What is $\frac{\partial^2 \ln(Z)}{\partial \beta^2}$?

Medium difficulty.

$$Z = \sum_i e^{-\beta E_i}$$

$$= \frac{\partial}{\partial \beta}\left[\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right] = -\frac{1}{Z^2}\left(\frac{\partial Z}{\partial \beta}\right)^2 + \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

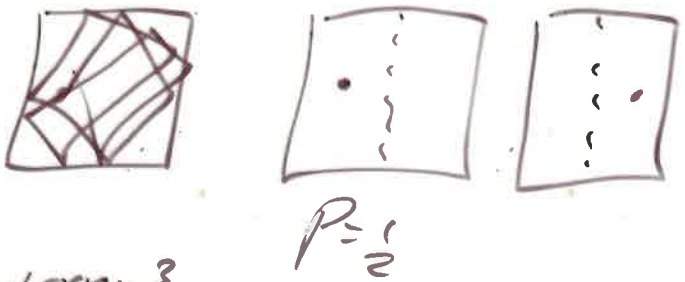
$$-(\bar{E})^2 \qquad \frac{1}{Z}\sum_i E_i^2 e^{-\beta E_i}$$

$$= \overline{E^2} - (\bar{E})^2 = \sigma_E^2 \qquad \overline{E^2} \qquad \sum_i p_i (E_i^2)$$

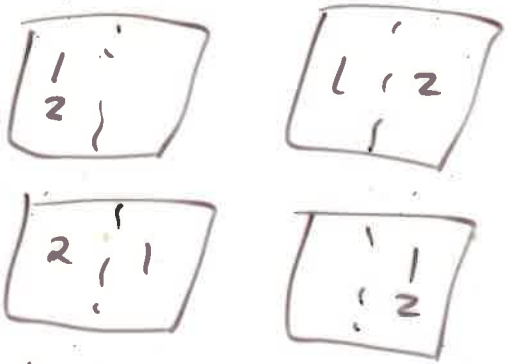
$$\frac{\partial}{\partial \beta}[\bar{E}] = \frac{\partial U}{\partial \beta} = \frac{\partial T}{\partial \beta}\frac{\partial}{\partial T}U = k_B T^2 C_V$$

$$\frac{dT}{d\beta} = \qquad \sigma_E^2$$

Easy Probability that all the air molecules will be found in half the room.



$P = \frac{1}{2}$



$P = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$PV = N k_B T$ ← 300k, ← $1000\,m^3$, ↑ 1×10^{-23} J/k

1 atm = 101,000 Pa

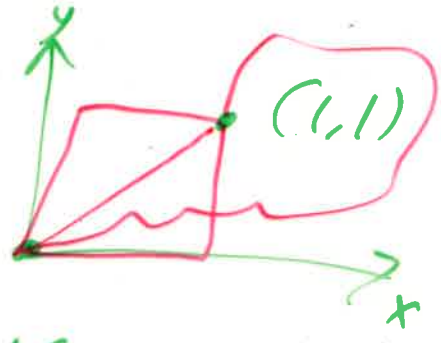
$$\frac{1}{2^N}$$

$dG = (a \ln|x| + \frac{b}{y})dx + \frac{bx}{y}dy$

$dQ = \left(\frac{\partial G}{\partial x}\right)dx + \left(\frac{\partial G}{\partial y}\right)dy$

$\frac{\partial^2 G}{\partial x \partial y} \stackrel{?}{=} \frac{\partial^2 G}{\partial y \partial x}$ ✗

Is dG exact or inexact



$\int_{0,0}^{1,1} dG$

path

If dG were exact

$\int_{0,0}^{1,1} dG = G\Big|_{0,0}^{1,1} = G(1,1) - G(0,0)$

$dS = \frac{dQ}{T}$

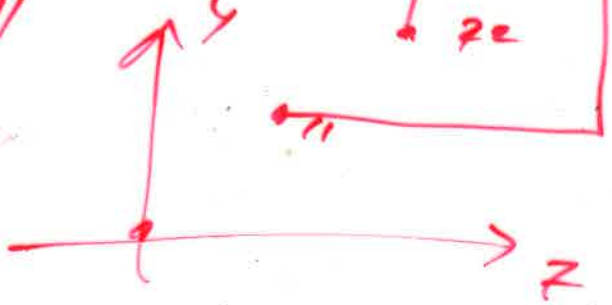
Find dF from dG when dF is exact

$dF = \frac{dG}{x} = \frac{a}{x}dx + \frac{b}{y}dy$

$F = a\ln|x| + b\ln|y|$

$\int_{0,0}^{1,1} dF = F(1,1) - F(0,0)$

$\frac{\partial^2 F}{\partial x \partial y} = 0$ ✓



100 distinguishable coins

states 3^{100} ← total # of microstates,

macrostates = ~~101~~ (0 heads, 1 head, ... 100 heads)

3 coins

H	H H	H T T, T H T, T T H
T	T T	H H T, H T H, T H H
S	S S	S T T, T S T, T T S
		S S T, T S S, S T S
		S H H, H S H, H H S
		S S H, S H S, S S H

1 coin: H, T, S
3 macrostates

HTS → 6 of these

2 coins:

H H	T H, T H
T T	T S, S T
S S	H S, S H

4 coins → 15 macrostates

10

6

			1			
		1		1		
	1		2		1	
1		3		3		1
	4		6		4	
1		5		10		5
	6		15		20	
1						1