Physics 3374/6351 Professor Scalise
Homework Assignment #7 Fall 2022

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1. Read Schroeder sections 7.1 and 7.2. Did you read all the pages?

2. A system comprises three distinguishable marbles (red, white, and blue) each of mass m which may be found on any step of a staircase whose steps are a distance h apart. The system is known to have energy $3mgh$ with respect to the bottom step (i.e. the bottom step has energy zero).

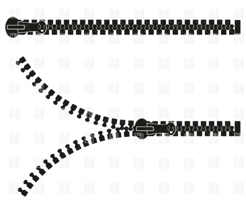
   (a) What kind of ensemble is this?

   (b) List the possible microstates. (A picture would work.)

   (c) What is the probability that all three marbles are on the same step?

   (d) What is the average height of the red marble?

3. Solve Schroeder problem 3.34. (See attached.)



4. Consider a horizontal zipper composed of N links which unzips from left to right. A zipped link has energy 0 and an unzipped link has energy ϵ. A link may only unzip if all of the links to its left are unzipped. (This model is used to study DNA molecules.)

   (a) What is the partition function?

   (b) What is the average number of unzipped links at temperature T?

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1. A system has energy levels that are equally spaced so that the nth energy level is $E_n = n\epsilon$, where $n = 0, 1, 2, 3, \ldots$. The degeneracy g_n of the nth energy level is $n + 1$. Calculate the energy U as a function of temperture. You should be able to evaluate the sum in the partition function exactly.

Bonus: Solve as much of the other class' assignment as you can.

Problem 3.34. Polymers, like rubber, are made of very long molecules, usually tangled up in a configuration that has lots of entropy. As a very crude model of a rubber band, consider a chain of N links, each of length ℓ (see Figure 3.17). Imagine that each link has only two possible states, pointing either left or right. The total length L of the rubber band is the net displacement from the beginning of the first link to the end of the last link.

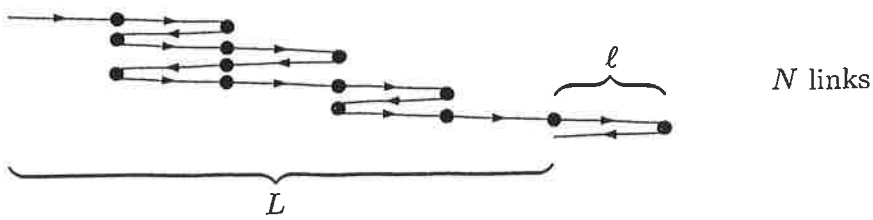

Figure 3.17. A crude model of a rubber band as a chain in which each link can only point left or right.

(a) Find an expression for the entropy of this system in terms of N and N_R, the number of links pointing to the right.

(b) Write down a formula for L in terms of N and N_R.

(c) For a one-dimensional system such as this, the length L is analogous to the volume V of a three-dimensional system. Similarly, the pressure P is replaced by the tension force F. Taking F to be positive when the rubber band is pulling inward, write down and explain the appropriate thermodynamic identity for this system.

(d) Using the thermodynamic identity, you can now express the tension force F in terms of a partial derivative of the entropy. From this expression, compute the tension in terms of L, T, N, and ℓ.

(e) Show that when $L \ll N\ell$, the tension force is directly proportional to L (Hooke's law).

(f) Discuss the dependence of the tension force on temperature. If you increase the temperature of a rubber band, does it tend to expand or contract? Does this behavior make sense?

(g) Suppose that you hold a relaxed rubber band in both hands and suddenly stretch it. Would you expect its temperature to increase or decrease? Explain. Test your prediction with a real rubber band (preferably a fairly heavy one with lots of stretch), using your lips or forehead as a thermometer. (Hint: The entropy you computed in part (a) is not the total entropy of the rubber band. There is additional entropy associated with the vibrational energy of the molecules; this entropy depends on U but is approximately independent of L.)