Isentropic Compressibility

$$\kappa_s \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = \frac{1}{\gamma p}$$

↑kappa

$$pV^{4/3} = B \implies V = C p^{-3/4}$$

$$\left(\frac{\partial V}{\partial p}\right)_s = \frac{d}{dp}\left[C p^{-3/4}\right] = C\left(-\frac{3}{4}\right)p^{-7/4}$$

$$\kappa_s = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = \frac{+\cancel{C}\left(\frac{3}{4}\right)p^{-7/4}}{\cancel{C}\,p^{-3/4}} = \frac{1}{\frac{4}{3}p} = \frac{1}{\gamma p}$$

Maxwell-Boltzmann distribution of speeds of molecules in an ideal gas at temp. T.

equipartition theorem: $\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} k_B T$

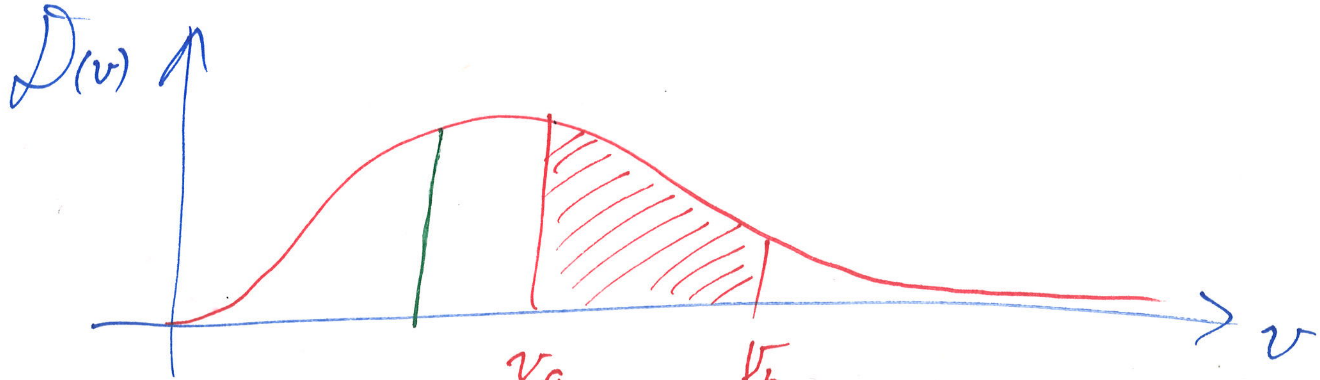
$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$

root mean square

We seek a probabity density function

$\frac{dP}{dv} = \mathcal{D}(v)$ The integral over $\mathcal{D}(v)$ gives the probability

$$P(v_a \le v \le v_b) = \int_{v_a}^{v_b} \mathcal{D}(v)\,dv = \int_{v_a}^{v_a} \frac{dP}{dv}\,dv = \int_{v_a}^{v_b} dP$$

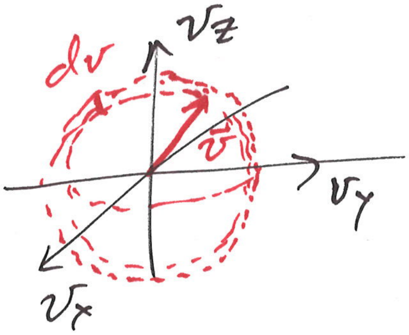


What is the probability that a molecule has speed exactly 4.0000... m/s $= 0$ $\int_4 \mathcal{D}(v)\,dv$

$$\mathcal{D}(v)\,dv = \left(\text{Normalization}\right)\left(\begin{array}{c}\text{Probability of}\\ \text{molecules having}\\ \text{speed } v\end{array}\right)\underbrace{\left(\begin{array}{c}\text{number of}\\ \text{vectors } \vec{v}\\ \text{corresponding to}\\ \text{speed } v \text{ in}\\ \text{3 dimensions}\end{array}\right)}_{\begin{array}{l}\text{degeneracy}\\ = \text{multiplicity}\\ = \#\text{ of microstates}\end{array}}$$

3-dimensional

$$= N_3 \cdot e^{-\frac{\frac{1}{2} m v^2}{k_B T}} \cdot 4\pi v^2\, d\underline{v}$$



$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

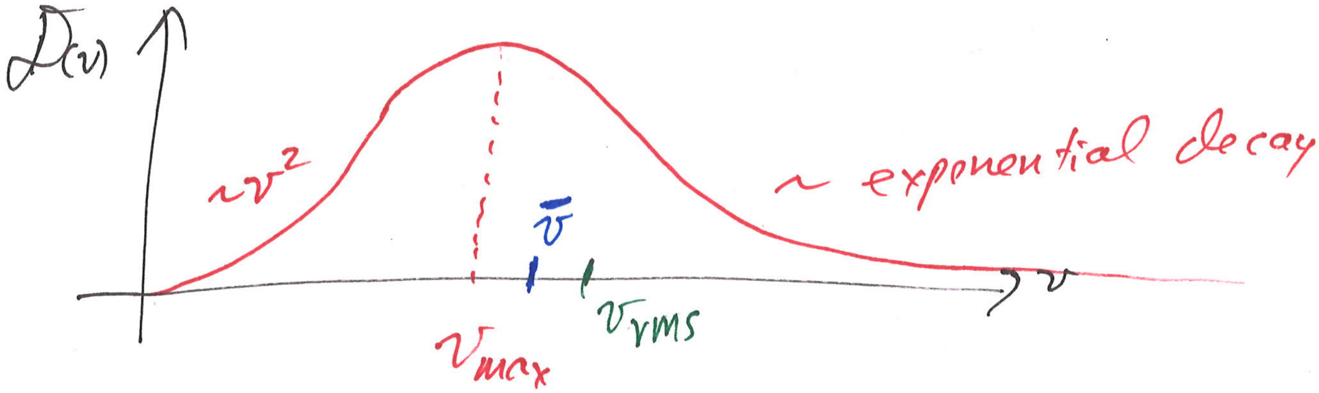
Get N from unitarity

$$P(0 \le v \le \infty) = 1 = \int_{v=0}^{\infty} \mathcal{D}(v)\, dv$$

$$\Rightarrow 1 = \int_{v=0}^{\infty} N_3 \exp\left[\frac{-\frac{1}{2} m v^2}{k_B T}\right] 4\pi v^2\, dv$$

$$N_3 = \left(\frac{m}{2\pi k_B T}\right)^{3/2}$$

$$\mathcal{D}(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[\frac{-\frac{1}{2}mv^2}{k_B T}\right] 4\pi v^2$$



$$\frac{d\mathcal{D}(v)}{dv} \stackrel{!}{=} 0 \Rightarrow v_{max} = \sqrt{\frac{2k_B T}{m}} \quad \text{(3-dimension)}$$

most likely speed – ~~median~~ mode

$$\langle v \rangle = \bar{v} = \int_{v=0}^{\infty} v\,\mathcal{D}(v)\,dv = \sqrt{\frac{8}{\pi}\frac{k_B T}{m}}$$

"Average" speed

Mean speed

RMS speed $v_{rms} = \sqrt{\int_{v=0}^{\infty} v^2 \mathcal{D}(v)\,dv} = \sqrt{\frac{3k_B T}{m}}$

$$= \sqrt{\langle v^2 \rangle}$$