Physics 3374/6351 Professor Scalise
Homework Assignment #12 Fall 2020

3374

1. Read Schroeder chapter 8. Did you read all the pages?

2. Consider a diatomic molecule with non-identical atoms like CO. Define the energy $\epsilon \equiv \frac{\hbar^2}{2I}$. Use a computer to sum the rotational quantum mechanical partition function numerically, keeping terms through $\ell = 10$. Calculate the average energy, and then get the heat capacity per diatomic molecule in units of k_B. Plot (not sketch) the heat capacity versus x, where $x \equiv k_B T/\epsilon$, for $x = 0$ to 7. Do you see the bump that I drew in lecture?

3. The radius of an atomic nucleus scales roughly as $R = (1.25 \text{ fm})A^{\frac{1}{3}}$, where A is the "mass number" or simply the number of nucleons.

   (a) What is the number density of nucleons?

   (b) What are the Fermi energy in MeV and Fermi temperature in K for a nucleus? Be careful about how you deal with protons and neutrons which are distinct particles not subject to Pauli exclusion.

4. For a system of fermions at room temperature, compute the probability of a single-particle state being occupied if its energy is

   (a) 1 eV less than μ

   (b) 0.01 eV less than μ

   (c) equal to μ

   (d) 0.01 eV greater than μ

   (e) 1 eV greater than μ

5. For a system of bosons at room temperature, compute the average oocupancy of a single-particle state and the probability of the state containing 0, 1, or 2 bosons if the energy of the state is

   (a) 0.001 eV greater than μ

   (b) 1 eV greater than μ

6351

1. In the van der Waals equation of state,

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = NK_bT$$

   the critical point is the unique point where both the first and second derivatives of pressure with respect to volume at fixed temperature are zero. Find the critical pressure, volume, and temperature in terms of a and b.

2. Rewrite the van der Waals equation of state in terms of the dimensionless variables $t \equiv \frac{T}{T_C}$, $p \equiv \frac{P}{P_C}$, and $v \equiv \frac{V}{V_C}$. Plot some isotherms above and below the critical point.

Bonus: Solve as much of the other class' assignment as you can.

3)

Prob 6.28

In[12]:=
```
z[β_] = Sum[(2 j + 1) Exp[-j (j + 1) β ϵ], {j, 0, 10}]
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Out[12]=
$$1 + 21\, e^{-110\beta\epsilon} + 19\, e^{-90\beta\epsilon} + 17\, e^{-72\beta\epsilon} + 15\, e^{-56\beta\epsilon} + 13\, e^{-42\beta\epsilon} + 11\, e^{-30\beta\epsilon} + 9\, e^{-20\beta\epsilon} + 7\, e^{-12\beta\epsilon} + 5\, e^{-6\beta\epsilon} + 3\, e^{-2\beta\epsilon}$$

In[13]:=
```
e[β_] = -D[Log[z[β]], β]
```

Out[13]=
$$-\left(-2310\, e^{-110\beta\epsilon}\,\epsilon - 1710\, e^{-90\beta\epsilon}\,\epsilon - 1224\, e^{-72\beta\epsilon}\,\epsilon - 840\, e^{-56\beta\epsilon}\,\epsilon - 546\, e^{-42\beta\epsilon}\,\epsilon - 330\, e^{-30\beta\epsilon}\,\epsilon - 180\, e^{-20\beta\epsilon}\,\epsilon - 84\, e^{-12\beta\epsilon}\,\epsilon - 30\, e^{-6\beta\epsilon}\,\epsilon - 6\, e^{-2\beta\epsilon}\,\epsilon\right) \Big/ \left(1 + 21\, e^{-110\beta\epsilon} + 19\, e^{-90\beta\epsilon} + 17\, e^{-72\beta\epsilon} + 15\, e^{-56\beta\epsilon} + 13\, e^{-42\beta\epsilon} + 11\, e^{-30\beta\epsilon} + 9\, e^{-20\beta\epsilon} + 7\, e^{-12\beta\epsilon} + 5\, e^{-6\beta\epsilon} + 3\, e^{-2\beta\epsilon}\right)$$

In[14]:=
```
e[1 / (k T)]
```

Out[14]=
$$-\left(-2310\, e^{-\frac{110\epsilon}{kT}}\,\epsilon - 1710\, e^{-\frac{90\epsilon}{kT}}\,\epsilon - 1224\, e^{-\frac{72\epsilon}{kT}}\,\epsilon - 840\, e^{-\frac{56\epsilon}{kT}}\,\epsilon - 546\, e^{-\frac{42\epsilon}{kT}}\,\epsilon - 330\, e^{-\frac{30\epsilon}{kT}}\,\epsilon - 180\, e^{-\frac{20\epsilon}{kT}}\,\epsilon - 84\, e^{-\frac{12\epsilon}{kT}}\,\epsilon - 30\, e^{-\frac{6\epsilon}{kT}}\,\epsilon - 6\, e^{-\frac{2\epsilon}{kT}}\,\epsilon\right) \Big/ \left(1 + 21\, e^{-\frac{110\epsilon}{kT}} + 19\, e^{-\frac{90\epsilon}{kT}} + 17\, e^{-\frac{72\epsilon}{kT}} + 15\, e^{-\frac{56\epsilon}{kT}} + 13\, e^{-\frac{42\epsilon}{kT}} + 11\, e^{-\frac{30\epsilon}{kT}} + 9\, e^{-\frac{20\epsilon}{kT}} + 7\, e^{-\frac{12\epsilon}{kT}} + 5\, e^{-\frac{6\epsilon}{kT}} + 3\, e^{-\frac{2\epsilon}{kT}}\right)$$

In[15]:=
```
c[T_] = D[e[1 / (k T)], T] // Simplify
```

Out[15]=
$$\left(12\, e^{\frac{20\epsilon}{kT}} \left(13\,300 + 42\,959\, e^{\frac{18\epsilon}{kT}} + 76\,545\, e^{\frac{34\epsilon}{kT}} + 8721\, e^{\frac{38\epsilon}{kT}} + 105\,196\, e^{\frac{48\epsilon}{kT}} + 27\,455\, e^{\frac{54\epsilon}{kT}} + 123\,200\, e^{\frac{60\epsilon}{kT}} + 47\,424\, e^{\frac{68\epsilon}{kT}} + 127\,575\, e^{\frac{70\epsilon}{kT}} + 5440\, e^{\frac{72\epsilon}{kT}} + 117\,649\, e^{\frac{78\epsilon}{kT}} + 62\,700\, e^{\frac{80\epsilon}{kT}} + 94\,640\, e^{\frac{84\epsilon}{kT}} + 16\,575\, e^{\frac{86\epsilon}{kT}} + 61\,236\, e^{\frac{88\epsilon}{kT}} + 91\,000\, e^{\frac{90\epsilon}{kT}} + 94\,920\, e^{\frac{98\epsilon}{kT}} + 3185\, e^{\frac{102\epsilon}{kT}} + 55\,860\, e^{\frac{104\epsilon}{kT}} + 71\,260\, e^{\frac{108\epsilon}{kT}} + 12\,825\, e^{\frac{110\epsilon}{kT}} + 9295\, e^{\frac{114\epsilon}{kT}} + 35\,700\, e^{\frac{116\epsilon}{kT}} + 30\,855\, e^{\frac{122\epsilon}{kT}} + 14\,580\, e^{\frac{124\epsilon}{kT}} + 20\,825\, e^{\frac{126\epsilon}{kT}} + 9060\, e^{\frac{128\epsilon}{kT}} + 16\,940\, e^{\frac{132\epsilon}{kT}} + 20\,344\, e^{\frac{138\epsilon}{kT}} + 10\,935\, e^{\frac{142\epsilon}{kT}} + 3920\, e^{\frac{144\epsilon}{kT}} + 6825\, e^{\frac{146\epsilon}{kT}} + 825\, e^{\frac{150\epsilon}{kT}} + 7020\, e^{\frac{152\epsilon}{kT}} + 5200\, e^{\frac{156\epsilon}{kT}} + 3990\, e^{\frac{158\epsilon}{kT}} + 2640\, e^{\frac{164\epsilon}{kT}} + 2492\, e^{\frac{168\epsilon}{kT}} + 825\, e^{\frac{170\epsilon}{kT}} + 735\, e^{\frac{174\epsilon}{kT}} + 729\, e^{\frac{178\epsilon}{kT}} + 300\, e^{\frac{180\epsilon}{kT}} + 105\, e^{\frac{182\epsilon}{kT}} + 175\, e^{\frac{186\epsilon}{kT}} + 84\, e^{\frac{188\epsilon}{kT}} + 20\, e^{\frac{192\epsilon}{kT}} + 15\, e^{\frac{194\epsilon}{kT}} + e^{\frac{198\epsilon}{kT}}\right) \epsilon^2\right) \Big/ \left(\left(21 + 19\, e^{\frac{20\epsilon}{kT}} + 17\, e^{\frac{38\epsilon}{kT}} + 15\, e^{\frac{54\epsilon}{kT}} + 13\, e^{\frac{68\epsilon}{kT}} + 11\, e^{\frac{80\epsilon}{kT}} + 9\, e^{\frac{90\epsilon}{kT}} + 7\, e^{\frac{98\epsilon}{kT}} + 5\, e^{\frac{104\epsilon}{kT}} + 3\, e^{\frac{108\epsilon}{kT}} + e^{\frac{110\epsilon}{kT}}\right)^2 k\, T^2\right)$$

In[16]:=
```
cc[x_] = c[T] / k /. {T → x ϵ / k}
```

Out[16]=
$$\left(12\, e^{20/x} \left(13\,300 + 42\,959\, e^{18/x} + 76\,545\, e^{34/x} + 8721\, e^{38/x} + 105\,196\, e^{48/x} + 27\,455\, e^{54/x} + 123\,200\, e^{60/x} + 47\,424\, e^{68/x} + 127\,575\, e^{70/x} + 5440\, e^{72/x} + 117\,649\, e^{78/x} + 62\,700\, e^{80/x} + 94\,640\, e^{84/x} + 16\,575\, e^{86/x} + 61\,236\, e^{88/x} + 91\,000\, e^{90/x} + 94\,920\, e^{98/x} + 3185\, e^{102/x} + 55\,860\, e^{104/x} + 71\,260\, e^{108/x} + 12\,825\, e^{110/x} + 9295\, e^{114/x} + 35\,700\, e^{116/x} + 30\,855\, e^{122/x} + 14\,580\, e^{124/x} + 20\,825\, e^{126/x} + 9060\, e^{128/x} + 16\,940\, e^{132/x} + 20\,344\, e^{138/x} + 10\,935\, e^{142/x} + 3920\, e^{144/x} + 6825\, e^{146/x} + 825\, e^{150/x} + 7020\, e^{152/x} + 5200\, e^{156/x} + 3990\, e^{158/x} + 2640\, e^{164/x} + 2492\, e^{168/x} + 825\, e^{170/x} + 735\, e^{174/x} + 729\, e^{178/x} + 300\, e^{180/x} + 105\, e^{182/x} + 175\, e^{186/x} + 84\, e^{188/x} + 20\, e^{192/x} + 15\, e^{194/x} + e^{198/x}\right)\right) \Big/ \left(\left(21 + 19\, e^{20/x} + 17\, e^{38/x} + 15\, e^{54/x} + 13\, e^{68/x} + 11\, e^{80/x} + 9\, e^{90/x} + 7\, e^{98/x} + 5\, e^{104/x} + 3\, e^{108/x} + e^{110/x}\right)^2 x^2\right)$$

In[17]:=
```
p1 = Plot[{cc[x], 1}, {x, 0, 7}, PlotRange → All]
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Out[17]=