N dipoles, two states each $S_k = \pm 1$

Nearest neighbor interactions

$$\text{energy} = \begin{cases} +\epsilon & \text{antiparallel spins } \uparrow\downarrow, \downarrow\uparrow \\ -\epsilon & \text{parallel spins } \uparrow\uparrow, \downarrow\downarrow \end{cases}$$

Interaction energy for a pair of dipoles

$$u_{jk} = -\epsilon S_j S_k$$

Total energy

$$U = -\epsilon \sum_{\substack{\text{nearest} \\ \text{neighbors}}} S_j S_k$$

$$\text{Partition function} = Z = \sum_{\{S_k\}=1}^{2^N} e^{-\beta U}$$

$\leftarrow$ all possible arrangements of the dipoles

One dimension

$$\uparrow_1 \uparrow_2 \downarrow \uparrow \uparrow \downarrow \downarrow \downarrow \uparrow \uparrow \downarrow \ldots \uparrow_N$$

each dipole has 2 nearest neighbors

$$U = -\epsilon \left(S_1 S_2 + S_2 S_3 + S_3 S_4 + \dots \quad S_{N-1} S_N \right)$$

$$Z = \sum_{S_1=-1}^{+1} \sum_{S_2=-1}^{+1} \cdots \sum_{S_N=-1}^{+1} e^{+\beta\epsilon S_1 S_2} \cdot e^{+\beta\epsilon S_2 S_3} \cdot \cdots \cdot e^{+\beta\epsilon S_{N-1} S_N}$$

$\uparrow$ not 0

Look at last sum

$$\sum_{S_N=-1}^{+1} e^{+\beta\epsilon S_{N-1} S_N} = e^{\beta\epsilon} + e^{-\beta\epsilon} = 2\cosh(\beta\epsilon)$$

Look at penultimate sum

$$\sum_{S_{N-1}=-1}^{+1} e^{+\beta\epsilon S_{N-2} S_{N-1}} = e^{\beta\epsilon} + e^{-\beta\epsilon} = 2\cosh(\beta\epsilon)$$

$$Z = 2^N \left[\cosh(\beta\epsilon) \right]^{N-1} \xrightarrow[N \gg 1]{\lim} \left[2\cosh(\beta\epsilon) \right]^N$$

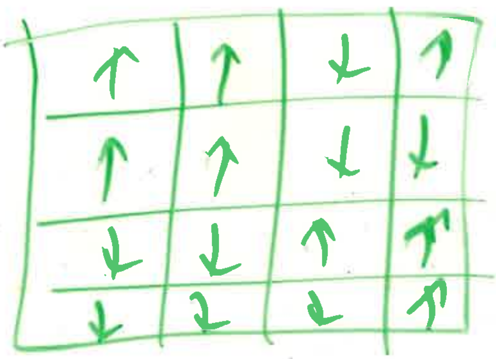
$$\langle U \rangle = -\frac{\partial \ln(Z)}{\partial \beta} = -\frac{1}{Z} \frac{\partial Z}{\partial \beta} = -N\epsilon \tanh(\beta\epsilon)$$

Looks like paramagnet (Lecture 14), but no external B field.

limit $T \to 0$ ($\beta \to \infty$), $\langle U \rangle = -N\epsilon$

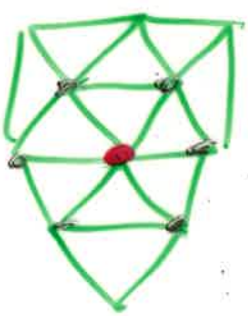
limit $T \to \infty$ ($\beta \to 0$), $\langle U \rangle = 0$

Two dimensions (square lattice) 17 wallpapers



each dipole has 4 nearest neighbors

2-dimensional hexagonal lattice



6 nearest neighbors
coordination number

3-dimensions — coordination number

simple cubic — 6

body-centered cubic bcc — 8

hexagonal close pack hcp
face-centered cubic fcc — 12