Physics 3374/6351 | Professor Scalise
Homework Assignment #3 | Fall 2022

3374

1. Read Schroeder chapter 4. Did you read all the pages?

2. You have just microwaved a cup of tea for too long and it is boiling, too hot to drink. You look around and see a punchbowl containing ice floating in water. You thoroughly mix one cup of water (no ice) from the punchbowl with your cup of tea in a thermos bottle. What is the change in entropy of the pint of liquid? Does the sign of the change make sense? Explain.

3. Suppose that

$$dU = (3 + 2xy^2)dx + (2x^2y + 3y^2z^3)dy + (3y^3z^2)dz$$

Is there a function $U(x, y, z)$ for which this is an exact differential? If not, prove it. If so, find the function U.

4. On episode 196 of MythBusters, Kari, Grant, and Tory test "tastes like chicken". There are 11 plates of fried chicken (C) and 9 plates of fried non-chicken (N) like snake, turtle, etc. numbered from 1 to 20 which are sampled one at a time by blindfolded tasters. An arrangement is an ordered set e.g. CCNNCNNNCNCC...; NCNCNCNCCCC...; etc.

   A) What is the probability of randomly guessing the correct arrangement if the blind tasters know that there are 11 Cs and 9 Ns?

   B) With the same knowledge, what is the probability of getting exactly one plate wrong?

   C) What is the probability of randomly guessing the correct arrangement if the blind tasters do not know how many of the plates are chicken?

   D) With the same knowledge, what is the probability of getting exactly one plate wrong?

5. The dihydrogen molecule can exist in either of two states: parahydrogen in which the nuclear spins are antiparallel giving the molecule a ground-state total spin angular momentum of zero; and orthohydrogen in which the nuclear spins are aligned giving the molecule a total spin angular momentum of 1 $\hbar$ and z-projections of spin $s_z = +1\ \hbar$, 0, or -1 $\hbar$. The transition between the two forms occurs very slowly in the absence of a catalyst.

   (a) An insulated, rigid box of volume $2V$ is prepared with n moles of dilute parahydrogen gas held in one half of the box by a partition. The pressure is P and the temperature is T. The partition is suddenly removed and the gas fills the entire box. After equilibrium is reached,

      i. What is the new pressure?
      ii. What is the new temperature?
      iii. How much heat Q was added to or removed from the gas?

iv. What is the change in entropy?

(b) An insulated, rigid box of volume $2V$ is prepared with n moles of dilute parahydrogen gas held in each half of the box by a partition. The pressure is P and the temperature is T. The partition is suddenly removed and the gas mixes in the entire box. After equilibrium is reached, what is the change in entropy?

(c) An insulated, rigid box of volume $2V$ is prepared with n moles of dilute parahydrogen gas held in one half of the box by a partition. The other half of the box contains n moles of orthohydrogen. The pressure is P and the temperature is T. The partition is suddenly removed and the gas mixes in the entire box. After equilibrium is reached, what is the change in entropy?

(d) Explain why changes in entropy may be calculated even if the system is not moving through equilibrium states.

(e) At room temperature, where k_BT is far above the energy difference between para- and orthohydrogen, and if you wait long enough for transitions to occur, what is the equilibrium ratio of the two forms of hydrogen.

6351

1. Show that the next term in Stirling's approximation is

$$N! \approx N^N e^{-N}\sqrt{2\pi N} + \frac{1}{12N} N^N e^{-N}\sqrt{2\pi N}$$

2. You are given a well-shuffled deck of 52 playing cards faces down.

   (a) What is the probability of drawing $7\spadesuit$, $10\spadesuit$, $3\heartsuit$, $K\diamondsuit$, and $7\heartsuit$, in that order?

   (b) Does it matter if you take the first five cards off the top of the deck or pull cards from random places in the deck?

   (c) What is the probability of drawing $7\spadesuit$, $10\spadesuit$, $3\heartsuit$, $K\diamondsuit$, and $7\heartsuit$, in any order?

   (d) How many microstates contain 5 clubs? In other words, what is the multiplicity of the macrostate with 5 clubs?

   (e) What is the probabilty of drawing a royal flush?

   (f) What is the probability that Jamy Ian Swiss (Google if necessary) can draw a spade royal flush from the deck?

3. A gas satisfies the relation $U = \dfrac{aS^4}{NV^2}$ where a is a constant, N is the number of particles which is held fixed, V is the volume, and S is the entropy.

   (a) Derive an expression for the temperature T.

   (b) Derive an expression for the pressure P.

   (c) What is the equation of state (an equation relating P, V, T, and N) for this gas?

Bonus: Solve as much of the other class' assignment as you can.