This is OK

$$P = -\left(\frac{\partial U}{\partial V}\right)_{SN}$$

S = ?

$$= -\left[\underbrace{\left(\frac{\partial U}{\partial V}\right)_{TN}}_{0} - \underbrace{\left(\frac{\partial U}{\partial S}\right)_{VN}}_{T}\left(\frac{\partial S}{\partial V}\right)_{TN}\right]$$

Entropy of the ideal gas

$$S = k_B \ln(\#\ \text{microstates}) = \infty\ ???$$

Phase space is chunky $\delta x \delta p \sim h$

$$Z = \frac{V^N}{N!}\left(\frac{2\pi m}{\beta}\right)^{\frac{3N}{2}}\left(\frac{1}{h^3}\right)^N \rightarrow \left[\frac{eV}{N}\left(\frac{2\pi m}{h^2\beta}\right)^{3/2}\right]^N$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{VN} = +\frac{\partial}{\partial T}\left[k_B T \ln(Z)\right]_{VN}$$

$$S = k_B \ln(Z) + Nk_B T\frac{\partial}{\partial T}\left[\ln\left(\frac{eV}{N}\left(\frac{2\pi m}{h^2\beta}\right)^{3/2}\right)\right]_{VN}$$

$$\frac{\partial}{\partial T}\left[\cancel{\ln(e)}+\cancel{\ln(V)}-\cancel{\ln(N)}+\cancel{\frac{3}{2}\ln\left(\frac{2\pi m k_B}{h^2}\right)}+\frac{3}{2}\ln(T)\right]_{VN}$$

$$=\frac{3}{2}\frac{1}{T}$$

Classical limit $h \to 0$

$$S = N k_B \ln\left[\frac{eV}{N}\left(\frac{2\pi m}{h^2\beta}\right)^{3/2}\right]+\frac{3}{2}N k_B$$

$$= N k_B\left\{\ln(e)+\ln\left[\frac{V}{N}\left(\frac{2\pi m}{h^2\beta}\right)^{3/2}\right]\right\}+\frac{3}{2}N k_B$$

$$= N k_B\left\{\ln\left[\frac{V}{N}\left(\frac{2\pi m}{h^2\beta}\right)^{3/2}\right]+\frac{5}{2}\right\}$$

Sackur-Tetrode

monatomic ideal gas

↑ from $\frac{1}{N!}$

Avoids the Gibbs paradox

$$\left(\frac{\partial S}{\partial V}\right)_{TN}=\frac{N k_B}{V}$$

$$S = N k_B\left\{\ln(V)-\ln(N)+\frac{3}{2}\ln\left(\frac{2\pi m}{h^2\beta}\right)+\frac{5}{2}\right\}$$

$$\left(\frac{\partial S}{\partial V}\right)_{TN}=\frac{N k_B}{V}$$

$$P = -\left[0 - T\left(\frac{N k_B}{V}\right)\right] \Longrightarrow PV = N k_B T \checkmark$$

Chemical Potential for an Ideal Gas

$$Z = \left[\frac{eV}{N} \left(\frac{\sqrt{2\pi m k_B T}}{h} \right)^3 \right]^N$$

Get a potential : $F = -k_B T \ln(Z)$

$$dF = -S\,dT + \mu\,dN - p\,dV$$

$$dF = \left(\frac{\partial F}{\partial T}\right)_{NV} dT + \left(\frac{\partial F}{\partial N}\right)_{TV} dN + \left(\frac{\partial F}{\partial V}\right)_{TN} dV$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{TV}$$

$$\mu = \frac{\partial}{\partial N}\left\{ -k_B T N \left[\ln(e) + \ln(V) - \ln(N) + \ln \ldots \right] \right\}$$

$$= -k_B T \ln\left(\frac{Ve}{N} \left(\frac{\sqrt{2\pi m k_B T}}{h} \right)^3 \right)$$

$$+ k_B T N \frac{\partial}{\partial N} \left[\cancel{\ln(e)} + \cancel{\ln(V)} \ldots + \ln(N) + \cancel{\ln \ldots} \right]$$

$$\mu = -k_B T \left[\ln\left(\frac{Ve}{N} \left[\frac{\sqrt{2\pi m k_B T}}{h} \right]^3 \right) - 1 \right]$$

$$\frac{1}{\lambda_{\omega}^3}$$

extensive → $\dfrac{V}{N} = v$ ← intensive

extensive → N

Define: $\lambda_Q \equiv \dfrac{h}{\sqrt{2\pi m k_B T}}$ thermal deBroglie wavelength

In analogy with deBroglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Define: Quantum volume $v_Q \equiv \lambda_Q^3$

intensive

$$\mu = -k_B T \ln\left(\frac{v}{v_Q}\right)$$