Physics 3374/6351 Professor Scalise
Homework Assignment #5 Fall 2022

# 3374

1. Read Schroeder sections 5.1-5.2 and chapter 3. Did you read all the pages?

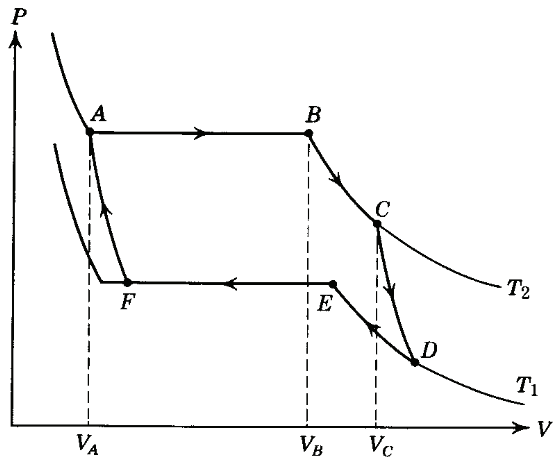
2. Starting with the definition of the factorial

$$N! = \int_{x=0}^{\infty} x^N \, e^{-x} \, dx = \Gamma(N+1)$$

show all the steps to derive the formula

$$\int_{r=0}^{\infty} r^{N-1} \, \exp(-r^2) \, dr = \frac{1}{2}\Gamma\left(\frac{N}{2}\right)$$

that we used in the lecture notes.




3. Two isotherms of 1 mole of a substance that can undergo a gas-liquid phase transition are shown in the PV diagram above. The substance goes through one cycle of the reversible transformation (ABCDEFA), where
1. (ABC) and (DEF) are isotherms;
2. FA and CD are adiabats;
3. In the gas phase (BCDE), the substance is an ideal gas. At (A), the substance is pure liquid;
4. Latent heat along (AB) is $L = 200$ cal/mol; $T_1 = 150$ K, $T_2 = 300$ K, $V_A = 0.5$ liter, $V_B = 1.0$ liter, and $V_C = 2.7$ liter.

   (a) What is the efficiency of the cycle $\eta$?

   (b) What is the relation between the net work done in the cycle $W$ to the heat $Q_2$ extracted from the hot reservoir during the isotherm stroke (ABC)?

   (c) What is the heat $Q_2$?

   (d) What is the net work $W$?

4. A computer bit has two states: 1 and 0. A byte is 8 bits. Your job is to erase a filled terabyte disk.
   (a) How much entropy is created in this irreversible process?
   (b) If this entropy is released to the environment at room temperature, how much heat must have been released to the room?
   (c) Is this amount of energy significant? Compare it to something.

5. Experimental measurements of the heat capacity of aluminum at low temperatures (below about 50 K) can be fit to the formula

$$C_V = aT + bT^3$$

where $C_V$ is the heat capacity of one mole of Al, $a = 0.00135\ J/K^2$, and $b = 2.48 \times 10^-5\ J/K^4$. Find a formula for the entropy per mole of Al in terms of temperature. What is the entropy of 1 gram of Al at 3 K? (Incidentally, the linear piece of $C_V$ is due to the conduction electrons and the cubic piece is due to phonons. Cf. Einstein-Debye specific heat.)

# 6351

1. Remember our convention for $n$-spheres: $n = 2$ dimensions corresponds to the film of a soap bubble and has surface area $S_2 = 4\pi r^2$; the volume of the solid spherical ball is $V_3 = \frac{4}{3}\pi r^3$; the "volume" of a disk is $V_2 = \pi r^2$. ($n$ is always the power of $r$).
   (a) For which (non-integer) dimension $n$ is the area of the unit ($r = 1$) hypersphere a maximum?
   (b) For which (non-integer) dimension $n$ is the volume of the unit hypersphere a maximum?
   (c) Plot graphs of $S_n$ and $V_n$ versus $n$.

2. (a) Using last week's problem on the entropy of a black hole, find the temperature of a black hole as a function of its mass.
   (b) Find the temperature of a solar-mass black hole.
   (c) Find the temperature of the largest known black hole.
   (d) Find the temperature of a Planck-mass black hole.

**Bonus:** Solve as much of the other class' assignment as you can.