

1. A gas satisfies the relation  $U = \frac{aS^4}{NV^2}$  where  $a$  is a constant,  $N$  is the number of particles which is held fixed,  $V$  is the volume, and  $S$  is the entropy.
  - (a) Derive an expression for the temperature  $T$ .
  - (b) Derive an expression for the pressure  $P$ .
  - (c) What is the equation of state (an equation relating  $P$ ,  $V$ ,  $T$ , and  $N$ ) for this gas?
  
2. Given 100 distinguishable coins which can show either heads or tails, (express numerical all answers in scientific notation, not formulae)
  - (a) How many possible states  $N_s$  are there?
  - (b) How many states have no heads showing?
  - (c) How many states have exactly one head showing?
  - (d) How many states have exactly two heads showing?
  - (e) If the coins are randomized, what is the most likely number of heads showing?
  - (f) How many states have this most likely number of heads showing?
  - (g) What fraction of the total number of states is this?
  - (h) If another 100 coins are added to the original 100, what is the new number of possible states in terms of the original number  $N_s$ ?
  - (i) Is the number of possible states extrinsic? If yes, explain why; if no, suggest an extrinsic quantity.
  
3. A peculiar gas is observed to be governed by two relations:

$$U(V, T) = Vf(T) \quad \text{and} \quad P(T) = \frac{1}{3}f(T)$$

where  $U$  is the energy,  $P$  is the pressure, and  $f(T)$  is a function of the absolute temperature  $T$ . Hints: find  $dU$ , use the first law to find  $dS$ , look at the mixed partial derivatives noting that  $dS$  is an exact differential, obtain a differential equation for  $f(T)$ .

- (a) How does the energy  $U$  depend on  $T$ ?
- (b) How does the pressure  $P$  depend on  $T$ ?
- (c) How does the entropy  $S$  depend on  $V$  and  $T$ ?