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1. Read Schroeder chapter 2. Did you read all the pages?

2. Use the Gibbs-Duhem relation to write

   (a) the enthalpy H

   (b) the Helmholtz free energy F or A

   in terms of T, S, P, V, μ, and N (and nothing else).

3. (a) Use a Legendre transformation to define a new thermodynamic potential X whose natural variables are (S,V,μ). What is dX?

   (b) Use the Gibbs-Duhem relation to write X in terms of T, S, P, V, μ, and N (and nothing else).

4. (a) Derive a Maxwell-like relation to write $\left(\frac{\partial \mu}{\partial P}\right)_{T,N}$ as another partial derivative of different variables. Be sure to specify which variables are held fixed (like T and N above).

   (b) Which mixed partial derivatives of which thermodynamic potential should be equated?

   (c) What is $\left(\frac{\partial \mu}{\partial P}\right)_{T,N}$ for an ideal gas? Simplify as much as possible.

5. According to the Sackur-Tetrode equation, the entropy of a monatomic ideal gas can become negative when its temperature (and hence its energy) is sufficiently low. Of course this is absurd, so the Sackur-Tetrode equation must be invalid at very low temperatures. Suppose that you start with a sample of helium at room temperature and atmospheric pressure, then lower the temperature while holding the density fixed. Pretend that the helium remains a gas and does not liquefy. Below what temperature would the Sackur-Tetrode equation predict that S is negative?

6351

1. A black hole is a region of space where gravity is so strong that nothing can escape. Throwing something into a black hole is therefore an irreversible process. Adding mass to a black hole increases the black hole's entropy. There is no way to tell what kind of matter has gone into making a black hole, therefore the entropy of a black hole must be greater than the entropy of any conceivable type of matter that could have been used to create it. Knowing this, estimate the entropy of a black hole.

   (a) Use dimensional analysis to find the radius of a black hole of mass M in terms of the fundamental constants G and c. (There is, in fact, a 2 in front of the expression which dimensional analysis can not give.)

(b) What is the entropy of a system (very roughly) in terms of the maximum number of particles N in a system. (Think of an ideal gas or an Einstein solid at high temperature and don't be too concerned with details. You can neglect any logarithms.)

(c) To make a black hole out of the maximum number of particles, you should use particles with the lowest possible energy: long-wavelength photons, but the wavelength can not be any longer than the size of the black hole. By setting the total energy of the photons equal to Mc^2, estimate the maximum number of photons that could be used to make a black hole of mass M.

(d) What is the entropy of a black hole of mass M in terms of its mass? What is the entropy of a black hole of mass M in terms of its area?

(e) What is the entropy of a solar-mass black hole?

(f) What is the entropy of the largest known black hole?

(g) What accounts for most of the entropy of the current Universe?

Bonus: Solve as much of the other class' assignment as you can.