C.f. Lecture #11

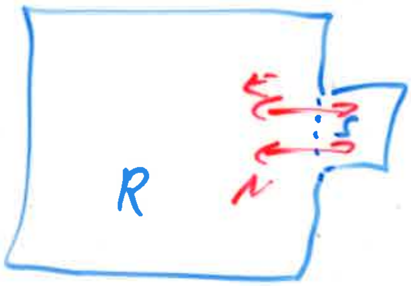
modification : maximize $\ln \Omega$

① $\lambda \left(\sum_i p_i = 1 \right)$ unitarity

② $\beta \left(\sum_i p_i E_i = U \right)$ energy $\beta = \frac{1}{k_B T}$

③ $(\beta\mu) \left(\sum_i \nu_i = N_{TOTAL} \right)$ enforces particle conservation.

Open system



Heat (energy) will flow until thermal equilibrium is reached. $(T_R = T_S)$

Particle flow will continue until diffusive equilibrium is reached $(\mu_R = \mu_S)$

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system : single particle wavefunction (state) either occupied , or unoccupied

reservoir : all the other states.

not spatially / temporally separated.

Probability that the system (state) is occupied by n particles

$$P_{(n)} = \frac{e^{-n\beta(\epsilon-\mu)}}{Q}$$

Fermions $n = 0$ or 1.

$$Q = \underbrace{1}_{n=0} + \underbrace{e^{-\beta(\epsilon-\mu)}}_{n=1}.$$

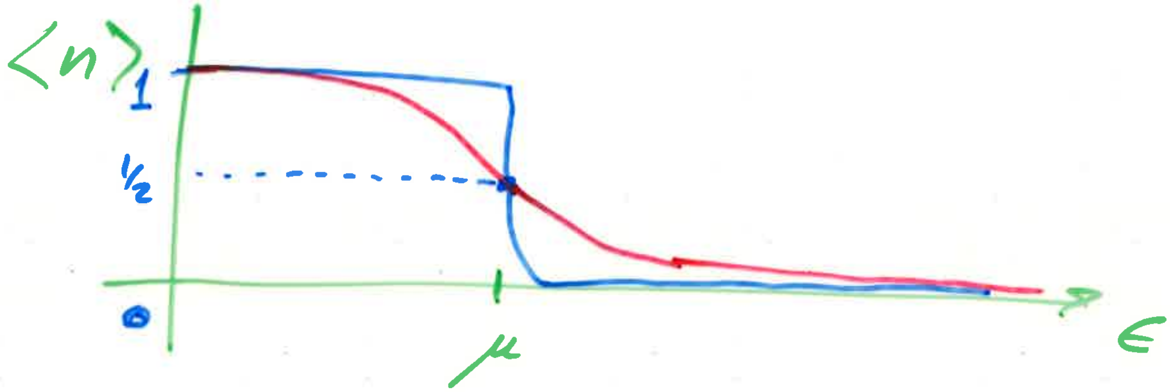
Prob. unoccupied $= P(0) = \dfrac{1}{1+e^{-\beta(\epsilon-\mu)}}$

Prob. occupied $= P(1) = \dfrac{e^{-\beta(\epsilon-\mu)}}{1+e^{-\beta(\epsilon-\mu)}}$

$$\langle n \rangle = \sum_{i=0}^{1} n_i P(i) = 0\,P(0) + 1\,P(1)$$

$$= \frac{1}{e^{+\beta(\epsilon-\mu)} + 1}$$

Fermi-Dirac distribution



Bosons Any number n can be in a state

$$\langle n \rangle = \sum_{j=1}^{\infty} j\, P(j) = 0\, P(0) + 1\, P(1) + 2P(2) + \dots$$

$$= \frac{\sum_j j\, e^{-j\beta(\epsilon-\mu)}}{Q} = \frac{\sum_j j\, e^{-j\beta(\epsilon-\mu)}}{\sum_k e^{-k\beta(\epsilon-\mu)}}$$

define $r = \beta(\epsilon-\mu)$

$$\sum_j j\, e^{-j\beta(\epsilon-\mu)} = \sum_j j\, e^{-jr} = -\frac{\partial}{\partial r}\left(\sum_j e^{-jr}\right)$$

$$= -\frac{\partial}{\partial r} Q$$

$$\langle n \rangle = \frac{-\frac{\partial}{\partial r} Q}{Q} \qquad Q = \sum_k e^{-kr} = \frac{1}{1-e^{-r}}$$

$$\langle n \rangle = \left[-\frac{\partial}{\partial \gamma} \left(1 - e^{-\gamma} \right)^{-1} \right] \left(1 - e^{-\gamma} \right)$$

$$= +\left(1 - e^{-\gamma} \right) \left(1 - e^{-\gamma} \right)^{-2} e^{-\gamma} = \frac{e^{-\gamma}}{1 - e^{-\gamma}} \left(\frac{e^{+\gamma}}{e^{+\gamma}} \right)$$

$$= \frac{1}{e^{+\gamma} - 1} = \frac{1}{e^{+\beta(\epsilon - \mu)} - 1}$$

Bose-Einstein distribution

Set $\mu = 0$: $\dfrac{1}{e^{+\beta\epsilon} - 1}$ Planck distribution (for photons)