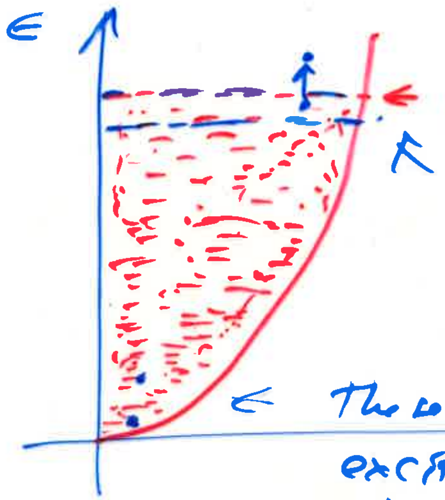
$T = 0K$



$\leftarrow \epsilon_F = \mu$ $\updownarrow k_B T$

These states can be excited.

account for low specific heat of metals (only a few degrees of freedom)

$\leftarrow$ These states can not be excited because the levels above are already filled.

From last time

$$U = \frac{\pi h^2}{8 m_e L^2} \int_{j=0}^{j_{max}} j^4 \, dj$$

change variables

$$\epsilon_j = \frac{h^2}{8 m L^2} j^2$$

$$j = \sqrt{\frac{8 m L^2}{h^2}} \sqrt{\epsilon}$$

$$dj = \sqrt{\frac{8 m L^2}{h^2}} \frac{d\epsilon}{2\sqrt{\epsilon}}$$

$$U = \int_{\epsilon=0}^{\epsilon_F} \epsilon \underbrace{\left[\frac{\pi}{2}\left(\frac{8 m L^2}{h^2}\right)^{3/2} \sqrt{\epsilon}\right]}_{g(\epsilon) \,=\, \text{density of states} \,=\, \text{degeneracy}} d\epsilon = \int_{\epsilon=0}^{\epsilon_F} \epsilon \, g(\epsilon) \, d\epsilon$$

$g(\epsilon)\,d\epsilon$ is the number of states with energy between ϵ and $\epsilon + d\epsilon$

$$g(\epsilon) = \frac{\pi}{2} \frac{(8m)^{3/2} V}{h^3} \sqrt{\epsilon} = \frac{3}{2} \frac{N}{\epsilon_F^{3/2}} \sqrt{\epsilon}$$

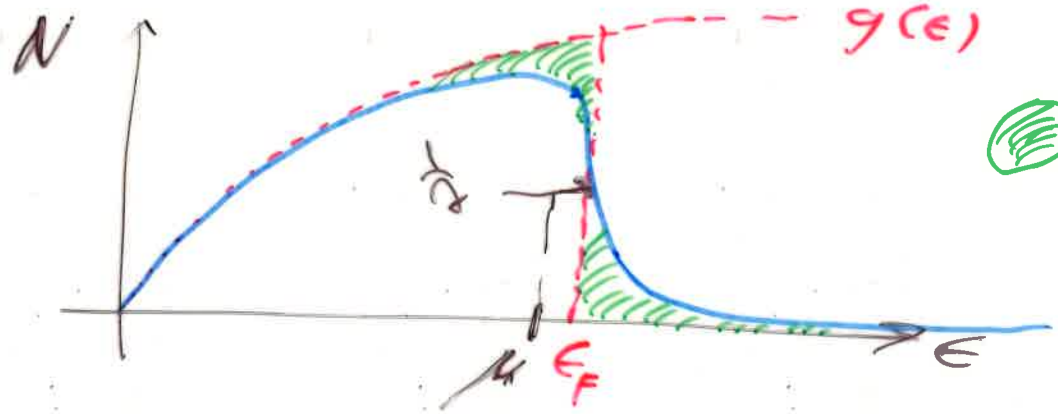
$$N = \int_{\epsilon=0}^{\epsilon_F} g(\epsilon) \, d\epsilon$$

$T \neq 0K$ "small"

$N = \sum$ (# states with energy ϵ) * (probability of that state being occupied)

$$= \int_{\epsilon=0}^{\infty} g(\epsilon) \langle n \rangle_{FD} \, d\epsilon = \int_{\epsilon=0}^{\infty} \frac{g(\epsilon)\, d\epsilon}{e^{+\beta(\epsilon-\mu)}+1}$$

$$U = \int_{\epsilon=0}^{\infty} \epsilon \, g(\epsilon) \langle n \rangle_{FD} \, d\epsilon = \int_{\epsilon=0}^{\infty} \frac{\epsilon \, g(\epsilon)\, d\epsilon}{e^{+\beta(\epsilon-\mu)}+1}$$



Plan: get μ from the N integral, plug μ into the U integral and get $U(N,T)$. $\Longrightarrow C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} \propto T$ near $T = 0K$

No approximations yet

$$N = \int_{\epsilon=0}^{\infty} g(\epsilon)\,\langle n\rangle_{FD}\, d\epsilon = \underbrace{\left(\frac{\pi(8m)^{3/2}V}{2h^3}\right)}_{g_0 = \frac{3N}{2\epsilon_F^{3/2}}}\int_{\epsilon=0}^{\infty}\sqrt{\epsilon}\,\langle n\rangle_{FD}\,d\epsilon$$

integrate by parts $\int_a^b v\,du = uv\Big|_a^b - \int_a^b v\,du$

surface term

$$N = \frac{2}{3}g_0\,\epsilon^{3/2}\langle n\rangle_{FD}\Big|_{\epsilon=0}^{\infty} - \frac{2}{3}g_0\int_{\epsilon=0}^{\infty}\epsilon^{3/2}\left(\frac{d\langle n\rangle_{FD}}{d\epsilon}\right)d\epsilon$$

$\epsilon^{3/2}\to 0$ at $\epsilon=0$

$\langle n\rangle_{FD}\to 0$ at $\epsilon=\infty$

$$\frac{d\langle n\rangle_{FD}}{d\epsilon} = \frac{d}{d\epsilon}\left[\frac{1}{e^{+\beta(\epsilon-\mu)}+1}\right] = \frac{-\beta\, e^{+\beta(\epsilon-\mu)}}{\left[e^{+\beta(\epsilon-\mu)}+1\right]^2}$$

$$N = \frac{2}{3}g_0\int_{\epsilon=0}^{\infty}\frac{\epsilon^{3/2}\beta e^{+\beta(\epsilon-\mu)}}{\left[e^{+\beta(\epsilon-\mu)}+1\right]^2}\,d\epsilon$$

change vars

$x = \beta(\epsilon-\mu)$

$dx = \beta\, d\epsilon$

$\epsilon = 0 \Rightarrow x = -\mu\beta$

$\epsilon = \infty \Rightarrow x = \infty$

$$N = \frac{2}{3}g_0\int_{x=-\mu\beta}^{\infty}\frac{e^{x}\epsilon^{3/2}}{(e^{x}+1)^2}\,dx$$

Next time: Sommerfeld expansion.