $T$, high $\beta$

$U \to 0$

High, low $\beta$

$U \to \infty$
One-dimensional chain w/ massless link

Teusim in chain? \( T = Mg \)

At temp T, what is \( L \)?